Computer Algebra Independent Integration Tests

Summer 2023 edition

2-Exponentials/54-2.2-c+d-x-^m-F^-g-e+f-x-^n-a+b-F^-g-e+f-x-^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [93]. This is test number [54].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (93)	0.00 (0)
Fricas	100.00 (93)	0.00 (0)
Mathematica	91.40 (85)	8.60 (8)
Maple	87.10 (81)	12.90 (12)
Maxima	83.87 (78)	16.13 (15)
Giac	58.06 (54)	41.94 (39)
Mupad	56.99 (53)	43.01 (40)
Sympy	54.84 (51)	45.16 (42)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

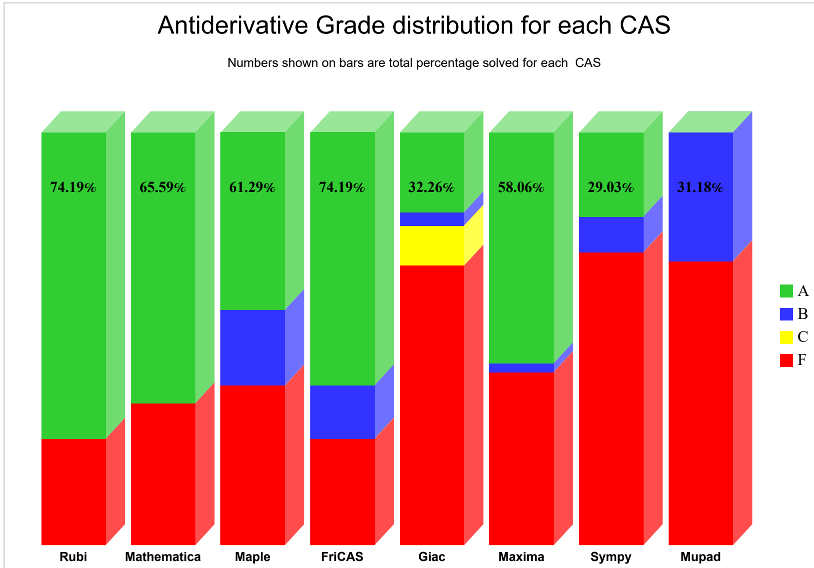
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

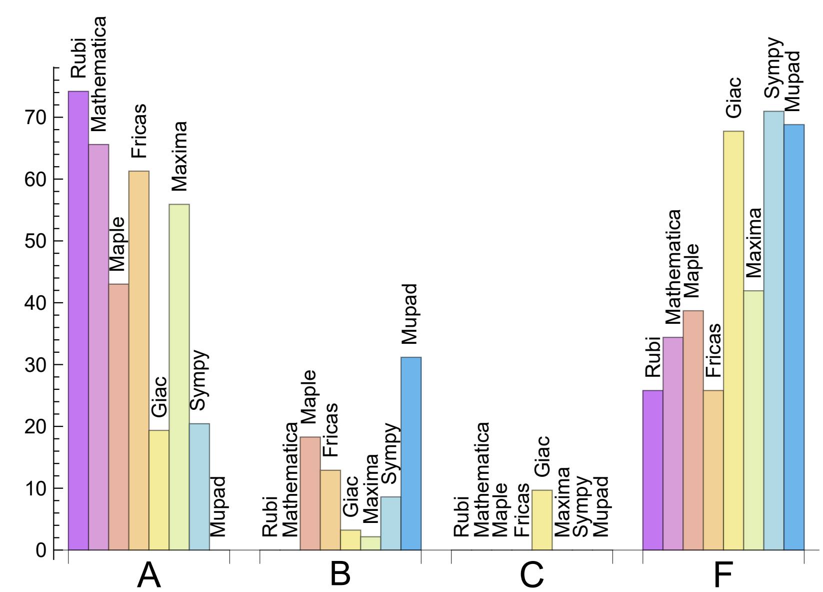
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.194	0.000	0.000	25.806
Mathematica	65.591	0.000	0.000	34.409
Fricas	61.290	12.903	0.000	25.806
Maxima	55.914	2.151	0.000	41.935
Maple	43.011	18.280	0.000	38.710
Sympy	20.430	8.602	0.000	70.968
Giac	19.355	3.226	9.677	67.742
Mupad	0.000	31.183	0.000	68.817

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Mathematica	8	100.00	0.00	0.00
Maple	12	100.00	0.00	0.00
Maxima	15	100.00	0.00	0.00
Giac	39	100.00	0.00	0.00
Mupad	40	0.00	100.00	0.00
Sympy	42	92.86	7.14	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.15
Maple	0.16
Mupad	0.22
Maxima	0.23
Fricas	0.26
Giac	0.40
Mathematica	0.51
Sympy	6.03

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	86.15	1.14	27.00	1.08
Mathematica	89.80	0.95	67.00	0.97
Rubi	127.51	1.00	84.00	1.00
Sympy	144.16	1.98	51.00	1.08
Maxima	179.22	2.50	94.00	1.14
Fricas	224.75	1.68	108.00	1.39
Maple	334.52	1.85	77.00	1.04
Giac	1165.43	5.16	29.00	1.08

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

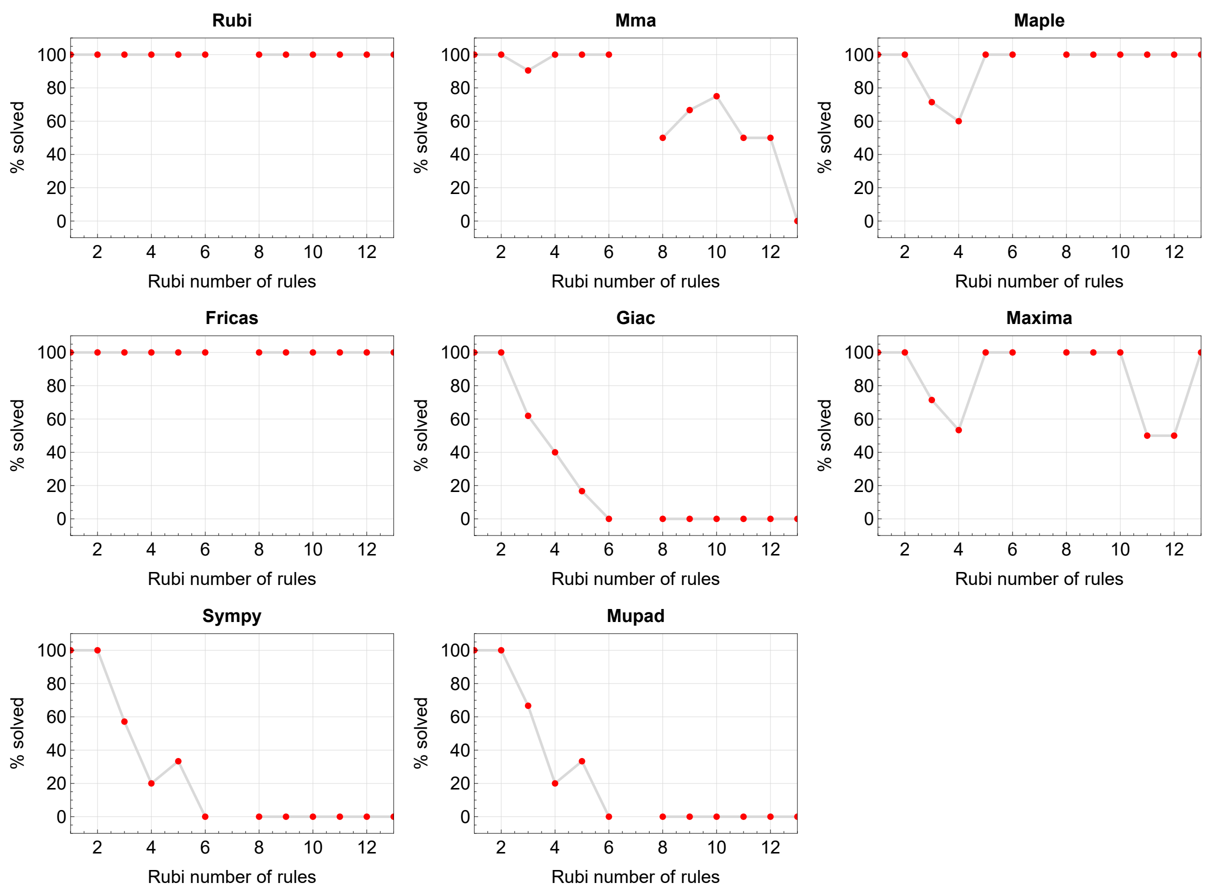


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

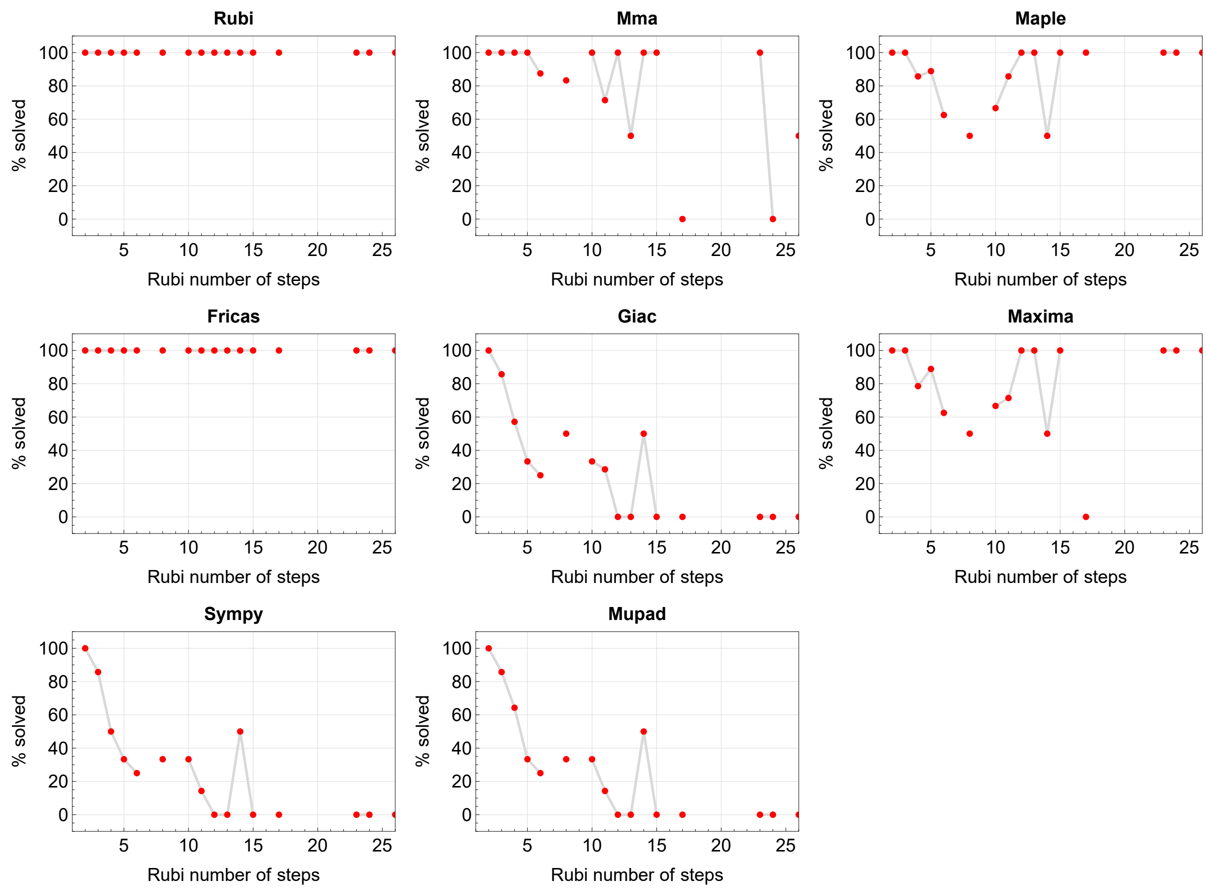


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

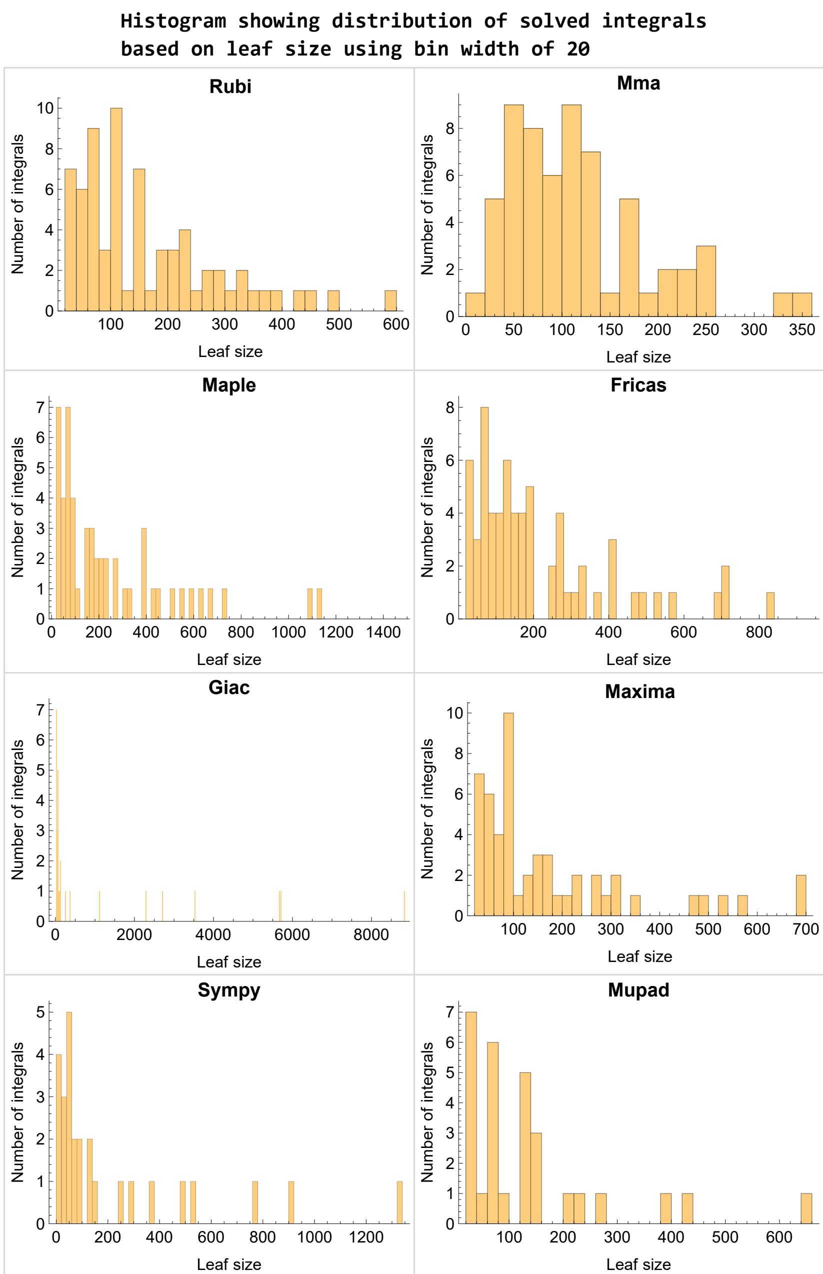


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

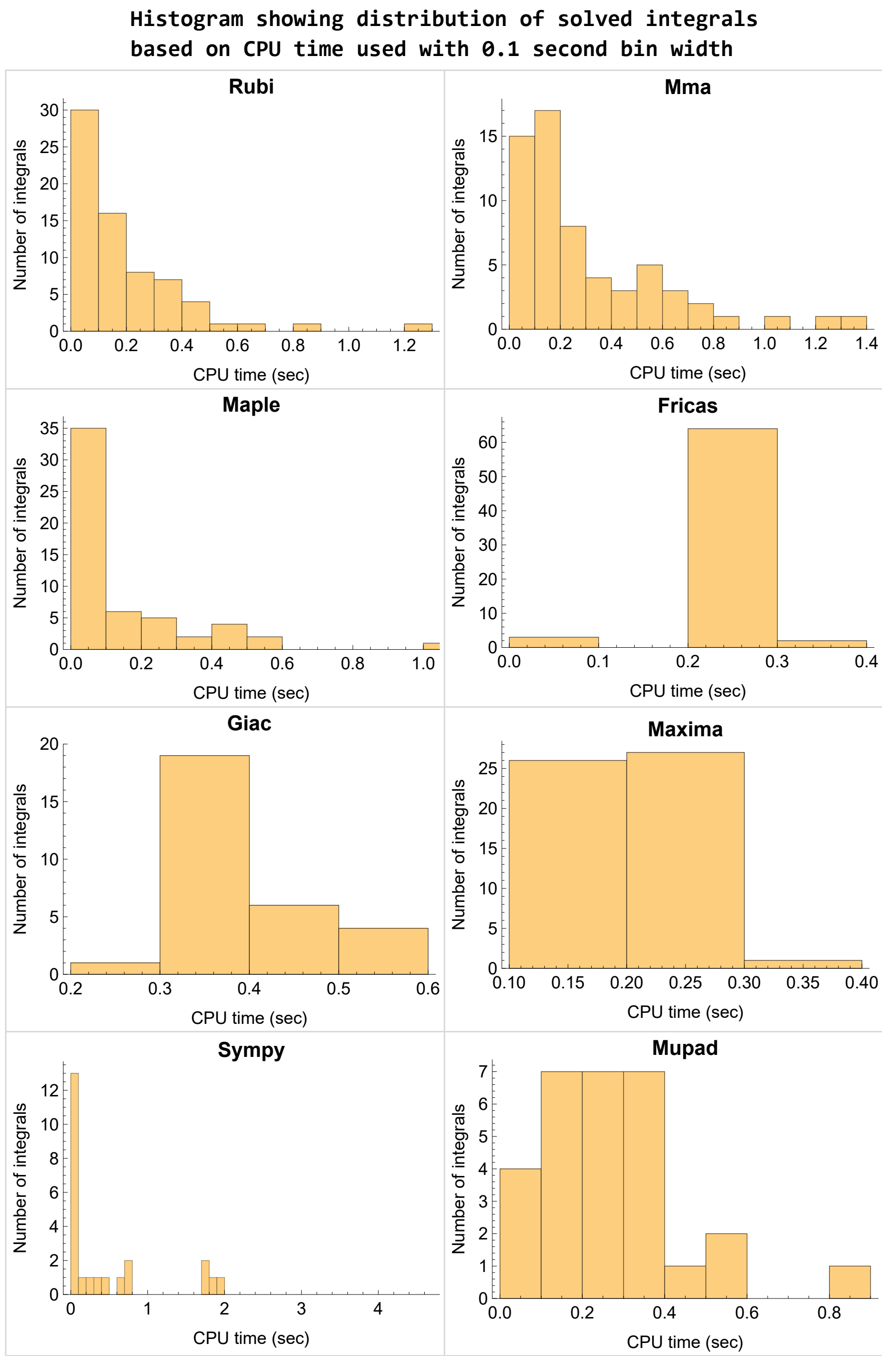


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

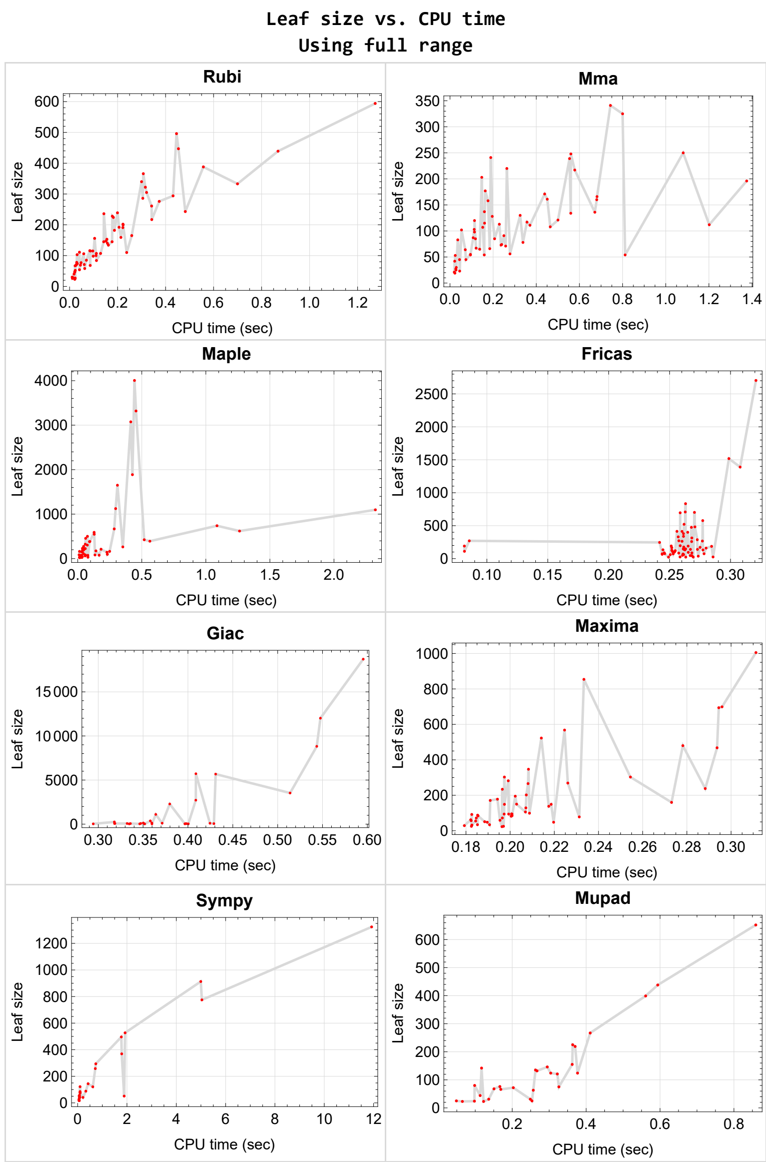


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{5, 6, 13, 14, 21, 22, 50, 51, 56, 57, 62, 63, 67, 68, 69, 73, 74, 75, 80, 81, 86, 87, 92, 93\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
      leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

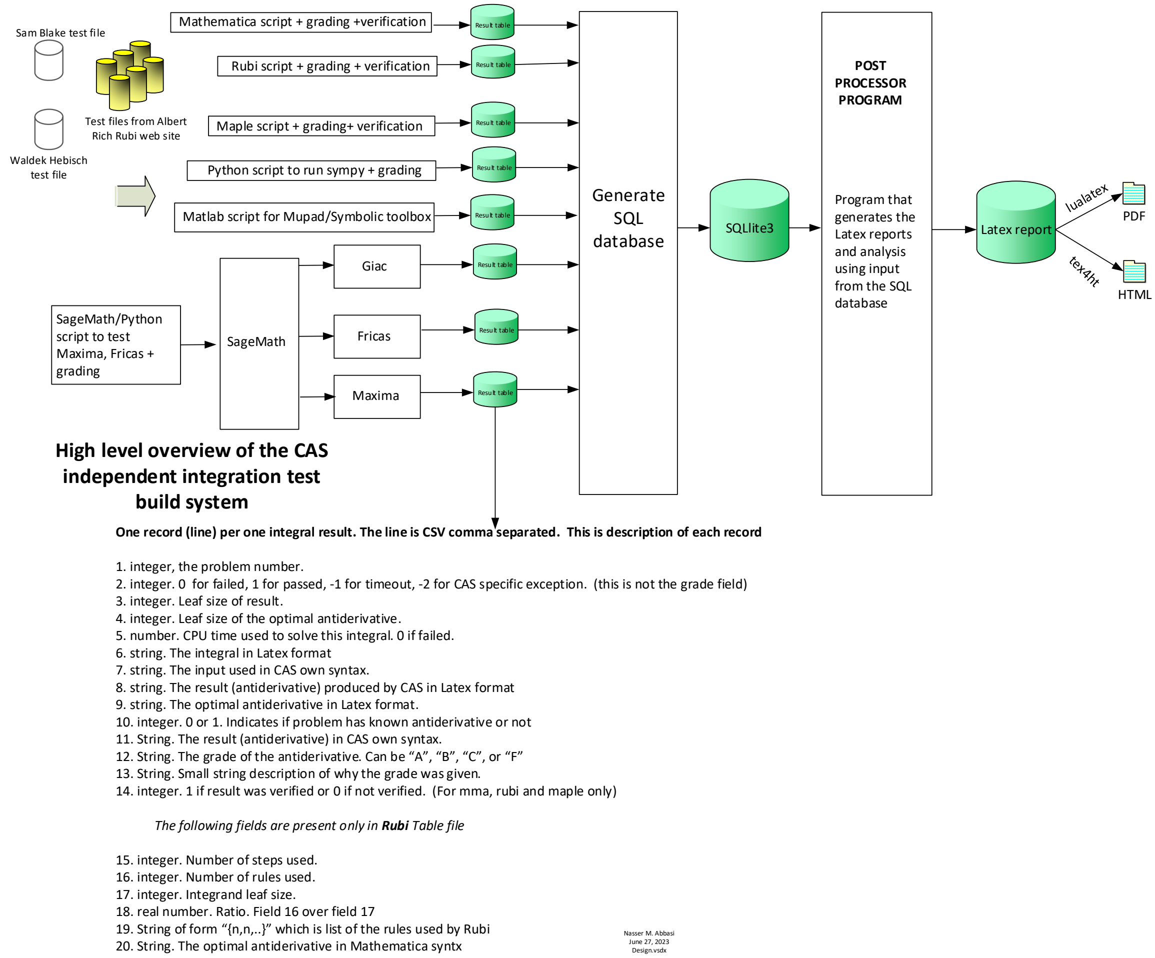
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 58, 59, 60, 61, 64, 65, 66, 70, 71, 72, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 55, 61, 64, 65, 66, 72, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91 }

B grade { }

C grade { }

F normal fail { 52, 53, 54, 58, 59, 60, 70, 71 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 3, 4, 7, 8, 9, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 26, 27, 28, 33, 34, 35, 40, 41, 42, 49, 55, 61, 64, 65, 66, 76, 79, 82, 84, 85, 88, 89, 90, 91 }

B grade { 2, 10, 25, 32, 39, 46, 47, 48, 52, 53, 54, 58, 59, 60, 77, 78, 83 }

C grade { }

F normal fail { 29, 30, 31, 36, 37, 38, 43, 44, 45, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 55, 61, 64, 65, 66, 70, 71, 72, 76, 77, 78, 79, 82, 83, 84, 85, 90, 91 }

B grade { 17, 18, 19, 46, 52, 53, 54, 58, 59, 60, 88, 89 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 52, 53, 55, 58, 59, 61, 64, 65, 66, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91 }

B grade { 46, 47 }

C grade { }

F normal fail { 29, 30, 31, 36, 37, 38, 43, 44, 45, 48, 54, 60, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 28, 35, 42, 49, 55, 61, 79, 85, 91 }

B grade { 64, 65, 66 }

C grade { 25, 26, 27, 32, 33, 34, 39, 40, 41 }

F normal fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 70, 71, 72, 76, 77, 78, 82, 83, 84, 88, 89, 90 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 55, 61, 79, 84, 85, 90, 91 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 64, 65, 66, 70, 71, 72, 76, 77, 78, 82, 83, 88, 89 }

F(-2) exception fail { }

Sympy

A grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 28, 34, 35, 41, 42, 49, 79, 84, 85, 90 }

B grade { 25, 26, 27, 32, 33, 39, 40, 91 }

C grade { }

F normal fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 55, 58, 59, 60, 61, 64, 65, 66, 70, 71, 72, 76, 77, 78, 82, 83, 88, 89 }

F(-1) timedout fail { 52, 53, 54 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	102	191	94	120	0	0	0
N.S.	1	1.00	0.93	1.74	0.85	1.09	0.00	0.00	0.00
time (sec)	N/A	0.238	0.053	0.038	0.199	0.255	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	162	72	100	0	0	0
N.S.	1	1.00	0.99	1.93	0.86	1.19	0.00	0.00	0.00
time (sec)	N/A	0.112	0.036	0.014	0.185	0.254	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	84	48	72	0	0	0
N.S.	1	1.00	0.91	1.45	0.83	1.24	0.00	0.00	0.00
time (sec)	N/A	0.063	0.024	0.011	0.190	0.253	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	42	25	32	24	17	31	24
N.S.	1	1.00	1.62	0.96	1.23	0.92	0.65	1.19	0.92
time (sec)	N/A	0.012	0.022	0.016	0.183	0.269	0.064	0.346	0.098

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	17	15	18	18
N.S.	1	1.00	1.12	0.94	1.06	1.00	0.88	1.06	1.06
time (sec)	N/A	0.029	0.132	0.063	0.215	0.249	1.172	0.321	0.086

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	17	18	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.00	1.06	1.06
time (sec)	N/A	0.032	0.208	0.046	0.221	0.251	0.897	0.394	0.089

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	21	24	34	23	17	33	23
N.S.	1	1.00	0.81	0.92	1.31	0.88	0.65	1.27	0.88
time (sec)	N/A	0.020	0.020	0.014	0.185	0.263	0.063	0.400	0.065

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	26	34	25	19	33	25
N.S.	1	1.00	0.68	0.93	1.21	0.89	0.68	1.18	0.89
time (sec)	N/A	0.015	0.023	0.014	0.191	0.259	0.068	0.397	0.049

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	158	382	195	330	0	0	0
N.S.	1	1.00	0.73	1.76	0.90	1.52	0.00	0.00	0.00
time (sec)	N/A	0.342	0.176	0.092	0.202	0.257	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	113	324	149	263	0	0	0
N.S.	1	1.00	0.68	1.96	0.90	1.59	0.00	0.00	0.00
time (sec)	N/A	0.259	0.229	0.059	0.197	0.260	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	85	154	95	176	0	0	0
N.S.	1	1.00	0.79	1.44	0.89	1.64	0.00	0.00	0.00
time (sec)	N/A	0.129	0.118	0.030	0.201	0.262	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	49	51	60	39	51	66
N.S.	1	1.00	0.98	1.07	1.11	1.30	0.85	1.11	1.43
time (sec)	N/A	0.021	0.044	0.020	0.188	0.251	0.074	0.397	0.169

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	53	35	70	18	18
N.S.	1	1.00	1.12	0.94	3.12	2.06	4.12	1.06	1.06
time (sec)	N/A	0.027	1.789	0.129	0.224	0.258	1.415	0.350	0.112

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	57	41	73	18	18
N.S.	1	1.00	1.12	0.94	3.35	2.41	4.29	1.06	1.06
time (sec)	N/A	0.027	0.920	0.125	0.218	0.250	1.331	0.349	0.119

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	53	55	65	39	54	68
N.S.	1	1.00	0.94	1.10	1.15	1.35	0.81	1.12	1.42
time (sec)	N/A	0.022	0.074	0.020	0.184	0.244	0.075	0.398	0.151

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	66	59	57	73	42	58	76
N.S.	1	1.00	1.27	1.13	1.10	1.40	0.81	1.12	1.46
time (sec)	N/A	0.024	0.184	0.019	0.182	0.244	0.089	0.429	0.167

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	241	548	303	702	0	0	0
N.S.	1	1.00	0.72	1.65	0.91	2.11	0.00	0.00	0.00
time (sec)	N/A	0.700	0.189	0.129	0.197	0.270	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	203	385	234	521	0	0	0
N.S.	1	1.00	0.84	1.58	0.96	2.14	0.00	0.00	0.00
time (sec)	N/A	0.482	0.147	0.095	0.196	0.262	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	120	216	149	338	0	0	0
N.S.	1	1.00	0.75	1.36	0.94	2.13	0.00	0.00	0.00
time (sec)	N/A	0.214	0.114	0.050	0.219	0.261	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	66	84	127	76	65	121
N.S.	1	1.00	0.93	0.96	1.22	1.84	1.10	0.94	1.75
time (sec)	N/A	0.027	0.071	0.030	0.185	0.252	0.105	0.336	0.322

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	128	53	165	18	18
N.S.	1	1.00	1.12	0.94	7.53	3.12	9.71	1.06	1.06
time (sec)	N/A	0.027	2.208	0.162	0.235	0.264	2.221	0.325	0.162

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	128	61	165	18	18
N.S.	1	1.00	1.12	0.94	7.53	3.59	9.71	1.06	1.06
time (sec)	N/A	0.028	1.167	0.159	0.233	0.247	2.081	0.373	0.155

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	71	88	132	78	71	124
N.S.	1	1.00	0.93	0.99	1.22	1.83	1.08	0.99	1.72
time (sec)	N/A	0.029	0.121	0.030	0.185	0.277	0.093	0.319	0.304

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	54	77	92	140	85	75	132
N.S.	1	1.00	0.69	0.99	1.18	1.79	1.09	0.96	1.69
time (sec)	N/A	0.030	0.812	0.028	0.183	0.267	0.098	0.347	0.267

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	130	392	282	267	496	5716	225
N.S.	1	1.00	0.85	2.56	1.84	1.75	3.24	37.36	1.47
time (sec)	N/A	0.156	0.325	0.563	0.199	0.277	1.772	0.409	0.364

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	91	163	171	166	294	2716	135
N.S.	1	1.00	0.79	1.42	1.49	1.44	2.56	23.62	1.17
time (sec)	N/A	0.097	0.250	0.249	0.191	0.252	0.736	0.409	0.263

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	73	84	85	87	144	1101	72
N.S.	1	1.00	0.95	1.09	1.10	1.13	1.87	14.30	0.94
time (sec)	N/A	0.048	0.237	0.059	0.201	0.246	0.425	0.364	0.203

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	29	37	41	31	31
N.S.	1	1.00	1.00	1.03	0.97	1.23	1.37	1.03	1.03
time (sec)	N/A	0.011	0.029	0.080	0.179	0.274	0.216	0.294	0.249

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	56	0	0	50	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.086	0.278	0.000	0.000	0.261	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	87	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.110	0.338	0.000	0.000	0.268	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	111	0	0	160	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.152	0.371	0.000	0.000	0.280	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	239	739	568	482	913	12013	438
N.S.	1	1.00	0.74	2.30	1.76	1.50	2.84	37.31	1.36
time (sec)	N/A	0.316	0.554	1.087	0.225	0.271	4.993	0.548	0.594

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	171	424	347	287	527	5675	267
N.S.	1	1.00	0.72	1.77	1.45	1.20	2.21	23.74	1.12
time (sec)	N/A	0.200	0.439	0.519	0.208	0.273	1.923	0.431	0.411

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	117	174	178	139	258	2284	146
N.S.	1	1.00	0.75	1.12	1.14	0.89	1.65	14.64	0.94
time (sec)	N/A	0.104	0.357	0.142	0.194	0.262	0.717	0.380	0.295

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	65	60	62	61	88	74	75
N.S.	1	1.00	0.97	0.90	0.93	0.91	1.31	1.10	1.12
time (sec)	N/A	0.023	0.138	0.058	0.182	0.262	0.333	0.318	0.326

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	108	0	0	95	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.162	0.465	0.000	0.000	0.265	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	136	0	0	171	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.222	0.671	0.000	0.000	0.275	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	286	286	217	0	0	314	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.305	0.578	0.000	0.000	0.268	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	341	1097	854	708	1324	18707	652
N.S.	1	1.00	0.69	2.21	1.72	1.43	2.67	37.72	1.31
time (sec)	N/A	0.446	0.744	2.323	0.233	0.263	11.921	0.595	0.859

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	248	620	523	414	775	8820	399
N.S.	1	1.00	0.68	1.69	1.43	1.13	2.12	24.10	1.09
time (sec)	N/A	0.307	0.560	1.263	0.214	0.256	5.039	0.544	0.561

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	161	263	269	192	369	3528	219
N.S.	1	1.00	0.68	1.11	1.14	0.81	1.56	14.95	0.93
time (sec)	N/A	0.143	0.451	0.352	0.226	0.251	1.787	0.514	0.370

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	87	82	94	84	121	98	124
N.S.	1	1.00	0.84	0.80	0.91	0.82	1.17	0.95	1.20
time (sec)	N/A	0.031	0.108	0.135	0.197	0.258	0.614	0.425	0.377

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	160	0	0	140	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.223	0.680	0.000	0.000	0.273	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	305	305	250	0	0	259	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.321	1.081	0.000	0.000	0.268	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	325	0	0	475	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.454	0.800	0.000	0.000	0.268	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	166	3075	480	413	0	0	0
N.S.	1	1.00	0.86	16.02	2.50	2.15	0.00	0.00	0.00
time (sec)	N/A	0.206	0.681	0.414	0.278	0.260	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	121	1124	303	271	0	0	0
N.S.	1	1.00	0.83	7.75	2.09	1.87	0.00	0.00	0.00
time (sec)	N/A	0.177	0.501	0.296	0.254	0.258	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	74	456	0	147	0	0	0
N.S.	1	1.00	0.76	4.65	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.098	0.240	0.062	0.000	0.261	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	55	44	59	44	51	72	44
N.S.	1	1.00	1.38	1.10	1.48	1.10	1.28	1.80	1.10
time (sec)	N/A	0.019	0.095	0.036	0.195	0.268	1.887	0.332	0.113

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	33	22	27	27
N.S.	1	1.00	1.08	1.00	1.04	1.32	0.88	1.08	1.08
time (sec)	N/A	0.106	1.703	0.037	0.239	0.252	1.826	0.345	0.134

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	57	24	27	27
N.S.	1	1.00	1.08	1.00	1.04	2.28	0.96	1.08	1.08
time (sec)	N/A	0.083	0.360	0.046	0.254	0.251	3.924	0.457	0.125

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	0	3319	699	1390	0	0	0
N.S.	1	1.00	0.00	8.55	1.80	3.58	0.00	0.00	0.00
time (sec)	N/A	0.557	0.000	0.455	0.296	0.308	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	0	1650	468	834	0	0	0
N.S.	1	1.00	0.00	5.61	1.59	2.84	0.00	0.00	0.00
time (sec)	N/A	0.431	0.000	0.310	0.294	0.263	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	0	591	0	400	0	0	0
N.S.	1	1.00	0.00	3.09	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.222	0.000	0.128	0.000	0.265	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	107	74	94	100	0	108	80
N.S.	1	1.00	1.45	1.00	1.27	1.35	0.00	1.46	1.08
time (sec)	N/A	0.032	0.152	0.079	0.201	0.254	0.000	0.371	0.098

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	192	67	24	27	27
N.S.	1	1.00	1.08	1.00	7.68	2.68	0.96	1.08	1.08
time (sec)	N/A	0.085	0.789	0.090	0.274	0.256	3.954	0.389	0.167

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	280	108	26	27	27
N.S.	1	1.00	1.08	1.00	11.20	4.32	1.04	1.08	1.08
time (sec)	N/A	0.074	0.724	0.004	0.304	0.267	23.593	0.528	0.179

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	A	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	594	594	0	4005	1005	2704	0	0	0
N.S.	1	1.00	0.00	6.74	1.69	4.55	0.00	0.00	0.00
time (sec)	N/A	1.273	0.000	0.443	0.311	0.321	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	A	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	0	1887	694	1518	0	0	0
N.S.	1	1.00	0.00	4.30	1.58	3.46	0.00	0.00	0.00
time (sec)	N/A	0.868	0.000	0.427	0.294	0.299	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	0	668	0	696	0	0	0
N.S.	1	1.00	0.00	2.42	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.374	0.000	0.285	0.000	0.259	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	128	96	138	190	0	132	142
N.S.	1	1.00	1.15	0.86	1.24	1.71	0.00	1.19	1.28
time (sec)	N/A	0.041	0.196	0.231	0.217	0.265	0.000	0.350	0.117

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	651	101	24	27	27
N.S.	1	1.00	1.08	1.00	26.04	4.04	0.96	1.08	1.08
time (sec)	N/A	0.076	1.212	0.006	0.314	0.265	12.850	0.417	0.225

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	807	159	26	27	27
N.S.	1	1.00	1.08	1.00	32.28	6.36	1.04	1.08	1.08
time (sec)	N/A	0.072	0.965	0.006	0.371	0.241	138.507	0.605	0.252

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	77	82	70	0	132	0
N.S.	1	1.00	1.00	1.08	1.15	0.99	0.00	1.86	0.00
time (sec)	N/A	0.061	0.258	0.167	0.200	0.261	0.000	0.360	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	134	144	160	133	0	250	0
N.S.	1	1.00	0.92	0.99	1.10	0.92	0.00	1.72	0.00
time (sec)	N/A	0.142	0.560	0.228	0.273	0.263	0.000	0.318	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	196	211	238	196	0	368	0
N.S.	1	1.00	0.88	0.94	1.06	0.88	0.00	1.64	0.00
time (sec)	N/A	0.182	1.375	0.182	0.288	0.259	0.000	0.358	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	18	18	15	18	18
N.S.	1	1.00	1.11	0.84	0.95	0.95	0.79	0.95	0.95
time (sec)	N/A	0.027	0.549	0.016	0.242	0.239	0.895	0.329	0.104

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	18	30	17	18	18
N.S.	1	1.00	1.11	0.84	0.95	1.58	0.89	0.95	0.95
time (sec)	N/A	0.026	1.650	0.004	0.291	0.237	5.027	0.377	0.243

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	18	42	17	18	18
N.S.	1	1.00	1.11	0.84	0.95	2.21	0.89	0.95	0.95
time (sec)	N/A	0.024	6.234	0.004	0.304	0.262	6.410	0.420	0.338

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	270	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.300	0.000	0.000	0.000	0.086	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	228	228	0	0	0	191	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.178	0.000	0.000	0.000	0.082	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	112	0	0	111	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.084	1.202	0.000	0.000	0.082	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	28	22	27	27
N.S.	1	1.00	1.08	1.00	1.04	1.12	0.88	1.08	1.08
time (sec)	N/A	0.074	0.454	0.017	0.254	0.242	3.406	0.393	0.134

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	50	24	27	27
N.S.	1	1.00	1.08	1.00	1.04	2.00	0.96	1.08	1.08
time (sec)	N/A	0.070	0.718	0.007	0.247	0.248	56.073	0.389	0.198

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	28	1	27	27
N.S.	1	1.00	1.08	1.00	1.04	1.12	0.04	1.08	1.08
time (sec)	N/A	0.071	0.420	0.032	0.282	0.261	0.000	0.687	0.167

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	115	225	106	134	0	0	0
N.S.	1	1.00	1.00	1.96	0.92	1.17	0.00	0.00	0.00
time (sec)	N/A	0.086	0.160	0.059	0.207	0.245	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	194	78	108	0	0	0
N.S.	1	1.00	1.00	2.28	0.92	1.27	0.00	0.00	0.00
time (sec)	N/A	0.070	0.207	0.046	0.231	0.252	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	154	48	75	0	0	0
N.S.	1	1.00	1.00	2.85	0.89	1.39	0.00	0.00	0.00
time (sec)	N/A	0.041	0.159	0.036	0.220	0.246	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	17	24	23
N.S.	1	1.00	1.00	1.04	1.00	1.00	0.74	1.04	1.00
time (sec)	N/A	0.022	0.045	0.016	0.196	0.249	0.061	0.352	0.122

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	32	26	19	26	26
N.S.	1	1.00	1.08	1.00	1.33	1.08	0.79	1.08	1.08
time (sec)	N/A	0.043	0.270	0.006	0.268	0.258	0.776	0.352	0.117

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	38	30	20	26	26
N.S.	1	1.00	1.08	1.00	1.58	1.25	0.83	1.08	1.08
time (sec)	N/A	0.041	0.242	0.006	0.255	0.306	0.693	0.323	0.130

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	137	274	129	246	0	0	0
N.S.	1	1.00	0.98	1.96	0.92	1.76	0.00	0.00	0.00
time (sec)	N/A	0.158	0.160	0.046	0.207	0.242	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	103	231	99	186	0	0	0
N.S.	1	1.00	0.96	2.16	0.93	1.74	0.00	0.00	0.00
time (sec)	N/A	0.112	0.112	0.037	0.209	0.284	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	54	67	72	74	58	0	63
N.S.	1	1.00	0.78	0.97	1.04	1.07	0.84	0.00	0.91
time (sec)	N/A	0.045	0.095	0.024	0.196	0.278	0.078	0.000	0.257

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	25	26	26	31
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.04	1.04	1.24
time (sec)	N/A	0.023	0.028	0.022	0.182	0.286	0.053	0.360	0.136

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	63	45	65	26	26
N.S.	1	1.00	1.08	1.00	2.62	1.88	2.71	1.08	1.08
time (sec)	N/A	0.072	0.173	0.006	0.265	0.272	0.784	0.457	0.143

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	67	51	70	26	26
N.S.	1	1.00	1.08	1.00	2.79	2.12	2.92	1.08	1.08
time (sec)	N/A	0.072	0.310	0.007	0.249	0.273	0.813	0.421	0.147

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	220	501	266	577	0	0	0
N.S.	1	1.00	0.84	1.92	1.02	2.21	0.00	0.00	0.00
time (sec)	N/A	0.342	0.264	0.076	0.208	0.277	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	177	304	202	379	0	0	0
N.S.	1	1.00	0.97	1.67	1.11	2.08	0.00	0.00	0.00
time (sec)	N/A	0.187	0.163	0.075	0.207	0.261	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	98	111	150	148	122	0	155
N.S.	1	1.00	0.92	1.05	1.42	1.40	1.15	0.00	1.46
time (sec)	N/A	0.059	0.114	0.045	0.203	0.263	0.098	0.000	0.362

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	46	53	26	25
N.S.	1	1.00	1.00	0.96	0.93	1.70	1.96	0.96	0.93
time (sec)	N/A	0.023	0.027	0.033	0.197	0.267	0.061	0.335	0.254

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	145	64	170	26	26
N.S.	1	1.00	1.08	1.00	6.04	2.67	7.08	1.08	1.08
time (sec)	N/A	0.072	0.783	0.007	0.264	0.253	2.759	0.431	0.161

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	147	72	172	26	26
N.S.	1	1.00	1.08	1.00	6.12	3.00	7.17	1.08	1.08
time (sec)	N/A	0.072	0.898	0.013	0.252	0.276	2.843	0.416	0.165

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [19] had the largest ratio of [.73329999999999999952]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	17	0.353
2	A	5	5	1.00	17	0.294
3	A	4	4	1.00	15	0.267
4	A	4	4	1.00	13	0.308
5	N/A	0	0	1.00	17	0.000
6	N/A	0	0	1.00	17	0.000
7	A	4	4	1.00	14	0.286
8	A	4	4	1.00	16	0.250
9	A	13	8	1.00	17	0.471
10	A	11	9	1.00	17	0.529
11	A	10	10	1.00	15	0.667
12	A	3	2	1.00	13	0.154
13	N/A	0	0	1.00	17	0.000
14	N/A	0	0	1.00	17	0.000
15	A	3	2	1.00	14	0.143
16	A	3	2	1.00	16	0.125
17	A	26	10	1.00	17	0.588
18	A	23	12	1.00	17	0.706
19	A	15	11	1.00	15	0.733
20	A	3	2	1.00	13	0.154
21	N/A	0	0	1.00	17	0.000
22	N/A	0	0	1.00	17	0.000
23	A	3	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	3	2	1.00	16	0.125
25	A	6	3	1.00	23	0.130
26	A	5	3	1.00	23	0.130
27	A	4	3	1.00	21	0.143
28	A	2	1	1.00	15	0.067
29	A	4	3	1.00	23	0.130
30	A	5	4	1.00	23	0.174
31	A	6	4	1.00	23	0.174
32	A	10	3	1.00	25	0.120
33	A	8	3	1.00	25	0.120
34	A	6	3	1.00	23	0.130
35	A	4	3	1.00	17	0.176
36	A	6	3	1.00	25	0.120
37	A	8	4	1.00	25	0.160
38	A	10	4	1.00	25	0.160
39	A	14	3	1.00	25	0.120
40	A	11	3	1.00	25	0.120
41	A	8	3	1.00	23	0.130
42	A	4	3	1.00	17	0.176
43	A	8	3	1.00	25	0.120
44	A	11	4	1.00	25	0.160
45	A	14	4	1.00	25	0.160
46	A	6	6	1.00	25	0.240
47	A	5	5	1.00	25	0.200
48	A	4	4	1.00	23	0.174
49	A	5	5	1.00	17	0.294
50	N/A	0	0	1.00	25	0.000
51	N/A	0	0	1.00	25	0.000
52	A	13	8	1.00	25	0.320
53	A	11	9	1.00	25	0.360
54	A	11	11	1.00	23	0.478
55	A	4	3	1.00	17	0.176
56	N/A	0	0	1.00	25	0.000
57	N/A	0	0	1.00	25	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
58	A	26	10	1.00	25	0.400
59	A	24	13	1.00	25	0.520
60	A	17	12	1.00	23	0.522
61	A	4	3	1.00	17	0.176
62	N/A	0	0	1.00	25	0.000
63	N/A	0	0	1.00	25	0.000
64	A	5	4	1.00	17	0.235
65	A	8	4	1.00	19	0.210
66	A	11	4	1.00	19	0.210
67	N/A	0	0	1.00	19	0.000
68	N/A	0	0	1.00	19	0.000
69	N/A	0	0	1.00	19	0.000
70	A	8	3	1.00	25	0.120
71	A	6	3	1.00	25	0.120
72	A	4	3	1.00	23	0.130
73	N/A	0	0	1.00	25	0.000
74	N/A	0	0	1.00	25	0.000
75	N/A	0	0	1.00	25	0.000
76	A	5	5	1.00	24	0.208
77	A	4	4	1.00	24	0.167
78	A	3	3	1.00	22	0.136
79	A	2	2	1.00	21	0.095
80	N/A	0	0	1.00	24	0.000
81	N/A	0	0	1.00	24	0.000
82	A	6	6	1.00	24	0.250
83	A	5	5	1.00	24	0.208
84	A	5	5	1.00	22	0.227
85	A	2	2	1.00	21	0.095
86	N/A	0	0	1.00	24	0.000
87	N/A	0	0	1.00	24	0.000
88	A	12	9	1.00	24	0.375
89	A	11	10	1.00	24	0.417
90	A	4	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	2	2	1.00	21	0.095
92	N/A	0	0	1.00	24	0.000
93	N/A	0	0	1.00	24	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{x^3}{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{ad^4}$$

```
[Out] 1/4*x^4/a-x^3*ln(1+b*exp(d*x+c)/a)/a/d-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a/d
^2+6*x*polylog(3,-b*exp(d*x+c)/a)/a/d^3-6*polylog(4,-b*exp(d*x+c)/a)/a/d^4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2215, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = -\frac{6\operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{ad^4} + \frac{6x\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2} - \frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{ad} + \frac{x^4}{4a}$$

```
[In] Int[x^3/(a + b*E^(c + d*x)),x]
```

```
[Out] x^4/(4*a) - (x^3*Log[1 + (b*E^(c + d*x))/a])/(a*d) - (3*x^2*PolyLog[2, -((b
*E^(c + d*x))/a])/(a*d^2) + (6*x*PolyLog[3, -((b*E^(c + d*x))/a])/(a*d^3)
 - (6*PolyLog[4, -((b*E^(c + d*x))/a])/(a*d^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{x^4}{4a} - \frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}}\,dx}{a}$$

$$= \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} + \frac{3\int x^2\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{ad}$$

$$= \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6\int x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)\,dx}{ad^2}$$

$$= \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6\int \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)\,dx}{ad^3}$$

$$= \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{ad^4}$$

$$= \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{ad^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \frac{-d^3x^3\log\left(1+\frac{ae^{-c-dx}}{b}\right) + 3d^2x^2\,\text{PolyLog}\left(2,-\frac{ae^{-c-dx}}{b}\right) + 6dx\,\text{PolyLog}\left(3,-\frac{ae^{-c-dx}}{b}\right) + 6\,\text{PolyLog}\left(4,-\right.}{ad^4}$$

```
[In] Integrate[x^3/(a + b*E^(c + d*x)),x]
```

```
[Out] (-(d^3*x^3*Log[1 + (a*E^(-c - d*x))/b]) + 3*d^2*x^2*PolyLog[2, -((a*E^(-c -
 d*x))/b)] + 6*d*x*PolyLog[3, -((a*E^(-c - d*x))/b)] + 6*PolyLog[4, -((a*E^
(-c - d*x))/b)])/(a*d^4)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.74

method	result
risch	$\frac{c^3 \ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{d^4 a}-\frac{c^3 \ln\left(\mathrm{e}^{dx+c}\right)}{d^4 a}+\frac{3c^4}{4d^4 a}-\frac{6\ \mathrm{Li}_4\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a\,d^4}+\frac{x^4}{4a}-\frac{\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)c^3}{d^4 a}-\frac{x^3 \ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{ad}$ −
derivativedivides	$-\frac{-\frac{(dx+c)^4}{4}+(dx+c)^3 \ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+3(dx+c)^2\,\mathrm{Li}_2\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)-6(dx+c)\,\mathrm{Li}_3\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+6\ \mathrm{Li}_4\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a}+\frac{c^3 \ln\left(a+b\,\mathrm{e}^{dx+c}\right.}{a}$
default	$-\frac{-\frac{(dx+c)^4}{4}+(dx+c)^3 \ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+3(dx+c)^2\,\mathrm{Li}_2\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)-6(dx+c)\,\mathrm{Li}_3\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+6\ \mathrm{Li}_4\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a}+\frac{c^3 \ln\left(a+b\,\mathrm{e}^{dx+c}\right.}{a}$

```
[In] int(x^3/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*c^3/a*ln(a+b*exp(d*x+c))-1/d^4*c^3/a*ln(exp(d*x+c))+3/4/d^4/a*c^4-6*p
olylog(4,-b*exp(d*x+c)/a)/a/d^4+1/4*x^4/a-1/d^4/a*ln(1+b*exp(d*x+c)/a)*c^3-
x^3*ln(1+b*exp(d*x+c)/a)/a/d-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a/d^2+6*x*pol
ylog(3,-b*exp(d*x+c)/a)/a/d^3+1/d^3/a*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \frac{d^4x^4 - 12\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + 4\,c^3\log\left(be^{(dx+c)}+a\right) + 24\,dx\mathrm{polylog}\left(3,-\frac{be^{(dx+c)}}{a}\right) - 4\left(d^3x^3+c^3\right)\mathrm{lo}}{4\,ad^4}$$

```
[In] integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(b*e^(d*x + c) + a)/a + 1) + 4*c^3*log(b*e
^(d*x + c) + a) + 24*d*x*polylog(3, -b*e^(d*x + c)/a) - 4*(d^3*x^3 + c^3)*l
og((b*e^(d*x + c) + a)/a) - 24*polylog(4, -b*e^(d*x + c)/a))/(a*d^4)
```

Sympy [F]

$$\int \frac{x^3}{a + be^{c+dx}}\,dx = \int \frac{x^3}{a + be^c e^{dx}}\,dx$$

```
[In] integrate(x**3/(a+b*exp(d*x+c)),x)
[Out] Integral(x**3/(a + b*exp(c)*exp(d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{a + be^{c+dx}}\,dx = \frac{x^4}{4\,a} - \frac{d^3x^3 \log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 3\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 6\,dx\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a}) + 6\,\mathrm{Li}_4(-\frac{be^{(dx+c)}}{a})}{ad^4}$$

```
[In] integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="maxima")
[Out] 1/4*x^4/a - (d^3*x^3*log(b*e^(d*x + c)/a + 1) + 3*d^2*x^2*dilog(-b*e^(d*x +
 c)/a) - 6*d*x*polylog(3, -b*e^(d*x + c)/a) + 6*polylog(4, -b*e^(d*x + c)/a
))/(a*d^4)
```

Giac [F]

$$\int \frac{x^3}{a + be^{c+dx}}\,dx = \int \frac{x^3}{be^{(dx+c)} + a}\,dx$$

```
[In] integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="giac")
[Out] integrate(x^3/(b*e^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \int \frac{x^3}{a+b\,\mathrm{e}^{c+d\,x}}\,dx$$

```
[In] int(x^3/(a + b*exp(c + d*x)),x)
[Out] int(x^3/(a + b*exp(c + d*x)), x)
```

3.2 $\int \frac{x^2}{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{x^2}{a+be^{c+dx}}\,dx=\frac{x^3}{3a}-\frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad}-\frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2}+\frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{ad^3}$$

```
[Out] 1/3*x^3/a-x^2*ln(1+b*exp(d*x+c)/a)/a/d-2*x*polylog(2,-b*exp(d*x+c)/a)/a/d^2
+2*polylog(3,-b*exp(d*x+c)/a)/a/d^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2215, 2221, 2611, 2320, 6724}

$$\int \frac{x^2}{a+be^{c+dx}}\,dx=\frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{ad^3}-\frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2}-\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{ad}+\frac{x^3}{3a}$$

```
[In] Int[x^2/(a + b*E^(c + d*x)),x]
```

```
[Out] x^3/(3*a) - (x^2*Log[1 + (b*E^(c + d*x))/a])/(a*d) - (2*x*PolyLog[2, -((b*E
^(c + d*x))/a)])/(a*d^2) + (2*PolyLog[3, -((b*E^(c + d*x))/a)])/(a*d^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3}{3a} - \frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{a} \\
&= \frac{x^3}{3a} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} + \frac{2\int x\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{ad} \\
&= \frac{x^3}{3a} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2\int \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)\,dx}{ad^2}
\end{aligned}$$

$$= \frac{x^3}{3a} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{ad^3}$$

$$= \frac{x^3}{3a} - \frac{x^2 \log\left(1 + \frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{ad^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{a + be^{c+dx}}\,dx = -\frac{x^2 \log\left(1 + \frac{ae^{-c-dx}}{b}\right)}{ad} + \frac{2x\,\text{PolyLog}\left(2, -\frac{ae^{-c-dx}}{b}\right)}{ad^2} + \frac{2\,\text{PolyLog}\left(3, -\frac{ae^{-c-dx}}{b}\right)}{ad^3}$$

```
[In] Integrate[x^2/(a + b*E^(c + d*x)),x]
```

```
[Out] -((x^2*Log[1 + (a*E^(-c - d*x))/b])/(a*d)) + (2*x*PolyLog[2, -((a*E^(-c - d
*x))/b)])/(a*d^2) + (2*PolyLog[3, -((a*E^(-c - d*x))/b)])/(a*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(79) = 158$.

Time = 0.01 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.93

method	result
derivativedivides	$\frac{c^2\left(-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a}+\frac{\ln\left(e^{dx+c}\right)}{a}\right)-\frac{-\frac{(dx+c)^3}{3}+(dx+c)^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+2(dx+c)\,\text{Li}_2\left(-\frac{b\,e^{dx+c}}{a}\right)-2\,\text{Li}_3\left(-\frac{b\,e^{dx+c}}{a}\right)}{a}+2c\left(\cdots\right.}{d^3}$
default	$\frac{c^2\left(-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a}+\frac{\ln\left(e^{dx+c}\right)}{a}\right)-\frac{-\frac{(dx+c)^3}{3}+(dx+c)^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+2(dx+c)\,\text{Li}_2\left(-\frac{b\,e^{dx+c}}{a}\right)-2\,\text{Li}_3\left(-\frac{b\,e^{dx+c}}{a}\right)}{a}+2c\left(\cdots\right.}{d^3}$
risch	$-\frac{c^2\ln\left(a+b\,e^{dx+c}\right)}{d^3a}+\frac{c^2\ln\left(e^{dx+c}\right)}{d^3a}+\frac{x^3}{3a}-\frac{c^2x}{d^2a}-\frac{2c^3}{3d^3a}-\frac{x^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{ad}+\frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c^2}{d^3a}-\frac{2x\,\text{Li}_2 \cdots}{}$

```
[In] int(x^2/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(c^2*(-1/a*ln(a+b*exp(d*x+c))+1/a*ln(exp(d*x+c)))-1/a*(-1/3*(d*x+c)^3
+(d*x+c)^2*ln(1+b*exp(d*x+c)/a)+2*(d*x+c)*polylog(2,-b*exp(d*x+c)/a)-2*poly
log(3,-b*exp(d*x+c)/a))+2*c/a*(-1/2*(d*x+c)^2+(d*x+c)*ln(1+b*exp(d*x+c)/a)+
polylog(2,-b*exp(d*x+c)/a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{a + be^{c+dx}}\,dx = \frac{d^3x^3 - 6\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a} + 1\right) - 3\,c^2\log\left(be^{(dx+c)} + a\right) - 3\left(d^2x^2 - c^2\right)\log\left(\frac{be^{(dx+c)}+a}{a}\right) + 6\,\mathrm{polylog}\left(3, -\frac{b}{}\right.}{3\,ad^3}$$

```
[In] integrate(x^2/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(d^3*x^3 - 6*d*x*dilog(-(b*e^(d*x + c) + a)/a + 1) - 3*c^2*log(b*e^(d*x
 + c) + a) - 3*(d^2*x^2 - c^2)*log((b*e^(d*x + c) + a)/a) + 6*polylog(3, -b
*e^(d*x + c)/a))/(a*d^3)
```

Sympy [F]

$$\int \frac{x^2}{a + be^{c+dx}}\,dx = \int \frac{x^2}{a + be^c e^{dx}}\,dx$$

```
[In] integrate(x**2/(a+b*exp(d*x+c)),x)
```

```
[Out] Integral(x**2/(a + b*exp(c)*exp(d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{a + be^{c+dx}}\,dx = \frac{x^3}{3\,a} - \frac{d^2x^2\log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\,\mathrm{Li}_3\left(-\frac{be^{(dx+c)}}{a}\right)}{ad^3}$$

```
[In] integrate(x^2/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/a - (d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)/
a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a*d^3)
```

Giac [F]

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \int \frac{x^2}{be^{(dx+c)}+a}\,dx$$

```
[In] integrate(x^2/(a+b*exp(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2/(b*e^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \int \frac{x^2}{a+b\,\mathrm{e}^{c+d\,x}}\,dx$$

```
[In] int(x^2/(a + b*exp(c + d*x)),x)
[Out] int(x^2/(a + b*exp(c + d*x)), x)
```

3.3 $\int \frac{x}{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{x}{a+be^{c+dx}}\,dx = \frac{x^2}{2a} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2}$$

```
[Out] 1/2*x^2/a-x*ln(1+b*exp(d*x+c)/a)/a/d-polylog(2,-b*exp(d*x+c)/a)/a/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2215, 2221, 2317, 2438}

$$\int \frac{x}{a+be^{c+dx}}\,dx = -\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2} - \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{ad} + \frac{x^2}{2a}$$

```
[In] Int[x/(a + b*E^(c + d*x)),x]
```

```
[Out] x^2/(2*a) - (x*Log[1 + (b*E^(c + d*x))/a])/(a*d) - PolyLog[2, -((b*E^(c + d
*x))/a)]/(a*d^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2}{2a} - \frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a} \\
&= \frac{x^2}{2a} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} + \frac{\int \log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{ad} \\
&= \frac{x^2}{2a} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{ad^2} \\
&= \frac{x^2}{2a} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{ad^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x}{a+be^{c+dx}}\,dx = -\frac{x\log\left(1+\frac{ae^{-c-dx}}{b}\right)}{ad} + \frac{\text{PolyLog}\left(2, -\frac{ae^{-c-dx}}{b}\right)}{ad^2}$$

```
[In] Integrate[x/(a + b*E^(c + d*x)),x]
[Out] -((x*Log[1 + (a*E^(-c - d*x))/b])/(a*d)) + PolyLog[2, -((a*E^(-c - d*x))/b)
]/(a*d^2)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{-\frac{(dx+c)^2}{2}+(dx+c)\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+\mathrm{Li}_2\left(-\frac{b\,e^{dx+c}}{a}\right)}{a}+\frac{c\ln\left(a+b\,e^{dx+c}\right)}{a}-\frac{c\ln\left(e^{dx+c}\right)}{a}}{d^2}$
default	$\frac{-\frac{-\frac{(dx+c)^2}{2}+(dx+c)\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+\mathrm{Li}_2\left(-\frac{b\,e^{dx+c}}{a}\right)}{a}+\frac{c\ln\left(a+b\,e^{dx+c}\right)}{a}-\frac{c\ln\left(e^{dx+c}\right)}{a}}{d^2}$
risch	$\frac{x^2}{2a}+\frac{cx}{da}+\frac{c^2}{2d^2a}-\frac{x\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{ad}-\frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c}{d^2a}-\frac{\mathrm{Li}_2\left(-\frac{b\,e^{dx+c}}{a}\right)}{a\,d^2}+\frac{c\ln(a+b\,e^{dx+c})}{d^2a}-\frac{c\ln(e^{dx+c})}{d^2a}$

```
[In] int(x/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-1/a*(-1/2*(d*x+c)^2+(d*x+c)*ln(1+b*exp(d*x+c)/a)+polylog(2,-b*exp(d
*x+c)/a))+c/a*ln(a+b*exp(d*x+c))-c/a*ln(exp(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \frac{x}{a+be^{c+dx}}\,dx = \frac{d^2x^2+2\,c\log\left(be^{(dx+c)}+a\right)-2\,(dx+c)\log\left(\frac{be^{(dx+c)}+a}{a}\right)-2\,\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right)}{2\,ad^2}$$

```
[In] integrate(x/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*x^2 + 2*c*log(b*e^(d*x + c) + a) - 2*(d*x + c)*log((b*e^(d*x + c)
+ a)/a) - 2*dilog(-(b*e^(d*x + c) + a)/a + 1))/(a*d^2)
```

Sympy [F]

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{a+be^ce^{dx}}\,dx$$

```
[In] integrate(x/(a+b*exp(d*x+c)),x)
```

```
[Out] Integral(x/(a + b*exp(c)*exp(d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+be^{c+dx}}\,dx = \frac{x^2}{2\,a} - \frac{dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{ad^2}$$

```
[In] integrate(x/(a+b*exp(d*x+c)),x, algorithm="maxima")
[Out] 1/2*x^2/a - (d*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a*d^2
)
```

Giac [F]

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{be^{(dx+c)}+a}\,dx$$

```
[In] integrate(x/(a+b*exp(d*x+c)),x, algorithm="giac")
[Out] integrate(x/(b*e^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{a+b\,\mathrm{e}^{c+d\,x}}\,dx$$

```
[In] int(x/(a + b*exp(c + d*x)),x)
[Out] int(x/(a + b*exp(c + d*x)), x)
```

3.4 $\int \frac{1}{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{1}{a+be^{c+dx}}\,dx=\frac{x}{a}-\frac{\log\left(a+be^{c+dx}\right)}{ad}$$

```
[Out] x/a-ln(a+b*exp(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2320, 36, 29, 31}

$$\int \frac{1}{a+be^{c+dx}}\,dx=\frac{x}{a}-\frac{\log\left(a+be^{c+dx}\right)}{ad}$$

```
[In] Int[(a + b*E^(c + d*x))^(-1),x]
```

```
[Out] x/a - Log[a + b*E^(c + d*x)]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{c+dx}\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{c+dx}\right)}{ad} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{c+dx}\right)}{ad}\\ &= \frac{x}{a} - \frac{\log\left(a+be^{c+dx}\right)}{ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{\log\left(e^{c+dx}\right)}{ad} - \frac{\log\left(a^2d+abde^{c+dx}\right)}{ad}$$

```
[In] Integrate[(a + b*E^(c + d*x))^(-1),x]
[Out] Log[E^(c + d*x)]/(a*d) - Log[a^2*d + a*b*d*E^(c + d*x)]/(a*d)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{-dx+\ln\left(a+b\,{\mathrm e}^{dx+c}\right)}{ad}$	25
norman	$\frac{x}{a}-\frac{\ln\left(a+b\,{\mathrm e}^{dx+c}\right)}{ad}$	26
derivativedivides	$\frac{-\frac{\ln\left(a+b\,{\mathrm e}^{dx+c}\right)}{a}+\frac{\ln\left({\mathrm e}^{dx+c}\right)}{a}}{d}$	33
default	$\frac{-\frac{\ln\left(a+b\,{\mathrm e}^{dx+c}\right)}{a}+\frac{\ln\left({\mathrm e}^{dx+c}\right)}{a}}{d}$	33
risch	$\frac{x}{a}+\frac{c}{ad}-\frac{\ln\left({\mathrm e}^{dx+c}+\frac{a}{b}\right)}{ad}$	36

```
[In] int(1/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -(-d*x+ln(a+b*exp(d*x+c)))/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{dx-\log\left(be^{(dx+c)}+a\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(d*x+c)),x, algorithm="fricas")
[Out] (d*x - log(b*e^(d*x + c) + a))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{x}{a}-\frac{\log\left(\frac{a}{b}+e^{c+dx}\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(d*x+c)),x)
[Out] x/a - log(a/b + exp(c + d*x))/(a*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{dx+c}{ad} - \frac{\log\left(be^{(dx+c)}+a\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(d*x+c)),x, algorithm="maxima")
[Out] (d*x + c)/(a*d) - log(b*e^(d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{\frac{dx+c}{a} - \frac{\log\left(\left|be^{(dx+c)}+a\right|\right)}{a}}{d}$$

```
[In] integrate(1/(a+b*exp(d*x+c)),x, algorithm="giac")
[Out] ((d*x + c)/a - log(abs(b*e^(d*x + c) + a))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+be^{c+dx}}\,dx = -\frac{\ln\left(a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)-d\,x}{a\,d}$$

```
[In] int(1/(a + b*exp(c + d*x)),x)
[Out] -(log(a + b*exp(d*x)*exp(c)) - d*x)/(a*d)
```

3.5 $\int \frac{1}{(a+be^{c+dx})x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{(a+be^{c+dx})\,x} \, dx = \text{Int}\left(\frac{1}{(a+be^{c+dx})\,x},x\right)$$

```
[Out] Unintegrable(1/(a+b*exp(d*x+c))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(a+be^{c+dx})\,x} \, dx = \int \frac{1}{(a+be^{c+dx})\,x} \, dx$$

```
[In] Int[1/((a + b*E^(c + d*x))*x),x]
```

```
[Out] Defer[Int][1/((a + b*E^(c + d*x))*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(a+be^{c+dx})\,x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx=\int \frac{1}{(a+be^{c+dx})\,x}\,dx$$

```
[In] Integrate[1/((a + b*E^(c + d*x))*x),x]
[Out] Integrate[1/((a + b*E^(c + d*x))*x), x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+b\,\mathrm{e}^{dx+c})\,x}dx$$

```
[In] int(1/(a+b*exp(d*x+c))/x,x)
[Out] int(1/(a+b*exp(d*x+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx=\int \frac{1}{(be^{(dx+c)}+a)x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x,x, algorithm="fricas")
[Out] integral(1/(b*x*e^(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx=\int \frac{1}{x\,(a+be^ce^{dx})}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x,x)
[Out] Integral(1/(x*(a + b*exp(c)*exp(d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x,x, algorithm="maxima")
[Out] integrate(1/((b*e^(d*x + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x,x, algorithm="giac")
[Out] integrate(1/((b*e^(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}\,dx$$

```
[In] int(1/(x*(a + b*exp(c + d*x))),x)
[Out] int(1/(x*(a + b*exp(c + d*x))), x)
```

3.6 $\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx=\text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)x^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*exp(d*x+c))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx=\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx$$

```
[In] Int[1/((a + b*E^(c + d*x))*x^2),x]
[Out] Defer[Int][1/((a + b*E^(c + d*x))*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx=\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx$$

```
[In] Integrate[1/((a + b*E^(c + d*x))*x^2),x]
[Out] Integrate[1/((a + b*E^(c + d*x))*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+b\,\mathrm{e}^{dx+c})\,x^2}dx$$

```
[In] int(1/(a+b*exp(d*x+c))/x^2,x)
[Out] int(1/(a+b*exp(d*x+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx=\int \frac{1}{(be^{(dx+c)}+a)x^2}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x^2,x, algorithm="fricas")
[Out] integral(1/(b*x^2*e^(d*x + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx=\int \frac{1}{x^2\,(a+be^ce^{dx})}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x**2,x)
[Out] Integral(1/(x**2*(a + b*exp(c)*exp(d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x^2}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x^2,x, algorithm="maxima")
[Out] integrate(1/((b*e^(d*x + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x^2}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))/x^2,x, algorithm="giac")
[Out] integrate(1/((b*e^(d*x + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x^2}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}\,dx$$

```
[In] int(1/(x^2*(a + b*exp(c + d*x))),x)
[Out] int(1/(x^2*(a + b*exp(c + d*x))), x)
```

3.7 $\int \frac{1}{a+be^{c-dx}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{1}{a+be^{c-dx}}\,dx=\frac{x}{a}+\frac{\log\left(a+be^{c-dx}\right)}{ad}$$

```
[Out] x/a+ln(a+b*exp(-d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2320, 36, 29, 31}

$$\int \frac{1}{a+be^{c-dx}}\,dx=\frac{\log\left(a+be^{c-dx}\right)}{ad}+\frac{x}{a}$$

```
[In] Int[(a + b*E^(c - d*x))^(-1),x]
```

```
[Out] x/a + Log[a + b*E^(c - d*x)]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{c-dx}\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{c-dx}\right)}{ad} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{c-dx}\right)}{ad} \\
&= \frac{x}{a} + \frac{\log\left(a+be^{c-dx}\right)}{ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\log\left(be^{c}+ae^{dx}\right)}{ad}$$

```
[In] Integrate[(a + b*E^(c - d*x))^(-1),x]
[Out] Log[b*E^c + a*E^(d*x)]/(a*d)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{dx+\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	24
norman	$\frac{x}{a}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	26
derivativedivides	$-\frac{\ln\left(\mathrm{e}^{-dx+c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	37
default	$-\frac{\ln\left(\mathrm{e}^{-dx+c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	37
risch	$\frac{x}{a}-\frac{c}{ad}+\frac{\ln\left(\mathrm{e}^{-dx+c}+\frac{a}{b}\right)}{ad}$	37

```
[In] int(1/(a+b*exp(-d*x+c)),x,method=_RETURNVERBOSE)
[Out] (d*x+ln(a+b*exp(-d*x+c)))/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+be^{c-dx}}\,dx=\frac{dx+\log\left(be^{(-dx+c)}+a\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(-d*x+c)),x, algorithm="fricas")
[Out] (d*x + log(b*e^(-d*x + c) + a))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{a+be^{c-dx}}\,dx=\frac{x}{a}+\frac{\log\left(\frac{a}{b}+e^{c-dx}\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(-d*x+c)),x)
[Out] x/a + log(a/b + exp(c - d*x))/(a*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{dx-c}{ad} + \frac{\log\left(be^{(-dx+c)}+a\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(-d*x+c)),x, algorithm="maxima")
[Out] (d*x - c)/(a*d) + log(b*e^(-d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\frac{dx-c}{a} + \frac{\log\left(\left|be^{(-dx+c)}+a\right|\right)}{a}}{d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c)),x, algorithm="giac")
[Out] ((d*x - c)/a + log(abs(b*e^(-d*x + c) + a))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^{-d\,x}\,\mathrm{e}^{c}\right)+d\,x}{a\,d}$$

```
[In] int(1/(a + b*exp(c - d*x)),x)
[Out] (log(a + b*exp(-d*x)*exp(c)) + d*x)/(a*d)
```

3.8 $\int \frac{1}{a+be^{-c-dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{1}{a+be^{-c-dx}}\,dx=\frac{x}{a}+\frac{\log\left(a+be^{-c-dx}\right)}{ad}$$

```
[Out] x/a+ln(a+b*exp(-d*x-c))/a/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 36, 29, 31}

$$\int \frac{1}{a+be^{-c-dx}}\,dx=\frac{\log\left(a+be^{-c-dx}\right)}{ad}+\frac{x}{a}$$

```
[In] Int[(a + b*E^(-c - d*x))^(-1),x]
```

```
[Out] x/a + Log[a + b*E^(-c - d*x)]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{-c-dx}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{-c-dx}\right)}{ad} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{-c-dx}\right)}{ad}\\
&= \frac{x}{a} + \frac{\log\left(a+be^{-c-dx}\right)}{ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{\log\left(b+ae^{c+dx}\right)}{ad}$$

```
[In] Integrate[(a + b*E^(-c - d*x))^(-1),x]
[Out] Log[b + a*E^(c + d*x)]/(a*d)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{dx+\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	26
norman	$\frac{x}{a}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	28
risch	$\frac{x}{a}+\frac{c}{ad}+\frac{\ln\left(\mathrm{e}^{-dx-c}+\frac{a}{b}\right)}{ad}$	38
derivativedivides	$-\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	41
default	$-\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	41

```
[In] int(1/(a+b*exp(-d*x-c)),x,method=_RETURNVERBOSE)
[Out] (d*x+ln(a+b*exp(-d*x-c)))/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{dx+\log\left(be^{(-dx-c)}+a\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(-d*x-c)),x, algorithm="fricas")
[Out] (d*x + log(b*e^(-d*x - c) + a))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{x}{a}+\frac{\log\left(\frac{a}{b}+e^{-c-dx}\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(-d*x-c)),x)
[Out] x/a + log(a/b + exp(-c - d*x))/(a*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1}{a + be^{-c-dx}}\,dx = \frac{dx + c}{ad} + \frac{\log\left(be^{(-dx-c)} + a\right)}{ad}$$

```
[In] integrate(1/(a+b*exp(-d*x-c)),x, algorithm="maxima")
[Out] (d*x + c)/(a*d) + log(b*e^(-d*x - c) + a)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{a + be^{-c-dx}}\,dx = \frac{\frac{dx+c}{a} + \frac{\log\left(\left|be^{(-dx-c)}+a\right|\right)}{a}}{d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c)),x, algorithm="giac")
[Out] ((d*x + c)/a + log(abs(b*e^(-d*x - c) + a))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{a + be^{-c-dx}}\,dx = \frac{\ln\left(a + b\,\mathrm{e}^{-c}\,\mathrm{e}^{-d\,x}\right) + d\,x}{a\,d}$$

```
[In] int(1/(a + b*exp(- c - d*x)),x)
[Out] (log(a + b*exp(-c)*exp(-d*x)) + d*x)/(a*d)
```

3.9 $\int \frac{x^3}{(a+be^{c+dx})^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 217

$$\begin{aligned}\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = &-\frac{x^3}{a^2d}+\frac{x^3}{ad\,(a+be^{c+dx})}+\frac{x^4}{4a^2}+\frac{3x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2}\\ &-\frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d}+\frac{6x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}\\ &-\frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2}-\frac{6\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}\\ &+\frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}-\frac{6\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}\end{aligned}$$

```
[Out] -x^3/a^2/d+x^3/a/d/(a+b*exp(d*x+c))+1/4*x^4/a^2+3*x^2*ln(1+b*exp(d*x+c)/a)/
a^2/d^2-x^3*ln(1+b*exp(d*x+c)/a)/a^2/d+6*x*polylog(2,-b*exp(d*x+c)/a)/a^2/d
^3-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2-6*polylog(3,-b*exp(d*x+c)/a)/a^
2/d^4+6*x*polylog(3,-b*exp(d*x+c)/a)/a^2/d^3-6*polylog(4,-b*exp(d*x+c)/a)/a
^2/d^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222}

$$\begin{aligned}\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = & -\frac{6\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}-\frac{6\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}\\ & +\frac{6x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}+\frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}\\ & -\frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2}+\frac{3x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^2d^2}\\ & -\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^2d}-\frac{x^3}{a^2d}+\frac{x^4}{4a^2}+\frac{x^3}{ad\,(a+be^{c+dx})}\end{aligned}$$

```
[In] Int[x^3/(a + b*E^(c + d*x))^2,x]
```

```
[Out] -(x^3/(a^2*d)) + x^3/(a*d*(a + b*E^(c + d*x))) + x^4/(4*a^2) + (3*x^2*Log[1
 + (b*E^(c + d*x))/a])/(a^2*d^2) - (x^3*Log[1 + (b*E^(c + d*x))/a])/(a^2*d)
 + (6*x*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^2*d^3) - (3*x^2*PolyLog[2, -((
b*E^(c + d*x))/a)])/(a^2*d^2) - (6*PolyLog[3, -((b*E^(c + d*x))/a)])/(a^2*d
^4) + (6*x*PolyLog[3, -((b*E^(c + d*x))/a)])/(a^2*d^3) - (6*PolyLog[4, -((b
*E^(c + d*x))/a)])/(a^2*d^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x^3}{a+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}\,dx}{a}$$

$$= \frac{x^3}{ad\left(a+be^{c+dx}\right)} + \frac{x^4}{4a^2} - \frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}}\,dx}{a^2} - \frac{3\int \frac{x^2}{a+be^{c+dx}}\,dx}{ad}$$

$$= -\frac{x^3}{a^2d} + \frac{x^3}{ad\left(a+be^{c+dx}\right)} + \frac{x^4}{4a^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} + \frac{3\int x^2\log\left(1+\frac{be^{c+dx}}{a}\right)dx}{a^2d} + \frac{(3b)\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{a^2d}$$

$$= -\frac{x^3}{a^2d} + \frac{x^3}{ad\left(a+be^{c+dx}\right)} + \frac{x^4}{4a^2} + \frac{3x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{6\int x\log\left(1+\frac{be^{c+dx}}{a}\right)dx}{a^2d^2} + \frac{6\int x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)dx}{a^2d^2}$$

$$= -\frac{x^3}{a^2d} + \frac{x^3}{ad\left(a+be^{c+dx}\right)} + \frac{x^4}{4a^2} + \frac{3x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} + \frac{6x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{6\int \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)dx}{a^2d^3} - \frac{6\int \text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)dx}{a^2d^3}$$

$$= -\frac{x^3}{a^2d} + \frac{x^3}{ad\left(a+be^{c+dx}\right)} + \frac{x^4}{4a^2} + \frac{3x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} + \frac{6x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{6\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^2d^4} - \frac{6\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^2d^4}$$

$$= -\frac{x^3}{a^2d} + \frac{x^3}{ad\left(a+be^{c+dx}\right)} + \frac{x^4}{4a^2} + \frac{3x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} + \frac{6x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{6\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^4} + \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx$$
$$= \frac{-\frac{4x^3}{d}+\frac{4ax^3}{ad+bde^{c+dx}}+x^4+\frac{12x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{4x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{d}-\frac{12x(-2+dx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d^3}+\frac{24(-1+dx)\operatorname{PolyLog}}{d^4}}{4a^2}$$

```
[In] Integrate[x^3/(a + b*E^(c + d*x))^2,x]
```

```
[Out] ((-4*x^3)/d + (4*a*x^3)/(a*d + b*d*E^(c + d*x)) + x^4 + (12*x^2*Log[1 + (b*
E^(c + d*x))/a])/d^2 - (4*x^3*Log[1 + (b*E^(c + d*x))/a])/d - (12*x*(-2 + d
*x)*PolyLog[2, -((b*E^(c + d*x))/a)])/d^3 + (24*(-1 + d*x)*PolyLog[3, -((b*
E^(c + d*x))/a)])/d^4 - (24*PolyLog[4, -((b*E^(c + d*x))/a)])/d^4)/(4*a^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.76

method	result
risch	$\frac{x^3}{ad(a+b\,e^{dx+c})}+\frac{c^3\ln(a+b\,e^{dx+c})}{d^4a^2}-\frac{c^3\ln(e^{dx+c})}{d^4a^2}+\frac{x^4}{4a^2}-\frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c^3}{d^4a^2}+\frac{2c^3}{d^4a^2}-\frac{x^3}{a^2d}-\frac{3\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c^2}{d^4a^2}+\frac{c^3}{d^3}$

```
[In] int(x^3/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^3/a/d/(a+b*exp(d*x+c))+1/d^4/a^2*c^3*ln(a+b*exp(d*x+c))-1/d^4/a^2*c^3*ln(
exp(d*x+c))+1/4*x^4/a^2-1/d^4/a^2*ln(1+b*exp(d*x+c)/a)*c^3+2/d^4/a^2*c^3-x^
3/a^2/d-3/d^4/a^2*ln(1+b*exp(d*x+c)/a)*c^2+1/d^3/a^2*c^3*x-x^3*ln(1+b*exp(d
*x+c)/a)/a^2/d-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2+6*x*polylog(3,-b*ex
p(d*x+c)/a)/a^2/d^3+3/d^3/a^2*c^2*x+3*x^2*ln(1+b*exp(d*x+c)/a)/a^2/d^2+6*x*
polylog(2,-b*exp(d*x+c)/a)/a^2/d^3-6*polylog(3,-b*exp(d*x+c)/a)/a^2/d^4+3/d
^4/a^2*c^2*ln(a+b*exp(d*x+c))-3/d^4/a^2*c^2*ln(exp(d*x+c))+3/4/d^4/a^2*c^4-
6*polylog(4,-b*exp(d*x+c)/a)/a^2/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx$$
$$= \frac{ad^4x^4 - ac^4 - 4\,ac^3 - 12\left(ad^2x^2 - 2\,adx + (bd^2x^2 - 2\,bdx)e^{(dx+c)}\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a} + 1\right) + (bd^4x^4 - 4\,bd^3x}{}$$

```
[In] integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^4*x^4 - a*c^4 - 4*a*c^3 - 12*(a*d^2*x^2 - 2*a*d*x + (b*d^2*x^2 - 2
*b*d*x)*e^(d*x + c))*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b*d^4*x^4 - 4*b*d
^3*x^3 - b*c^4 - 4*b*c^3)*e^(d*x + c) + 4*(a*c^3 + 3*a*c^2 + (b*c^3 + 3*b*c
^2)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 4*(a*d^3*x^3 - 3*a*d^2*x^2 + a*c^
3 + 3*a*c^2 + (b*d^3*x^3 - 3*b*d^2*x^2 + b*c^3 + 3*b*c^2)*e^(d*x + c))*log(
(b*e^(d*x + c) + a)/a) - 24*(b*e^(d*x + c) + a)*polylog(4, -b*e^(d*x + c)/a
) + 24*(a*d*x + (b*d*x - b)*e^(d*x + c) - a)*polylog(3, -b*e^(d*x + c)/a))/
(a^2*b*d^4*e^(d*x + c) + a^3*d^4)
```

Sympy [F]

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = \frac{x^3}{a^2d + abde^{c+dx}} + \frac{\int \left(-\frac{3x^2}{a+be^ce^{dx}}\right)dx + \int \frac{dx^3}{a+be^ce^{dx}}\,dx}{ad}$$

```
[In] integrate(x**3/(a+b*exp(d*x+c))**2,x)
```

```
[Out] x**3/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(-3*x**2/(a + b*exp(c)*exp(d*
x)), x) + Integral(d*x**3/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx$$
$$= \frac{x^3}{abde^{(dx+c)} + a^2d} + \frac{d^4x^4 - 4\,d^3x^3}{4\,a^2d^4}$$
$$- \frac{d^3x^3\log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 3\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 6\,dx\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a}) + 6\,\mathrm{Li}_4(-\frac{be^{(dx+c)}}{a})}{a^2d^4}$$
$$+ \frac{3\left(d^2x^2\log\left(\frac{be^{(dx+c)}}{a} + 1\right) + 2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})\right)}{a^2d^4}$$

```
[In] integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] x^3/(a*b*d*e^(d*x + c) + a^2*d) + 1/4*(d^4*x^4 - 4*d^3*x^3)/(a^2*d^4) - (d^
3*x^3*log(b*e^(d*x + c)/a + 1) + 3*d^2*x^2*dilog(-b*e^(d*x + c)/a) - 6*d*x*
polylog(3, -b*e^(d*x + c)/a) + 6*polylog(4, -b*e^(d*x + c)/a))/(a^2*d^4) +
3*(d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)/a) - 2*pol
ylog(3, -b*e^(d*x + c)/a))/(a^2*d^4)
```

Giac [F]

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = \int \frac{x^3}{\left(be^{(dx+c)}+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*e^(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = \int \frac{x^3}{\left(a+b\,\mathrm{e}^{c+d\,x}\right)^2}\,dx$$

```
[In] int(x^3/(a + b*exp(c + d*x))^2,x)
```

```
[Out] int(x^3/(a + b*exp(c + d*x))^2, x)
```

3.10 $\int \frac{x^2}{(a+be^{c+dx})^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 165

$$\int \frac{x^2}{(a+be^{c+dx})^2} dx = -\frac{x^2}{a^2d} + \frac{x^2}{ad(a+be^{c+dx})} + \frac{x^3}{3a^2} + \frac{2x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} + \frac{2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}$$

```
[Out] -x^2/a^2/d+x^2/a/d/(a+b*exp(d*x+c))+1/3*x^3/a^2+2*x*ln(1+b*exp(d*x+c)/a)/a^
2/d^2-x^2*ln(1+b*exp(d*x+c)/a)/a^2/d+2*polylog(2,-b*exp(d*x+c)/a)/a^2/d^3-2
*x*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2+2*polylog(3,-b*exp(d*x+c)/a)/a^2/d^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438}

$$\int \frac{x^2}{(a+be^{c+dx})^2} dx = \frac{2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} + \frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{2x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^2d^2} - \frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^2d} - \frac{x^2}{a^2d} + \frac{x^3}{3a^2} + \frac{x^2}{ad(a+be^{c+dx})}$$

```
[In] Int[x^2/(a + b*E^(c + d*x))^2,x]
```

```
[Out] -(x^2/(a^2*d)) + x^2/(a*d*(a + b*E^(c + d*x))) + x^3/(3*a^2) + (2*x*Log[1 +
 (b*E^(c + d*x))/a])/(a^2*d^2) - (x^2*Log[1 + (b*E^(c + d*x))/a])/(a^2*d) +
 (2*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^2*d^3) - (2*x*PolyLog[2, -((b*E^(c
 + d*x))/a)])/(a^2*d^2) + (2*PolyLog[3, -((b*E^(c + d*x))/a)])/(a^2*d^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{x^2}{a+be^{c+dx}}\,dx}{a}-\frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}\,dx}{a}\\
&= \frac{x^2}{ad\left(a+be^{c+dx}\right)}+\frac{x^3}{3a^2}-\frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{a^2}-\frac{2\int \frac{x}{a+be^{c+dx}}\,dx}{ad}\\
&= -\frac{x^2}{a^2d}+\frac{x^2}{ad\left(a+be^{c+dx}\right)}+\frac{x^3}{3a^2}-\frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d}\\
&\quad+\frac{2\int x\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^2d}+\frac{(2b)\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a^2d}\\
&= -\frac{x^2}{a^2d}+\frac{x^2}{ad\left(a+be^{c+dx}\right)}+\frac{x^3}{3a^2}+\frac{2x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2}-\frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d}\\
&\quad-\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2}-\frac{2\int \log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^2d^2}+\frac{2\int \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)\,dx}{a^2d^2}
\end{aligned}$$

$$= -\frac{x^2}{a^2d} + \frac{x^2}{ad(a+be^{c+dx})} + \frac{x^3}{3a^2} + \frac{2x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{2\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^2d^3} + \frac{2\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^2d^3}$$

$$= -\frac{x^2}{a^2d} + \frac{x^2}{ad(a+be^{c+dx})} + \frac{x^3}{3a^2} + \frac{2x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} + \frac{2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \frac{\frac{d^2x^2(adx+be^{c+dx}(-3+dx))}{a+be^{c+dx}} - 3dx(-2+dx)\log\left(1+\frac{be^{c+dx}}{a}\right) + (6-6dx)\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right) + 6\,\text{PolyLog}\left(3,\right.}{3a^2d^3}$$

```
[In] Integrate[x^2/(a + b*E^(c + d*x))^2,x]
```

```
[Out] ((d^2*x^2*(a*d*x + b*E^(c + d*x)*(-3 + d*x)))/(a + b*E^(c + d*x)) - 3*d*x*(
-2 + d*x)*Log[1 + (b*E^(c + d*x))/a] + (6 - 6*d*x)*PolyLog[2, -((b*E^(c + d
*x))/a)] + 6*PolyLog[3, -((b*E^(c + d*x))/a)])/(3*a^2*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(157) = 314.

Time = 0.06 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.96

method	result
risch	$\frac{x^2}{ad(a+b\,e^{dx+c})} - \frac{2c\ln(a+b\,e^{dx+c})}{d^3a^2} + \frac{2c\ln(e^{dx+c})}{d^3a^2} + \frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c^2}{d^3a^2} - \frac{x^2}{a^2d} + \frac{x^3}{3a^2} + \frac{2x\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{a^2d^2} + \frac{2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right.}{d^3a^2}$

```
[In] int(x^2/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2/a/d/(a+b*exp(d*x+c))-2/d^3/a^2*c*ln(a+b*exp(d*x+c))+2/d^3/a^2*c*ln(exp(
d*x+c))+1/d^3/a^2*ln(1+b*exp(d*x+c)/a)*c^2-x^2/a^2/d+1/3*x^3/a^2+2*x*ln(1+b
```

```
*exp(d*x+c)/a)/a^2/d^2+2/d^3/a^2*ln(1+b*exp(d*x+c)/a)*c-1/d^2/a^2*c^2*x-x^2
*ln(1+b*exp(d*x+c)/a)/a^2/d-2*x*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2-2/d^2/a^
2*c*x-2/3/d^3/a^2*c^3+2*polylog(3,-b*exp(d*x+c)/a)/a^2/d^3-1/d^3/a^2*c^2-1/
d^3/a^2*c^2*ln(a+b*exp(d*x+c))+1/d^3/a^2*c^2*ln(exp(d*x+c))+2*polylog(2,-b*
exp(d*x+c)/a)/a^2/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.59

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx$$
$$= \frac{ad^3x^3 + ac^3 + 3\,ac^2 - 6\left(adx + (bdx-b)e^{(dx+c)} - a\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + \left(bd^3x^3 - 3\,bd^2x^2 + bc^3 + 3\,bc\right.}{}$$

```
[In] integrate(x^2/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^3*x^3 + a*c^3 + 3*a*c^2 - 6*(a*d*x + (b*d*x - b)*e^(d*x + c) - a)*
dilog(-(b*e^(d*x + c) + a)/a + 1) + (b*d^3*x^3 - 3*b*d^2*x^2 + b*c^3 + 3*b*
c^2)*e^(d*x + c) - 3*(a*c^2 + 2*a*c + (b*c^2 + 2*b*c)*e^(d*x + c))*log(b*e^
(d*x + c) + a) - 3*(a*d^2*x^2 - a*c^2 - 2*a*d*x - 2*a*c + (b*d^2*x^2 - b*c^
2 - 2*b*d*x - 2*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) + 6*(b*e^(d*x
+ c) + a)*polylog(3, -b*e^(d*x + c)/a))/(a^2*b*d^3*e^(d*x + c) + a^3*d^3)
```

Sympy [F]

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \frac{x^2}{a^2d + abde^{c+dx}} + \frac{\int \left(-\frac{2x}{a+be^ce^{dx}}\right)dx + \int \frac{dx^2}{a+be^ce^{dx}}\,dx}{ad}$$

```
[In] integrate(x**2/(a+b*exp(d*x+c))**2,x)
```

```
[Out] x**2/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(-2*x/(a + b*exp(c)*exp(d*x))
, x) + Integral(d*x**2/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \frac{x^2}{abde^{(dx+c)}+a^2d}+\frac{d^3x^3-3\,d^2x^2}{3\,a^2d^3} - \frac{d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right)+2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)-2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})}{a^2d^3} + \frac{2\left(dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^2d^3}$$

```
[In] integrate(x^2/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] x^2/(a*b*d*e^(d*x + c) + a^2*d) + 1/3*(d^3*x^3 - 3*d^2*x^2)/(a^2*d^3) - (d^
2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)/a) - 2*polylog(
3, -b*e^(d*x + c)/a))/(a^2*d^3) + 2*(d*x*log(b*e^(d*x + c)/a + 1) + dilog(-
b*e^(d*x + c)/a))/(a^2*d^3)
```

Giac [F]

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \int \frac{x^2}{\left(be^{(dx+c)}+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*e^(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{e}^{c+d\,x}\right)^2}\,dx$$

```
[In] int(x^2/(a + b*exp(c + d*x))^2,x)
```

```
[Out] int(x^2/(a + b*exp(c + d*x))^2, x)
```

3.11 $\int \frac{x}{(a+be^{c+dx})^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{x}{(a+be^{c+dx})^2} dx = -\frac{x}{a^2d} + \frac{x}{ad(a+be^{c+dx})} + \frac{x^2}{2a^2} + \frac{\log(a+be^{c+dx})}{a^2d^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} - \frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2}$$

```
[Out] -x/a^2/d+x/a/d/(a+b*exp(d*x+c))+1/2*x^2/a^2+ln(a+b*exp(d*x+c))/a^2/d^2-x*ln
(1+b*exp(d*x+c)/a)/a^2/d-polylog(2,-b*exp(d*x+c)/a)/a^2/d^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2216, 2215, 2221, 2317, 2438, 2222, 2320, 36, 29, 31}

$$\int \frac{x}{(a+be^{c+dx})^2} dx = -\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2} + \frac{\log(a+be^{c+dx})}{a^2d^2} - \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^2d} - \frac{x}{a^2d} + \frac{x^2}{2a^2} + \frac{x}{ad(a+be^{c+dx})}$$

```
[In] Int[x/(a + b*E^(c + d*x))^2,x]
```

```
[Out] -(x/(a^2*d)) + x/(a*d*(a + b*E^(c + d*x))) + x^2/(2*a^2) + Log[a + b*E^(c +
 d*x)]/(a^2*d^2) - (x*Log[1 + (b*E^(c + d*x))/a])/(a^2*d) - PolyLog[2, -((b
*E^(c + d*x))/a)]/(a^2*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{x}{a+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}\,dx}{a}\\
&= \frac{x}{ad\left(a+be^{c+dx}\right)} + \frac{x^2}{2a^2} - \frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a^2} - \frac{\int \frac{1}{a+be^{c+dx}}\,dx}{ad}\\
&= \frac{x}{ad\left(a+be^{c+dx}\right)} + \frac{x^2}{2a^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{c+dx}\right)}{ad^2} + \frac{\int \log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^2d}\\
&= \frac{x}{ad\left(a+be^{c+dx}\right)} + \frac{x^2}{2a^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} - \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{c+dx}\right)}{a^2d^2}\\
&\quad + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^2d^2} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{c+dx}\right)}{a^2d^2}\\
&= -\frac{x}{a^2d} + \frac{x}{ad\left(a+be^{c+dx}\right)} + \frac{x^2}{2a^2} + \frac{\log\left(a+be^{c+dx}\right)}{a^2d^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^2d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{x}{(a+be^{c+dx})^2}\,dx = \frac{\frac{dx(adx+be^{c+dx}(-2+dx))}{a+be^{c+dx}} - 2(-1+dx)\log\left(1+\frac{be^{c+dx}}{a}\right) - 2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{2a^2d^2}$$

```
[In] Integrate[x/(a + b*E^(c + d*x))^2,x]
```

```
[Out] ((d*x*(a*d*x + b*E^(c + d*x)*(-2 + d*x)))/(a + b*E^(c + d*x)) - 2*(-1 + d*x
)*Log[1 + (b*E^(c + d*x))/a] - 2*PolyLog[2, -((b*E^(c + d*x))/a)])/(2*a^2*d
^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{\operatorname{dilog}\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2}-\frac{(dx+c)\ln\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2}+\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2}-\frac{b(dx+c)\mathrm{e}^{dx+c}}{a^2\left(a+b\,\mathrm{e}^{dx+c}\right)}+\frac{(dx+c)^2}{2a^2}-c\left(-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{dx+}\right.}\right.}{d^2}$
default	$\frac{-\frac{\operatorname{dilog}\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2}-\frac{(dx+c)\ln\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2}+\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2}-\frac{b(dx+c)\mathrm{e}^{dx+c}}{a^2\left(a+b\,\mathrm{e}^{dx+c}\right)}+\frac{(dx+c)^2}{2a^2}-c\left(-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{dx+}\right.}\right.}{d^2}$
risch	$\frac{x}{ad\left(a+b\,\mathrm{e}^{dx+c}\right)}+\frac{x^2}{2a^2}+\frac{cx}{d\,a^2}+\frac{c^2}{2d^2a^2}-\frac{x\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2d}-\frac{\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)c}{d^2a^2}-\frac{\operatorname{Li}_2\left(-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2d^2}+\frac{\ln\left(a+b\,\mathrm{e}^{dx}\right.}{a^2d^2}$

```
[In] int(x/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-1/a^2*dilog((a+b*exp(d*x+c))/a)-1/a^2*(d*x+c)*ln((a+b*exp(d*x+c))/a
)+1/a^2*ln(a+b*exp(d*x+c))-1/a^2*b*(d*x+c)*exp(d*x+c)/(a+b*exp(d*x+c))+1/2/
a^2*(d*x+c)^2-c*(-1/a^2*ln(a+b*exp(d*x+c))+1/a/(a+b*exp(d*x+c))+1/a^2*ln(ex
p(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.64

$$\int \frac{x}{(a+be^{c+dx})^2}\,dx$$
$$= \frac{ad^2x^2 - ac^2 - 2\,ac - 2\left(be^{(dx+c)} + a\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a} + 1\right) + (bd^2x^2 - bc^2 - 2\,bdx - 2\,bc)e^{(dx+c)} + 2\,(ac +}{2\,(a^2bd^2e^{(dx+c)} + a^3}$$

```
[In] integrate(x/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(a*d^2*x^2 - a*c^2 - 2*a*c - 2*(b*e^(d*x + c) + a)*dilog(-(b*e^(d*x + c
) + a)/a + 1) + (b*d^2*x^2 - b*c^2 - 2*b*d*x - 2*b*c)*e^(d*x + c) + 2*(a*c
+ (b*c + b)*e^(d*x + c) + a)*log(b*e^(d*x + c) + a) - 2*(a*d*x + a*c + (b*d
*x + b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a))/(a^2*b*d^2*e^(d*x + c) +
 a^3*d^2)
```

Sympy [F]

$$\int \frac{x}{(a+be^{c+dx})^2}\,dx = \frac{x}{a^2d + abde^{c+dx}} + \frac{\int \frac{dx}{a+be^ce^{dx}}\,dx + \int \left(-\frac{1}{a+be^ce^{dx}}\right)dx}{ad}$$

```
[In] integrate(x/(a+b*exp(d*x+c))**2,x)
[Out] x/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(d*x/(a + b*exp(c)*exp(d*x)), x)
 + Integral(-1/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{x}{(a+be^{c+dx})^2}\,dx = \frac{x}{abde^{(dx+c)} + a^2d} + \frac{x^2}{2\,a^2} - \frac{x}{a^2d} - \frac{dx\log\left(\frac{be^{(dx+c)}}{a} + 1\right) + \mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{a^2d^2} + \frac{\log\left(be^{(dx+c)} + a\right)}{a^2d^2}$$

```
[In] integrate(x/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
[Out] x/(a*b*d*e^(d*x + c) + a^2*d) + 1/2*x^2/a^2 - x/(a^2*d) - (d*x*log(b*e^(d*x
 + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^2*d^2) + log(b*e^(d*x + c) + a)/
(a^2*d^2)
```

Giac [F]

$$\int \frac{x}{(a+be^{c+dx})^2}\,dx = \int \frac{x}{(be^{(dx+c)}+a)^2}\,dx$$

```
[In] integrate(x/(a+b*exp(d*x+c))^2,x, algorithm="giac")
[Out] integrate(x/(b*e^(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+be^{c+dx})^2}\,dx = \int \frac{x}{(a+b\,\mathrm{e}^{c+d\,x})^2}\,dx$$

```
[In] int(x/(a + b*exp(c + d*x))^2,x)
[Out] int(x/(a + b*exp(c + d*x))^2, x)
```

3.12 $\int \frac{1}{(a+be^{c+dx})^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{1}{(a+be^{c+dx})^2} dx = \frac{1}{ad(a+be^{c+dx})} + \frac{x}{a^2} - \frac{\log(a+be^{c+dx})}{a^2d}$$

```
[Out] 1/a/d/(a+b*exp(d*x+c))+x/a^2-ln(a+b*exp(d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2320, 46}

$$\int \frac{1}{(a+be^{c+dx})^2} dx = -\frac{\log(a+be^{c+dx})}{a^2d} + \frac{x}{a^2} + \frac{1}{ad(a+be^{c+dx})}$$

```
[In] Int[(a + b*E^(c + d*x))^(-2),x]
```

```
[Out] 1/(a*d*(a + b*E^(c + d*x))) + x/a^2 - Log[a + b*E^(c + d*x)]/(a^2*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, e^{c+dx}\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)\,dx, x, e^{c+dx}\right)}{d}\\ &= \frac{1}{ad\,(a+be^{c+dx})} + \frac{x}{a^2} - \frac{\log\left(a+be^{c+dx}\right)}{a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{\frac{a}{a+be^{c+dx}} + \log\left(e^{c+dx}\right) - \log\left(a+be^{c+dx}\right)}{a^2d}$$

```
[In] Integrate[(a + b*E^(c + d*x))^(-2),x]
[Out] (a/(a + b*E^(c + d*x)) + Log[E^(c + d*x)] - Log[a + b*E^(c + d*x)])/(a^2*d)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{dx+c}\right)}+\frac{\ln\left(\mathrm{e}^{dx+c}\right)}{a^2}}{d}$	49
default	$\frac{-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{dx+c}\right)}+\frac{\ln\left(\mathrm{e}^{dx+c}\right)}{a^2}}{d}$	49
risch	$\frac{x}{a^2}+\frac{c}{a^2d}+\frac{1}{ad\left(a+b\,\mathrm{e}^{dx+c}\right)}-\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a}{b}\right)}{a^2d}$	55
norman	$\frac{\frac{x}{a}+\frac{bx\,\mathrm{e}^{dx+c}}{a^2}-\frac{b\,\mathrm{e}^{dx+c}}{a^2d}}{a+b\,\mathrm{e}^{dx+c}}-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2d}$	67
parallelrisch	$-\frac{-b^2\mathrm{e}^{dx+c}xd+\ln\left(a+b\,\mathrm{e}^{dx+c}\right)\mathrm{e}^{dx+c}b^2-xabd+\ln\left(a+b\,\mathrm{e}^{dx+c}\right)ab-ab}{a^2bd\left(a+b\,\mathrm{e}^{dx+c}\right)}$	83

```
[In] int(1/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(a+b*exp(d*x+c))+1/a/(a+b*exp(d*x+c))+1/a^2*ln(exp(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+be^{c+dx})^2}\,dx = \frac{bdxe^{(dx+c)} + adx - \left(be^{(dx+c)} + a\right)\log\left(be^{(dx+c)} + a\right) + a}{a^2bde^{(dx+c)} + a^3d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*e^(d*x + c) + a*d*x - (b*e^(d*x + c) + a)*log(b*e^(d*x + c) + a) + a
)/(a^2*b*d*e^(d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+be^{c+dx})^2}\,dx = \frac{1}{a^2d + abde^{c+dx}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{c+dx}\right)}{a^2d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))**2,x)
```

```
[Out] 1/(a**2*d + a*b*d*exp(c + d*x)) + x/a**2 - log(a/b + exp(c + d*x))/(a**2*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+be^{c+dx})^2}\,dx = \frac{1}{(abe^{(dx+c)} + a^2)d} + \frac{dx+c}{a^2d} - \frac{\log\left(be^{(dx+c)} + a\right)}{a^2d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/((a*b*e^(d*x + c) + a^2)*d) + (d*x + c)/(a^2*d) - log(b*e^(d*x + c) + a)/
(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+be^{c+dx})^2}\,dx = \frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(dx+c)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(dx+c)}+a)ab}\right)}{d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*e^(d*x + c) + a) + 1))/(a^2*b) + 1/((b*e^(d*x + c) + a)*a*
b))/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+be^{c+dx})^2}\,dx = \frac{\frac{x}{a} + \frac{b\,x\,\mathrm{e}^{c+d\,x}}{a^2} - \frac{b\,\mathrm{e}^{c+d\,x}}{a^2\,d}}{a+b\,\mathrm{e}^{c+d\,x}} - \frac{\ln\left(a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a^2\,d}$$

```
[In] int(1/(a + b*exp(c + d*x))^2,x)
```

```
[Out] (x/a + (b*x*exp(c + d*x))/a^2 - (b*exp(c + d*x))/(a^2*d))/(a + b*exp(c + d*
x)) - log(a + b*exp(d*x)*exp(c))/(a^2*d)
```

3.13 $\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \operatorname{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^2 x}, x\right)$$

```
[Out] Unintegrable(1/(a+b*exp(d*x+c))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx$$

```
[In] Int[1/((a + b*E^(c + d*x))^2*x),x]
```

```
[Out] Defer[Int][1/((a + b*E^(c + d*x))^2*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx=\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx$$

```
[In] Integrate[1/((a + b*E^(c + d*x))^2*x),x]
[Out] Integrate[1/((a + b*E^(c + d*x))^2*x), x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)^{2} x}dx$$

```
[In] int(1/(a+b*exp(d*x+c))^2/x,x)
[Out] int(1/(a+b*exp(d*x+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx=\int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2/x,x, algorithm="fricas")
[Out] integral(1/(b^2*x*e^(2*d*x + 2*c) + 2*a*b*x*e^(d*x + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx=\frac{1}{a^2dx+abdxe^{c+dx}}+\frac{\int \frac{dx}{ax^2+bx^2e^ce^{dx}}\,dx+\int \frac{1}{ax^2+bx^2e^ce^{dx}}\,dx}{ad}$$

```
[In] integrate(1/(a+b*exp(d*x+c))**2/x,x)
[Out] 1/(a**2*d*x + a*b*d*x*exp(c + d*x)) + (Integral(d*x/(a*x**2 + b*x**2*exp(c)
*exp(d*x)), x) + Integral(1/(a*x**2 + b*x**2*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.12

$$\int \frac{1}{(a+be^{c+dx})^2 x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)^2 x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2/x,x, algorithm="maxima")
```

```
[Out] 1/(a*b*d*x*e^(d*x + c) + a^2*d*x) + integrate((d*x + 1)/(a*b*d*x^2*e^(d*x +
 c) + a^2*d*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})^2 x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)^2 x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2/x,x, algorithm="giac")
```

```
[Out] integrate(1/((b*e^(d*x + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})^2 x}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{e}^{c+d\,x})^2}\,dx$$

```
[In] int(1/(x*(a + b*exp(c + d*x))^2),x)
```

```
[Out] int(1/(x*(a + b*exp(c + d*x))^2), x)
```

3.14 $\int \frac{1}{\left(a+be^{c+dx}\right)^2x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2x^2}\,dx=\operatorname{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^2x^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*exp(d*x+c))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2x^2}\,dx=\int \frac{1}{\left(a+be^{c+dx}\right)^2x^2}\,dx$$

```
[In] Int[1/((a + b*E^(c + d*x))^2*x^2),x]
```

```
[Out] Defer[Int][1/((a + b*E^(c + d*x))^2*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+be^{c+dx}\right)^2x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{c+dx})^2 x^2} \, dx = \int \frac{1}{(a+be^{c+dx})^2 x^2} \, dx$$

```
[In] Integrate[1/((a + b*E^(c + d*x))^2*x^2),x]
[Out] Integrate[1/((a + b*E^(c + d*x))^2*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)^2 x^2}dx$$

```
[In] int(1/(a+b*exp(d*x+c))^2/x^2,x)
[Out] int(1/(a+b*exp(d*x+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{1}{(a+be^{c+dx})^2 x^2} \, dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x^2} \, dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2/x^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*e^(2*d*x + 2*c) + 2*a*b*x^2*e^(d*x + c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.29

$$\int \frac{1}{(a+be^{c+dx})^2 x^2} \, dx = \frac{1}{a^2dx^2+abdx^2e^{c+dx}} + \frac{\int \frac{dx}{ax^3+bx^3e^ce^{dx}}\, dx + \int \frac{2}{ax^3+bx^3e^ce^{dx}}\, dx}{ad}$$

```
[In] integrate(1/(a+b*exp(d*x+c))**2/x**2,x)
[Out] 1/(a**2*d*x**2 + a*b*d*x**2*exp(c + d*x)) + (Integral(d*x/(a*x**3 + b*x**3*
exp(c)*exp(d*x)), x) + Integral(2/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x))/(a
*d)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2} \, dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x^2} \, dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*d*x^2*e^(d*x + c) + a^2*d*x^2) + integrate((d*x + 2)/(a*b*d*x^3*e^(d
*x + c) + a^2*d*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2} \, dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x^2} \, dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*e^(d*x + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2} \, dx = \int \frac{1}{x^2 \left(a+b\,\mathrm{e}^{c+d\,x}\right)^2} \, dx$$

```
[In] int(1/(x^2*(a + b*exp(c + d*x))^2),x)
```

```
[Out] int(1/(x^2*(a + b*exp(c + d*x))^2), x)
```

3.15 $\int \frac{1}{(a+be^{c-dx})^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = -\frac{1}{ad\,(a+be^{c-dx})}+\frac{x}{a^2}+\frac{\log\left(a+be^{c-dx}\right)}{a^2d}$$

```
[Out] -1/a/d/(a+b*exp(-d*x+c))+x/a^2+ln(a+b*exp(-d*x+c))/a^2/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2320, 46}

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = \frac{\log\left(a+be^{c-dx}\right)}{a^2d}+\frac{x}{a^2}-\frac{1}{ad\,(a+be^{c-dx})}$$

```
[In] Int[(a + b*E^(c - d*x))^(-2),x]
```

```
[Out] -(1/(a*d*(a + b*E^(c - d*x)))) + x/a^2 + Log[a + b*E^(c - d*x)]/(a^2*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, e^{c-dx}\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, e^{c-dx}\right)}{d}\\ &= -\frac{1}{ad\left(a+be^{c-dx}\right)}+\frac{x}{a^2}+\frac{\log\left(a+be^{c-dx}\right)}{a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = \frac{\frac{be^c}{be^c+ae^{dx}}+\log\left(ad\left(be^c+ae^{dx}\right)\right)}{a^2d}$$

```
[In] Integrate[(a + b*E^(c - d*x))^(-2),x]
[Out] ((b*E^c)/(b*E^c + a*E^(d*x)) + Log[a*d*(b*E^c + a*E^(d*x))])/(a^2*d)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^2}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{-dx+c}\right)}}{d}$	53
default	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^2}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{-dx+c}\right)}}{d}$	53
risch	$\frac{x}{a^2}-\frac{c}{a^2d}-\frac{1}{ad\left(a+b\,e^{-dx+c}\right)}+\frac{\ln\left(e^{-dx+c}+\frac{a}{b}\right)}{a^2d}$	58
norman	$\frac{\frac{x}{a}+\frac{bx\,e^{-dx+c}}{a^2}-\frac{1}{ad}}{a+b\,e^{-dx+c}}+\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^2d}$	62
parallelrisch	$\frac{b^2e^{-dx+c}xd+\ln\left(a+b\,e^{-dx+c}\right)e^{-dx+c}b^2+xabd+\ln\left(a+b\,e^{-dx+c}\right)ab-ab}{a^2bd\left(a+b\,e^{-dx+c}\right)}$	85

```
[In] int(1/(a+b*exp(-d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/a^2*ln(exp(-d*x+c))-1/a^2*ln(a+b*exp(-d*x+c))+1/a/(a+b*exp(-d*x+c))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = \frac{bdxe^{(-dx+c)} + adx + \left(be^{(-dx+c)} + a\right)\log\left(be^{(-dx+c)} + a\right) - a}{a^2bde^{(-dx+c)} + a^3d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*e^(-d*x + c) + a*d*x + (b*e^(-d*x + c) + a)*log(b*e^(-d*x + c) + a)
- a)/(a^2*b*d*e^(-d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = -\frac{1}{a^2d + abde^{c-dx}} + \frac{x}{a^2} + \frac{\log\left(\frac{a}{b} + e^{c-dx}\right)}{a^2d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))**2,x)
```

```
[Out] -1/(a**2*d + a*b*d*exp(c - d*x)) + x/a**2 + log(a/b + exp(c - d*x))/(a**2*d
)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = -\frac{1}{(abe^{(-dx+c)} + a^2)d} + \frac{dx - c}{a^2d} + \frac{\log\left(be^{(-dx+c)} + a\right)}{a^2d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((a*b*e^(-d*x + c) + a^2)*d) + (d*x - c)/(a^2*d) + log(b*e^(-d*x + c) +
a)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = -\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(-dx+c)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(-dx+c)}+a)ab}\right)}{d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="giac")
```

```
[Out] -b*(log(abs(-a/(b*e^(-d*x + c) + a) + 1))/(a^2*b) + 1/((b*e^(-d*x + c) + a)
*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+be^{c-dx})^2}\,dx = \frac{\frac{x}{a}+\frac{b\,x\,\mathrm{e}^{c-d\,x}}{a^2}+\frac{b\,\mathrm{e}^{c-d\,x}}{a^2\,d}}{a+b\,\mathrm{e}^{c-d\,x}} + \frac{\ln\left(a+b\,\mathrm{e}^{-d\,x}\,\mathrm{e}^{c}\right)}{a^2\,d}$$

```
[In] int(1/(a + b*exp(c - d*x))^2,x)
```

```
[Out] (x/a + (b*x*exp(c - d*x))/a^2 + (b*exp(c - d*x))/(a^2*d))/(a + b*exp(c - d*
x)) + log(a + b*exp(-d*x)*exp(c))/(a^2*d)
```

3.16 $\int \frac{1}{\left(a+be^{-c-dx}\right)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2} dx = -\frac{1}{ad\left(a+be^{-c-dx}\right)} + \frac{x}{a^2} + \frac{\log\left(a+be^{-c-dx}\right)}{a^2d}$$

```
[Out] -1/a/d/(a+b*exp(-d*x-c))+x/a^2+ln(a+b*exp(-d*x-c))/a^2/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2320, 46}

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2} dx = \frac{\log\left(a+be^{-c-dx}\right)}{a^2d} + \frac{x}{a^2} - \frac{1}{ad\left(a+be^{-c-dx}\right)}$$

```
[In] Int[(a + b*E^(-c - d*x))^(-2),x]
```

```
[Out] -(1/(a*d*(a + b*E^(-c - d*x)))) + x/a^2 + Log[a + b*E^(-c - d*x)]/(a^2*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, e^{-c-dx}\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, e^{-c-dx}\right)}{d}\\ &= -\frac{1}{ad\,(a+be^{-c-dx})}+\frac{x}{a^2}+\frac{\log\left(a+be^{-c-dx}\right)}{a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+be^{-c-dx})^2}\,dx = \frac{b+bdx+ade^{c+dx}x+2\left(b+ae^{c+dx}\right)\text{arctanh}\left(1+\frac{2ae^{c+dx}}{b}\right)}{a^2d\,(b+ae^{c+dx})}$$

```
[In] Integrate[(a + b*E^(-c - d*x))^(-2),x]
[Out] (b + b*d*x + a*d*E^(c + d*x)*x + 2*(b + a*E^(c + d*x))*ArcTanh[1 + (2*a*E^(
c + d*x))/b])/(a^2*d*(b + a*E^(c + d*x)))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$-\frac{\frac{\ln\left({e}^{-dx-c}\right)}{a^{2}}-\frac{\ln\left(a+b\,{e}^{-dx-c}\right)}{a^{2}}+\frac{1}{a\left(a+b\,{e}^{-dx-c}\right)}}{d}$	59
default	$-\frac{\frac{\ln\left({e}^{-dx-c}\right)}{a^{2}}-\frac{\ln\left(a+b\,{e}^{-dx-c}\right)}{a^{2}}+\frac{1}{a\left(a+b\,{e}^{-dx-c}\right)}}{d}$	59
risch	$\frac{x}{a^{2}}+\frac{c}{a^{2}d}-\frac{1}{ad\left(a+b\,{e}^{-dx-c}\right)}+\frac{\ln\left({e}^{-dx-c}+\frac{a}{b}\right)}{a^{2}d}$	61
norman	$\frac{\frac{x}{a}+\frac{bx\,{e}^{-dx-c}}{a^{2}}-\frac{1}{ad}}{a+b\,{e}^{-dx-c}}+\frac{\ln\left(a+b\,{e}^{-dx-c}\right)}{a^{2}d}$	68
parallelrisch	$\frac{b^{2}{e}^{-dx-c}xd+\ln\left(a+b\,{e}^{-dx-c}\right){e}^{-dx-c}b^{2}+xabd+\ln\left(a+b\,{e}^{-dx-c}\right)ab-ab}{a^{2}bd\left(a+b\,{e}^{-dx-c}\right)}$	95

```
[In] int(1/(a+b*exp(-d*x-c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/a^2*ln(exp(-d*x-c))-1/a^2*ln(a+b*exp(-d*x-c))+1/a/(a+b*exp(-d*x-c))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx=\frac{bdxe^{(-dx-c)}+adx+\left(be^{(-dx-c)}+a\right)\log\left(be^{(-dx-c)}+a\right)-a}{a^2bde^{(-dx-c)}+a^3d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*e^(-d*x - c) + a*d*x + (b*e^(-d*x - c) + a)*log(b*e^(-d*x - c) + a)
- a)/(a^2*b*d*e^(-d*x - c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx=-\frac{1}{a^2d+abde^{-c-dx}}+\frac{x}{a^2}+\frac{\log\left(\frac{a}{b}+e^{-c-dx}\right)}{a^2d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))**2,x)
```

```
[Out] -1/(a**2*d + a*b*d*exp(-c - d*x)) + x/a**2 + log(a/b + exp(-c - d*x))/(a**2
*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx=-\frac{1}{\left(abe^{(-dx-c)}+a^2\right)d}+\frac{dx+c}{a^2d}+\frac{\log\left(be^{(-dx-c)}+a\right)}{a^2d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="maxima")
```

```
[Out] -1/((a*b*e^(-d*x - c) + a^2)*d) + (d*x + c)/(a^2*d) + log(b*e^(-d*x - c) +
a)/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{-c-dx})^2}\,dx = -\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(-dx-c)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(-dx-c)}+a)ab}\right)}{d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="giac")
```

```
[Out] -b*(log(abs(-a/(b*e^(-d*x - c) + a) + 1))/(a^2*b) + 1/((b*e^(-d*x - c) + a)
*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a+be^{-c-dx})^2}\,dx = \frac{\frac{x}{a} + \frac{b\,\mathrm{e}^{-c-d\,x}}{a^2\,d} + \frac{b\,x\,\mathrm{e}^{-c-d\,x}}{a^2}}{a+b\,\mathrm{e}^{-c-d\,x}} + \frac{\ln\left(a+b\,\mathrm{e}^{-c}\,\mathrm{e}^{-d\,x}\right)}{a^2\,d}$$

```
[In] int(1/(a + b*exp(- c - d*x))^2,x)
```

```
[Out] (x/a + (b*exp(- c - d*x))/(a^2*d) + (b*x*exp(- c - d*x))/a^2)/(a + b*exp(-
c - d*x)) + log(a + b*exp(-c)*exp(-d*x))/(a^2*d)
```

3.17 $\int \frac{x^3}{(a+be^{c+dx})^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 333

$$\begin{aligned}\int \frac{x^3}{(a+be^{c+dx})^3}\,dx &= \frac{3x^2}{2a^3d^2} - \frac{3x^2}{2a^2d^2(a+be^{c+dx})} - \frac{3x^3}{2a^3d} + \frac{x^3}{2ad(a+be^{c+dx})^2} \\ &\quad + \frac{x^3}{a^2d(a+be^{c+dx})} + \frac{x^4}{4a^3} - \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^3} \\ &\quad + \frac{9x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{2a^3d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} \\ &\quad - \frac{3\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^4} + \frac{9x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^3} \\ &\quad - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{9\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^4} \\ &\quad + \frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{6\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^3d^4}\end{aligned}$$

```
[Out] 3/2*x^2/a^3/d^2-3/2*x^2/a^2/d^2/(a+b*exp(d*x+c))-3/2*x^3/a^3/d+1/2*x^3/a/d/
(a+b*exp(d*x+c))^2+x^3/a^2/d/(a+b*exp(d*x+c))+1/4*x^4/a^3-3*x*ln(1+b*exp(d*
x+c)/a)/a^3/d^3+9/2*x^2*ln(1+b*exp(d*x+c)/a)/a^3/d^2-x^3*ln(1+b*exp(d*x+c)/
a)/a^3/d-3*polylog(2,-b*exp(d*x+c)/a)/a^3/d^4+9*x*polylog(2,-b*exp(d*x+c)/a
)/a^3/d^3-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^3/d^2-9*polylog(3,-b*exp(d*x+c
)/a)/a^3/d^4+6*x*polylog(3,-b*exp(d*x+c)/a)/a^3/d^3-6*polylog(4,-b*exp(d*x+
c)/a)/a^3/d^4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222, 2317, 2438}

$$\int \frac{x^3}{(a+be^{c+dx})^3}\,dx = -\frac{3\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^4} - \frac{9\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^4} - \frac{6\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^3d^4} + \frac{9x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{3x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d^3} - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{9x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{2a^3d^2} - \frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d} + \frac{3x^2}{2a^3d^2} - \frac{3x^3}{2a^3d} + \frac{x^4}{4a^3} - \frac{3x^2}{2a^2d^2\left(a+be^{c+dx}\right)} + \frac{x^3}{a^2d\left(a+be^{c+dx}\right)} + \frac{x^3}{2ad\left(a+be^{c+dx}\right)^2}$$

```
[In] Int[x^3/(a + b*E^(c + d*x))^3,x]
```

```
[Out] (3*x^2)/(2*a^3*d^2) - (3*x^2)/(2*a^2*d^2*(a + b*E^(c + d*x))) - (3*x^3)/(2*
a^3*d) + x^3/(2*a*d*(a + b*E^(c + d*x))^2) + x^3/(a^2*d*(a + b*E^(c + d*x))
) + x^4/(4*a^3) - (3*x*Log[1 + (b*E^(c + d*x))/a])/(a^3*d^3) + (9*x^2*Log[1
 + (b*E^(c + d*x))/a])/(2*a^3*d^2) - (x^3*Log[1 + (b*E^(c + d*x))/a])/(a^3*
d) - (3*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^3*d^4) + (9*x*PolyLog[2, -((b*
E^(c + d*x))/a)])/(a^3*d^3) - (3*x^2*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^3
*d^2) - (9*PolyLog[3, -((b*E^(c + d*x))/a)])/(a^3*d^4) + (6*x*PolyLog[3, -(
(b*E^(c + d*x))/a)])/(a^3*d^3) - (6*PolyLog[4, -((b*E^(c + d*x))/a)])/(a^3*
d^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
```

```
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{x^3}{(a+be^{c+dx})^2}\,dx}{a} - \frac{b\int \frac{e^{c+dx}x^3}{(a+be^{c+dx})^3}\,dx}{a} \\
&= \frac{x^3}{2ad\,(a+be^{c+dx})^2} + \frac{\int \frac{x^3}{a+be^{c+dx}}\,dx}{a^2} - \frac{b\int \frac{e^{c+dx}x^3}{(a+be^{c+dx})^2}\,dx}{a^2} - \frac{3\int \frac{x^2}{(a+be^{c+dx})^2}\,dx}{2ad} \\
&= \frac{x^3}{2ad\,(a+be^{c+dx})^2} + \frac{x^3}{a^2d\,(a+be^{c+dx})} + \frac{x^4}{4a^3} - \frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}}\,dx}{a^3} \\
&\quad - \frac{3\int \frac{x^2}{a+be^{c+dx}}\,dx}{2a^2d} - \frac{3\int \frac{x^2}{a+be^{c+dx}}\,dx}{a^2d} + \frac{(3b)\int \frac{e^{c+dx}x^2}{(a+be^{c+dx})^2}\,dx}{2a^2d} \\
&= -\frac{3x^2}{2a^2d^2\,(a+be^{c+dx})} - \frac{3x^3}{2a^3d} + \frac{x^3}{2ad\,(a+be^{c+dx})^2} \\
&\quad + \frac{x^3}{a^2d\,(a+be^{c+dx})} + \frac{x^4}{4a^3} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} + \frac{3\int \frac{x}{a+be^{c+dx}}\,dx}{a^2d^2} \\
&\quad + \frac{3\int x^2\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d} + \frac{(3b)\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{2a^3d} + \frac{(3b)\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{a^3d} \\
&= \frac{3x^2}{2a^3d^2} - \frac{3x^2}{2a^2d^2\,(a+be^{c+dx})} - \frac{3x^3}{2a^3d} + \frac{x^3}{2ad\,(a+be^{c+dx})^2} \\
&\quad + \frac{x^3}{a^2d\,(a+be^{c+dx})} + \frac{x^4}{4a^3} + \frac{9x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{2a^3d^2} \\
&\quad - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{3\int x\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d^2} \\
&\quad - \frac{6\int x\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d^2} + \frac{6\int x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)\,dx}{a^3d^2} - \frac{(3b)\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a^3d^2}
\end{aligned}
$$

$$= \frac{3x^2}{2a^3d^2} - \frac{3x^2}{2a^2d^2(a+be^{c+dx})} - \frac{3x^3}{2a^3d} + \frac{x^3}{2ad(a+be^{c+dx})^2} + \frac{x^3}{a^2d(a+be^{c+dx})} + \frac{x^4}{4a^3} - \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{9x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{2a^3d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} + \frac{9x\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{6x\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{3\int\log\left(1+\frac{be^{c+dx}}{a}\right)dx}{a^3d^3} - \frac{3\int\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)dx}{a^3d^3} - \frac{6\int\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)dx}{a^3d^3} - \frac{6\int\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a}\right)dx}{a^3d^3}$$

$$= \frac{3x^2}{2a^3d^2} - \frac{3x^2}{2a^2d^2(a+be^{c+dx})} - \frac{3x^3}{2a^3d} + \frac{x^3}{2ad(a+be^{c+dx})^2} + \frac{x^3}{a^2d(a+be^{c+dx})} + \frac{x^4}{4a^3} - \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{9x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{2a^3d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} + \frac{9x\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{6x\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{3\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}dx, x, e^{c+dx}\right)}{a^3d^4} - \frac{3\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{x}dx, x, e^{c+dx}\right)}{a^3d^4} - \frac{6\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{x}dx, x, e^{c+dx}\right)}{a^3d^4} - \frac{6\mathrm{Subst}\left(\int\frac{\mathrm{Li}_3\left(-\frac{bx}{a}\right)}{x}dx, x, e^{c+dx}\right)}{a^3d^4}$$

$$= \frac{3x^2}{2a^3d^2} - \frac{3x^2}{2a^2d^2(a+be^{c+dx})} - \frac{3x^3}{2a^3d} + \frac{x^3}{2ad(a+be^{c+dx})^2} + \frac{x^3}{a^2d(a+be^{c+dx})} + \frac{x^4}{4a^3} - \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^3} + \frac{9x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{2a^3d^2} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{3\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4} + \frac{9x\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{9\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4} + \frac{6x\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{6\mathrm{Li}_4\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^4}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(a+be^{c+dx})^3}\,dx = \frac{\frac{6x^2}{d^2} - \frac{6ax^2}{d^2(a+be^{c+dx})} - \frac{6x^3}{d} + \frac{2a^2x^3}{d(a+be^{c+dx})^2} + \frac{4ax^3}{ad+bde^{c+dx}} + x^4 - \frac{12x\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^3} + \frac{18x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^2} - \frac{4x^3\log\left(1+\frac{be}{}\right.}{d}}{4a^3}$$

```
[In] Integrate[x^3/(a + b*E^(c + d*x))^3,x]
```

```
[Out] ((6*x^2)/d^2 - (6*a*x^2)/(d^2*(a + b*E^(c + d*x))) - (6*x^3)/d + (2*a^2*x^3
)/(d*(a + b*E^(c + d*x))^2) + (4*a*x^3)/(a*d + b*d*E^(c + d*x)) + x^4 - (12
*x*Log[1 + (b*E^(c + d*x))/a])/d^3 + (18*x^2*Log[1 + (b*E^(c + d*x))/a])/d^
2 - (4*x^3*Log[1 + (b*E^(c + d*x))/a])/d - (12*(1 - 3*d*x + d^2*x^2)*PolyLo
g[2, -((b*E^(c + d*x))/a)])/d^4 + (12*(-3 + 2*d*x)*PolyLog[3, -((b*E^(c + d
*x))/a)])/d^4 - (24*PolyLog[4, -((b*E^(c + d*x))/a)])/d^4)/(4*a^3)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.65

method	result
risch	$\frac{x^2(2xbd\,e^{dx+c}+3xad-3b\,e^{dx+c}-3a)}{2a^2d^2(a+b\,e^{dx+c})^2}-\frac{x^3\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{a^3d}+\frac{9c^2x}{2d^3a^3}+\frac{c^3x}{d^3a^3}+\frac{3cx}{d^3a^3}+\frac{3c\ln(a+b\,e^{dx+c})}{d^4a^3}-\frac{3c\ln(e^{dx+c})}{d^4a^3}+$

```
[In] int(x^3/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*(2*x*b*d*exp(d*x+c)+3*x*a*d-3*b*exp(d*x+c)-3*a)/a^2/d^2/(a+b*exp(d*
x+c))^2-x^3*ln(1+b*exp(d*x+c)/a)/a^3/d+9/2/d^3/a^3*c^2*x+1/d^3/a^3*c^3*x+3/
d^3/a^3*c*x+3/d^4/a^3*c*ln(a+b*exp(d*x+c))-3/d^4/a^3*c*ln(exp(d*x+c))+9/2/d
^4/a^3*c^2*ln(a+b*exp(d*x+c))-9/2/d^4/a^3*c^2*ln(exp(d*x+c))+1/d^4/a^3*c^3*
ln(a+b*exp(d*x+c))-1/d^4/a^3*c^3*ln(exp(d*x+c))-3/d^4/a^3*ln(1+b*exp(d*x+c)
/a)*c-9/2/d^4/a^3*ln(1+b*exp(d*x+c)/a)*c^2-1/d^4/a^3*ln(1+b*exp(d*x+c)/a)*c
^3+9*x*polylog(2,-b*exp(d*x+c)/a)/a^3/d^3-3*x^2*polylog(2,-b*exp(d*x+c)/a)/
a^3/d^2+6*x*polylog(3,-b*exp(d*x+c)/a)/a^3/d^3-3*x*ln(1+b*exp(d*x+c)/a)/a^3
/d^3+9/2*x^2*ln(1+b*exp(d*x+c)/a)/a^3/d^2+1/4*x^4/a^3+3/2/d^4/a^3*c^2+3/d^4
/a^3*c^3+3/4/d^4/a^3*c^4+3/2*x^2/a^3/d^2-3/2*x^3/a^3/d-3*polylog(2,-b*exp(d
*x+c)/a)/a^3/d^4-9*polylog(3,-b*exp(d*x+c)/a)/a^3/d^4-6*polylog(4,-b*exp(d*
x+c)/a)/a^3/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(306) = 612.

Time = 0.27 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.11

$$\int \frac{x^3}{(a+be^{c+dx})^3}\,dx$$
$$=\frac{a^2d^4x^4-a^2c^4-6\,a^2c^3-6\,a^2c^2-12\left(a^2d^2x^2-3\,a^2dx+a^2+(b^2d^2x^2-3\,b^2dx+b^2)e^{(2\,dx+2\,c)}+2\,(abd^2x^2-\right.}{}$$

```
[In] integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d^4*x^4 - a^2*c^4 - 6*a^2*c^3 - 6*a^2*c^2 - 12*(a^2*d^2*x^2 - 3*a^
2*d*x + a^2 + (b^2*d^2*x^2 - 3*b^2*d*x + b^2)*e^(2*d*x + 2*c) + 2*(a*b*d^2*
```

```
x^2 - 3*a*b*d*x + a*b)*e^(d*x + c))*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b^
2*d^4*x^4 - 6*b^2*d^3*x^3 - b^2*c^4 + 6*b^2*d^2*x^2 - 6*b^2*c^3 - 6*b^2*c^2
)*e^(2*d*x + 2*c) + 2*(a*b*d^4*x^4 - 4*a*b*d^3*x^3 - a*b*c^4 + 3*a*b*d^2*x^
2 - 6*a*b*c^3 - 6*a*b*c^2)*e^(d*x + c) + 2*(2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*c
 + (2*b^2*c^3 + 9*b^2*c^2 + 6*b^2*c)*e^(2*d*x + 2*c) + 2*(2*a*b*c^3 + 9*a*b
*c^2 + 6*a*b*c)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 2*(2*a^2*d^3*x^3 - 9*
a^2*d^2*x^2 + 2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*d*x + 6*a^2*c + (2*b^2*d^3*x^3
- 9*b^2*d^2*x^2 + 2*b^2*c^3 + 9*b^2*c^2 + 6*b^2*d*x + 6*b^2*c)*e^(2*d*x + 2
*c) + 2*(2*a*b*d^3*x^3 - 9*a*b*d^2*x^2 + 2*a*b*c^3 + 9*a*b*c^2 + 6*a*b*d*x
+ 6*a*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) - 24*(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + a^2)*polylog(4, -b*e^(d*x + c)/a) + 12*(2*a^2*d*x -
 3*a^2 + (2*b^2*d*x - 3*b^2)*e^(2*d*x + 2*c) + 2*(2*a*b*d*x - 3*a*b)*e^(d*x
 + c))*polylog(3, -b*e^(d*x + c)/a))/(a^3*b^2*d^4*e^(2*d*x + 2*c) + 2*a^4*b
*d^4*e^(d*x + c) + a^5*d^4)
```

Sympy [F]

$$\int \frac{x^3}{(a+be^{c+dx})^3}\,dx = \frac{3adx^3 - 3ax^2 + (2bdx^3 - 3bx^2)\,e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \frac{6x}{a+be^ce^{dx}}\,dx + \int \left(-\frac{9dx^2}{a+be^ce^{dx}}\right)dx + \int \frac{2d^2x^3}{a+be^ce^{dx}}\,dx}{2a^2d^2}$$

```
[In] integrate(x**3/(a+b*exp(d*x+c))**3,x)
```

```
[Out] (3*a*d*x**3 - 3*a*x**2 + (2*b*d*x**3 - 3*b*x**2)*exp(c + d*x))/(2*a**4*d**2
 + 4*a**3*b*d**2*exp(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integ
ral(6*x/(a + b*exp(c)*exp(d*x)), x) + Integral(-9*d*x**2/(a + b*exp(c)*exp(
d*x)), x) + Integral(2*d**2*x**3/(a + b*exp(c)*exp(d*x)), x))/(2*a**2*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{3\,adx^3-3\,ax^2+\left(2\,bdx^3e^c-3\,bx^2e^c\right)e^{(dx)}}{2\left(a^2b^2d^2e^{(2\,dx+2\,c)}+2\,a^3bd^2e^{(dx+c)}+a^4d^2\right)}+\frac{d^4x^4-6\,d^3x^3+6\,d^2x^2}{4\,a^3d^4} - \frac{d^3x^3\log\left(\frac{be^{(dx+c)}}{a}+1\right)+3\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)-6\,dx\mathrm{Li}_3\left(-\frac{be^{(dx+c)}}{a}\right)+6\,\mathrm{Li}_4\left(-\frac{be^{(dx+c)}}{a}\right)}{a^3d^4} + \frac{9\left(d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right)+2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)-2\,\mathrm{Li}_3\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{2\,a^3d^4} - \frac{3\left(dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^3d^4}$$

```
[In] integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*d*x^3 - 3*a*x^2 + (2*b*d*x^3*e^c - 3*b*x^2*e^c)*e^(d*x))/(a^2*b^2*
d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + a^4*d^2) + 1/4*(d^4*x^4 - 6
*d^3*x^3 + 6*d^2*x^2)/(a^3*d^4) - (d^3*x^3*log(b*e^(d*x + c)/a + 1) + 3*d^2
*x^2*dilog(-b*e^(d*x + c)/a) - 6*d*x*polylog(3, -b*e^(d*x + c)/a) + 6*polyl
og(4, -b*e^(d*x + c)/a))/(a^3*d^4) + 9/2*(d^2*x^2*log(b*e^(d*x + c)/a + 1)
+ 2*d*x*dilog(-b*e^(d*x + c)/a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a^3*d^4)
 - 3*(d*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^3*d^4)
```

Giac [F]

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \int \frac{x^3}{\left(be^{(dx+c)}+a\right)^3}\,dx$$

```
[In] integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*e^(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+be^{c+dx})^3}\,dx = \int \frac{x^3}{\left(a+b\,\mathrm{e}^{c+d\,x}\right)^3}\,dx$$

```
[In] int(x^3/(a + b*exp(c + d*x))^3,x)
[Out] int(x^3/(a + b*exp(c + d*x))^3, x)
```

3.18 $\int \frac{x^2}{(a+be^{c+dx})^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 243

$$\int \frac{x^2}{(a+be^{c+dx})^3} dx = \frac{x}{a^3d^2} - \frac{x}{a^2d^2(a+be^{c+dx})} - \frac{3x^2}{2a^3d} + \frac{x^2}{2ad(a+be^{c+dx})^2} + \frac{x^2}{a^2d(a+be^{c+dx})} + \frac{x^3}{3a^3} - \frac{\log(a+be^{c+dx})}{a^3d^3} + \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} + \frac{3\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^3}$$

```
[Out] x/a^3/d^2-x/a^2/d^2/(a+b*exp(d*x+c))-3/2*x^2/a^3/d+1/2*x^2/a/d/(a+b*exp(d*x
+c))^2+x^2/a^2/d/(a+b*exp(d*x+c))+1/3*x^3/a^3-ln(a+b*exp(d*x+c))/a^3/d^3+3*
x*ln(1+b*exp(d*x+c)/a)/a^3/d^2-x^2*ln(1+b*exp(d*x+c)/a)/a^3/d+3*polylog(2,-
b*exp(d*x+c)/a)/a^3/d^3-2*x*polylog(2,-b*exp(d*x+c)/a)/a^3/d^2+2*polylog(3,
-b*exp(d*x+c)/a)/a^3/d^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules

used = {2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438, 36, 29, 31}

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \frac{3\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^3}+\frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{\log\left(a+be^{c+dx}\right)}{a^3d^3}-\frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{3x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d^2}-\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d}+\frac{x}{a^3d^2}-\frac{3x^2}{2a^3d}+\frac{x^3}{3a^3} - \frac{x}{a^2d^2\left(a+be^{c+dx}\right)}+\frac{x^2}{a^2d\left(a+be^{c+dx}\right)}+\frac{x^2}{2ad\left(a+be^{c+dx}\right)^2}$$

```
[In] Int[x^2/(a + b*E^(c + d*x))^3,x]
```

```
[Out] x/(a^3*d^2) - x/(a^2*d^2*(a + b*E^(c + d*x))) - (3*x^2)/(2*a^3*d) + x^2/(2*
a*d*(a + b*E^(c + d*x))^2) + x^2/(a^2*d*(a + b*E^(c + d*x))) + x^3/(3*a^3)
- Log[a + b*E^(c + d*x)]/(a^3*d^3) + (3*x*Log[1 + (b*E^(c + d*x))/a])/(a^3*
d^2) - (x^2*Log[1 + (b*E^(c + d*x))/a])/(a^3*d) + (3*PolyLog[2, -((b*E^(c +
 d*x))/a)])/(a^3*d^3) - (2*x*PolyLog[2, -((b*E^(c + d*x))/a)])/(a^3*d^2) +
(2*PolyLog[3, -((b*E^(c + d*x))/a)])/(a^3*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
```

```
f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^3}\,dx}{a} \\
&= \frac{x^2}{2ad\left(a+be^{c+dx}\right)^2} + \frac{\int \frac{x^2}{a+be^{c+dx}}\,dx}{a^2} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}\,dx}{a^2} - \frac{\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx}{ad} \\
&= \frac{x^2}{2ad\left(a+be^{c+dx}\right)^2} + \frac{x^2}{a^2d\left(a+be^{c+dx}\right)} + \frac{x^3}{3a^3} - \frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{a^3} \\
&\quad - \frac{\int \frac{x}{a+be^{c+dx}}\,dx}{a^2d} - \frac{2\int \frac{x}{a+be^{c+dx}}\,dx}{a^2d} + \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}\,dx}{a^2d} \\
&= -\frac{x}{a^2d^2\left(a+be^{c+dx}\right)} - \frac{3x^2}{2a^3d} + \frac{x^2}{2ad\left(a+be^{c+dx}\right)^2} \\
&\quad + \frac{x^2}{a^2d\left(a+be^{c+dx}\right)} + \frac{x^3}{3a^3} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} + \frac{\int \frac{1}{a+be^{c+dx}}\,dx}{a^2d^2} \\
&\quad + \frac{2\int x\log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d} + \frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a^3d} + \frac{(2b)\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a^3d} \\
&= -\frac{x}{a^2d^2\left(a+be^{c+dx}\right)} - \frac{3x^2}{2a^3d} + \frac{x^2}{2ad\left(a+be^{c+dx}\right)^2} + \frac{x^2}{a^2d\left(a+be^{c+dx}\right)} \\
&\quad + \frac{x^3}{3a^3} + \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{c+dx}\right)}{a^2d^3} - \frac{\int \log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d^2} \\
&\quad - \frac{2\int \log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d^2} + \frac{2\int \text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)\,dx}{a^3d^2}
\end{aligned}
$$

$$= -\frac{x}{a^2d^2\left(a+be^{c+dx}\right)} - \frac{3x^2}{2a^3d} + \frac{x^2}{2ad\left(a+be^{c+dx}\right)^2} + \frac{x^2}{a^2d\left(a+be^{c+dx}\right)} + \frac{x^3}{3a^3} + \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{c+dx}\right)}{a^3d^3} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^3d^3} - \frac{2\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^3d^3} + \frac{2\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^3d^3} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{c+dx}\right)}{a^3d^3}$$

$$= \frac{x}{a^3d^2} - \frac{x}{a^2d^2\left(a+be^{c+dx}\right)} - \frac{3x^2}{2a^3d} + \frac{x^2}{2ad\left(a+be^{c+dx}\right)^2} + \frac{x^2}{a^2d\left(a+be^{c+dx}\right)} + \frac{x^3}{3a^3} - \frac{\log\left(a+be^{c+dx}\right)}{a^3d^3} + \frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^2} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} + \frac{3\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3} - \frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2} + \frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx$$

$$= \frac{\frac{6x}{d^2} - \frac{6ax}{d^2\left(a+be^{c+dx}\right)} - \frac{9x^2}{d} + \frac{3a^2x^2}{d\left(a+be^{c+dx}\right)^2} + \frac{6ax^2}{ad+bde^{c+dx}} + 2x^3 - \frac{6\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^3} + \frac{18x\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^2} - \frac{6x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{d}}{6a^3}$$

```
[In] Integrate[x^2/(a + b*E^(c + d*x))^3,x]
```

```
[Out] ((6*x)/d^2 - (6*a*x)/(d^2*(a + b*E^(c + d*x))) - (9*x^2)/d + (3*a^2*x^2)/(d
*(a + b*E^(c + d*x))^2) + (6*a*x^2)/(a*d + b*d*E^(c + d*x)) + 2*x^3 - (6*Lo
g[1 + (b*E^(c + d*x))/a])/d^3 + (18*x*Log[1 + (b*E^(c + d*x))/a])/d^2 - (6*
x^2*Log[1 + (b*E^(c + d*x))/a])/d - (6*(-3 + 2*d*x)*PolyLog[2, -((b*E^(c +
d*x))/a)])/d^3 + (12*PolyLog[3, -((b*E^(c + d*x))/a)])/d^3)/(6*a^3)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.58

method	result
risch	$\frac{x\left(2xbd\,\mathrm{e}^{dx+c}+3xad-2b\,\mathrm{e}^{dx+c}-2a\right)}{2a^2d^2\left(a+b\,\mathrm{e}^{dx+c}\right)^2}-\frac{3c\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{d^3a^3}+\frac{3c\ln\left(\mathrm{e}^{dx+c}\right)}{d^3a^3}+\frac{x^3}{3a^3}+\frac{\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)c^2}{d^3a^3}-\frac{3x^2}{2a^3d}+\frac{3\ln\left(1+\frac{b\,\mathrm{e}^{dx}}{a}\right.}{d^3a^3}$

```
[In] int(x^2/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(2*x*b*d*exp(d*x+c)+3*x*a*d-2*b*exp(d*x+c)-2*a)/a^2/d^2/(a+b*exp(d*x+
c))^2-3/d^3/a^3*c*ln(a+b*exp(d*x+c))+3/d^3/a^3*c*ln(exp(d*x+c))+1/3*x^3/a^3
+1/d^3/a^3*ln(1+b*exp(d*x+c)/a)*c^2-3/2*x^2/a^3/d+3/d^3/a^3*ln(1+b*exp(d*x+
c)/a)*c-3/d^2/a^3*c*x+3*x*ln(1+b*exp(d*x+c)/a)/a^3/d^2-1/d^2/a^3*c^2*x-x^2*
ln(1+b*exp(d*x+c)/a)/a^3/d-2*x*polylog(2,-b*exp(d*x+c)/a)/a^3/d^2-ln(a+b*ex
p(d*x+c))/a^3/d^3+1/d^3/a^3*ln(exp(d*x+c))-2/3/d^3/a^3*c^3+2*polylog(3,-b*e
xp(d*x+c)/a)/a^3/d^3-3/2/d^3/a^3*c^2-1/d^3/a^3*c^2*ln(a+b*exp(d*x+c))+1/d^3
/a^3*c^2*ln(exp(d*x+c))+3*polylog(2,-b*exp(d*x+c)/a)/a^3/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(226) = 452$.

Time = 0.26 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.14

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx$$
$$= \frac{2\,a^2d^3x^3 + 2\,a^2c^3 + 9\,a^2c^2 + 6\,a^2c - 6\,\left(2\,a^2dx - 3\,a^2 + (2\,b^2dx - 3\,b^2)e^{(2\,dx+2\,c)} + 2\,(2\,abdx - 3\,ab)e^{(dx+c)}\right)}{}$$

```
[In] integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*d^3*x^3 + 2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*c - 6*(2*a^2*d*x - 3*a^2
 + (2*b^2*d*x - 3*b^2)*e^(2*d*x + 2*c) + 2*(2*a*b*d*x - 3*a*b)*e^(d*x + c))
*dilog(-(b*e^(d*x + c) + a)/a + 1) + (2*b^2*d^3*x^3 - 9*b^2*d^2*x^2 + 2*b^2
*c^3 + 9*b^2*c^2 + 6*b^2*d*x + 6*b^2*c)*e^(2*d*x + 2*c) + 2*(2*a*b*d^3*x^3
- 6*a*b*d^2*x^2 + 2*a*b*c^3 + 9*a*b*c^2 + 3*a*b*d*x + 6*a*b*c)*e^(d*x + c)
- 6*(a^2*c^2 + 3*a^2*c + a^2 + (b^2*c^2 + 3*b^2*c + b^2)*e^(2*d*x + 2*c) +
2*(a*b*c^2 + 3*a*b*c + a*b)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 6*(a^2*d^
2*x^2 - a^2*c^2 - 3*a^2*d*x - 3*a^2*c + (b^2*d^2*x^2 - b^2*c^2 - 3*b^2*d*x
- 3*b^2*c)*e^(2*d*x + 2*c) + 2*(a*b*d^2*x^2 - a*b*c^2 - 3*a*b*d*x - 3*a*b*c
)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) + 12*(b^2*e^(2*d*x + 2*c) + 2*a*b
*e^(d*x + c) + a^2)*polylog(3, -b*e^(d*x + c)/a))/(a^3*b^2*d^3*e^(2*d*x + 2
*c) + 2*a^4*b*d^3*e^(d*x + c) + a^5*d^3)
```

Sympy [F]

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \frac{3adx^2-2ax+(2bdx^2-2bx)\,e^{c+dx}}{2a^4d^2+4a^3bd^2e^{c+dx}+2a^2b^2d^2e^{2c+2dx}} + \frac{\int\left(-\frac{3dx}{a+be^ce^{dx}}\right)dx+\int\frac{d^2x^2}{a+be^ce^{dx}}\,dx+\int\frac{1}{a+be^ce^{dx}}\,dx}{a^2d^2}$$

```
[In] integrate(x**2/(a+b*exp(d*x+c))**3,x)
```

```
[Out] (3*a*d*x**2 - 2*a*x + (2*b*d*x**2 - 2*b*x)*exp(c + d*x))/(2*a**4*d**2 + 4*a
**3*b*d**2*exp(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integral(-3
*d*x/(a + b*exp(c)*exp(d*x)), x) + Integral(d**2*x**2/(a + b*exp(c)*exp(d*x
)), x) + Integral(1/(a + b*exp(c)*exp(d*x)), x))/(a**2*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \frac{3\,adx^2-2\,ax+2\,(bdx^2e^c-bxe^c)e^{(dx)}}{2\,(a^2b^2d^2e^{(2\,dx+2\,c)}+2\,a^3bd^2e^{(dx+c)}+a^4d^2)} + \frac{x}{a^3d^2} + \frac{2\,d^3x^3-9\,d^2x^2}{6\,a^3d^3} - \frac{d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right)+2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)-2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})}{a^3d^3} + \frac{3\left(dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^3d^3} - \frac{\log\left(be^{(dx+c)}+a\right)}{a^3d^3}$$

```
[In] integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*d*x^2 - 2*a*x + 2*(b*d*x^2*e^c - b*x*e^c)*e^(d*x))/(a^2*b^2*d^2*e^
(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + a^4*d^2) + x/(a^3*d^2) + 1/6*(2*d
^3*x^3 - 9*d^2*x^2)/(a^3*d^3) - (d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*d
ilog(-b*e^(d*x + c)/a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a^3*d^3) + 3*(d*x
*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^3*d^3) - log(b*e^(d
*x + c) + a)/(a^3*d^3)
```

Giac [F]

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \int \frac{x^2}{(be^{(dx+c)}+a)^3}\,dx$$

```
[In] integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="giac")
[Out] integrate(x^2/(b*e^(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \int \frac{x^2}{(a+b\,e^{c+d\,x})^3}\,dx$$

```
[In] int(x^2/(a + b*exp(c + d*x))^3,x)
[Out] int(x^2/(a + b*exp(c + d*x))^3, x)
```

3.19 $\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 159

$$\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx = -\frac{1}{2a^2d^2\left(a+be^{c+dx}\right)}-\frac{3x}{2a^3d}+\frac{x}{2ad\left(a+be^{c+dx}\right)^2}+\frac{x}{a^2d\left(a+be^{c+dx}\right)}+\frac{x^2}{2a^3}+\frac{3\log\left(a+be^{c+dx}\right)}{2a^3d^2}-\frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d}-\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2}$$

```
[Out] -1/2/a^2/d^2/(a+b*exp(d*x+c))-3/2*x/a^3/d+1/2*x/a/d/(a+b*exp(d*x+c))^2+x/a^
2/d/(a+b*exp(d*x+c))+1/2*x^2/a^3+3/2*ln(a+b*exp(d*x+c))/a^3/d^2-x*ln(1+b*ex
p(d*x+c)/a)/a^3/d-polylog(2,-b*exp(d*x+c)/a)/a^3/d^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {2216, 2215, 2221, 2317, 2438, 2222, 2320, 36, 29, 31, 46}

$$\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx = -\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2}+\frac{3\log\left(a+be^{c+dx}\right)}{2a^3d^2}-\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{a^3d}-\frac{3x}{2a^3d}+\frac{x^2}{2a^3}-\frac{1}{2a^2d^2\left(a+be^{c+dx}\right)}+\frac{x}{a^2d\left(a+be^{c+dx}\right)}+\frac{x}{2ad\left(a+be^{c+dx}\right)^2}$$

```
[In] Int[x/(a + b*E^(c + d*x))^3,x]
```

```
[Out] -1/2*1/(a^2*d^2*(a + b*E^(c + d*x))) - (3*x)/(2*a^3*d) + x/(2*a*d*(a + b*E^
(c + d*x))^2) + x/(a^2*d*(a + b*E^(c + d*x))) + x^2/(2*a^3) + (3*Log[a + b*
E^(c + d*x)])/(2*a^3*d^2) - (x*Log[1 + (b*E^(c + d*x))/a])/(a^3*d) - PolyLo
g[2, -((b*E^(c + d*x))/a)]/(a^3*d^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx}{a}-\frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^3}\,dx}{a}\\
&= \frac{x}{2ad\left(a+be^{c+dx}\right)^2}+\frac{\int \frac{x}{a+be^{c+dx}}\,dx}{a^2}-\frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}\,dx}{a^2}-\frac{\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx}{2ad}\\
&= \frac{x}{2ad\left(a+be^{c+dx}\right)^2}+\frac{x}{a^2d\left(a+be^{c+dx}\right)}+\frac{x^2}{2a^3}-\frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}\,dx}{a^3}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx,x,e^{c+dx}\right)}{2ad^2}-\frac{\int \frac{1}{a+be^{c+dx}}\,dx}{a^2d}\\
&= \frac{x}{2ad\left(a+be^{c+dx}\right)^2}+\frac{x}{a^2d\left(a+be^{c+dx}\right)}+\frac{x^2}{2a^3}\\
&\quad-\frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d}-\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,e^{c+dx}\right)}{a^2d^2}\\
&\quad-\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx,x,e^{c+dx}\right)}{2ad^2}+\frac{\int \log\left(1+\frac{be^{c+dx}}{a}\right)\,dx}{a^3d}
\end{aligned}$$

$$= -\frac{1}{2a^2d^2(a+be^{c+dx})} - \frac{x}{2a^3d} + \frac{x}{2ad(a+be^{c+dx})^2} + \frac{x}{a^2d(a+be^{c+dx})} + \frac{x^2}{2a^3} + \frac{\log(a+be^{c+dx})}{2a^3d^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{c+dx}\right)}{a^3d^2} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^{c+dx}\right)}{a^3d^2} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{c+dx}\right)}{a^3d^2}$$

$$= -\frac{1}{2a^2d^2(a+be^{c+dx})} - \frac{3x}{2a^3d} + \frac{x}{2ad(a+be^{c+dx})^2} + \frac{x}{a^2d(a+be^{c+dx})} + \frac{x^2}{2a^3} + \frac{3\log(a+be^{c+dx})}{2a^3d^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a}\right)}{a^3d^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \frac{\frac{adx}{(a+be^{c+dx})^2} + \frac{-1+2dx}{a+be^{c+dx}} + \frac{-3dx+3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a} + \frac{dx\left(dx-2\log\left(1+\frac{be^{c+dx}}{a}\right)\right)-2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a}}{2a^2d^2}$$

```
[In] Integrate[x/(a + b*E^(c + d*x))^3,x]
```

```
[Out] ((a*d*x)/(a + b*E^(c + d*x))^2 + (-1 + 2*d*x)/(a + b*E^(c + d*x)) + (-3*d*x
 + 3*Log[1 + (b*E^(c + d*x))/a])/a + (d*x*(d*x - 2*Log[1 + (b*E^(c + d*x))/
a]) - 2*PolyLog[2, -((b*E^(c + d*x))/a)])/a)/(2*a^2*d^2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36

method	result
risch	$\frac{2xbd\,\mathrm{e}^{dx+c}+3xad-b\,\mathrm{e}^{dx+c}-a}{2a^2d^2(a+b\,\mathrm{e}^{dx+c})^2}+\frac{3\ln(a+b\,\mathrm{e}^{dx+c})}{2a^3d^2}-\frac{3\ln(\mathrm{e}^{dx+c})}{2d^2a^3}+\frac{c\ln(a+b\,\mathrm{e}^{dx+c})}{d^2a^3}-\frac{c\ln(\mathrm{e}^{dx+c})}{d^2a^3}+\frac{x^2}{2a^3}+\frac{c}{d}$
derivativedivides	$\frac{-\frac{\mathrm{dilog}\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3}-\frac{(dx+c)\ln\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3}+\frac{3\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{2a^3}-\frac{1}{2a^2\left(a+b\,\mathrm{e}^{dx+c}\right)}-\frac{b^2(dx+c)\mathrm{e}^{2dx+2c}}{2a^3\left(a+b\,\mathrm{e}^{dx+c}\right)^2}-\frac{b(dx+c)\mathrm{e}^{dx+c}}{a^2\left(a+b\,\mathrm{e}^{dx+c}\right)^2}-}{d^2}$
default	$\frac{-\frac{\mathrm{dilog}\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3}-\frac{(dx+c)\ln\left(\frac{a+b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3}+\frac{3\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{2a^3}-\frac{1}{2a^2\left(a+b\,\mathrm{e}^{dx+c}\right)}-\frac{b^2(dx+c)\mathrm{e}^{2dx+2c}}{2a^3\left(a+b\,\mathrm{e}^{dx+c}\right)^2}-\frac{b(dx+c)\mathrm{e}^{dx+c}}{a^2\left(a+b\,\mathrm{e}^{dx+c}\right)^2}-}{d^2}$

```
[In] int(x/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*x*b*d*exp(d*x+c)+3*x*a*d-b*exp(d*x+c)-a)/a^2/d^2/(a+b*exp(d*x+c))^2+
3/2*ln(a+b*exp(d*x+c))/a^3/d^2-3/2/d^2/a^3*ln(exp(d*x+c))+1/d^2/a^3*c*ln(a+
b*exp(d*x+c))-1/d^2/a^3*c*ln(exp(d*x+c))+1/2*x^2/a^3+1/d/a^3*c*x+1/2/d^2/a^
3*c^2-x*ln(1+b*exp(d*x+c)/a)/a^3/d-1/d^2/a^3*ln(1+b*exp(d*x+c)/a)*c-polylog
(2,-b*exp(d*x+c)/a)/a^3/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(142) = 284$.

Time = 0.26 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.13

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx$$
$$= \frac{a^2d^2x^2 - a^2c^2 - 3\,a^2c - a^2 - 2\left(b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + a^2\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a} + 1\right) + \left(b^2d^2x^2 - b^2c^2 - 3\,b^2\right.}{}$$

```
[In] integrate(x/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*x^2 - a^2*c^2 - 3*a^2*c - a^2 - 2*(b^2*e^(2*d*x + 2*c) + 2*a*b
*e^(d*x + c) + a^2)*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b^2*d^2*x^2 - b^2*
c^2 - 3*b^2*d*x - 3*b^2*c)*e^(2*d*x + 2*c) + (2*a*b*d^2*x^2 - 2*a*b*c^2 - 4
*a*b*d*x - 6*a*b*c - a*b)*e^(d*x + c) + (2*a^2*c + 3*a^2 + (2*b^2*c + 3*b^2
)*e^(2*d*x + 2*c) + 2*(2*a*b*c + 3*a*b)*e^(d*x + c))*log(b*e^(d*x + c) + a)
 - 2*(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*e^(2*d*x + 2*c) + 2*(a*b*d*x + a*
b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a))/(a^3*b^2*d^2*e^(2*d*x + 2*c)
+ 2*a^4*b*d^2*e^(d*x + c) + a^5*d^2)
```

Sympy [F]

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx$$
$$= \frac{3adx - a + (2bdx - b)\,e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \frac{2dx}{a+be^ce^{dx}}\,dx + \int \left(-\frac{3}{a+be^ce^{dx}}\right)\,dx}{2a^2d}$$

```
[In] integrate(x/(a+b*exp(d*x+c))**3,x)
```

```
[Out] (3*a*d*x - a + (2*b*d*x - b)*exp(c + d*x))/(2*a**4*d**2 + 4*a**3*b*d**2*exp
(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integral(2*d*x/(a + b*exp
(c)*exp(d*x)), x) + Integral(-3/(a + b*exp(c)*exp(d*x)), x))/(2*a**2*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \frac{3\,adx + (2\,bdxe^c - be^c)e^{(dx)} - a}{2\,(a^2b^2d^2e^{(2\,dx+2\,c)} + 2\,a^3bd^2e^{(dx+c)} + a^4d^2)} + \frac{x^2}{2\,a^3} - \frac{3\,x}{2\,a^3d} - \frac{dx\log\left(\frac{be^{(dx+c)}}{a}+1\right) + \mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{a^3d^2} + \frac{3\,\log\left(be^{(dx+c)}+a\right)}{2\,a^3d^2}$$

```
[In] integrate(x/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(3*a*d*x + (2*b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*b^2*d^2*e^(2*d*x + 2
*c) + 2*a^3*b*d^2*e^(d*x + c) + a^4*d^2) + 1/2*x^2/a^3 - 3/2*x/(a^3*d) - (d
*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^3*d^2) + 3/2*log(
b*e^(d*x + c) + a)/(a^3*d^2)
```

Giac [F]

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \int \frac{x}{\left(be^{(dx+c)}+a\right)^3}\,dx$$

```
[In] integrate(x/(a+b*exp(d*x+c))^3,x, algorithm="giac")
[Out] integrate(x/(b*e^(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \int \frac{x}{\left(a+b\,\mathrm{e}^{c+d\,x}\right)^3}\,dx$$

```
[In] int(x/(a + b*exp(c + d*x))^3,x)
[Out] int(x/(a + b*exp(c + d*x))^3, x)
```

3.20 $\int \frac{1}{(a+be^{c+dx})^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{(a+be^{c+dx})^3} dx = \frac{1}{2ad(a+be^{c+dx})^2} + \frac{1}{a^2d(a+be^{c+dx})} + \frac{x}{a^3} - \frac{\log(a+be^{c+dx})}{a^3d}$$

```
[Out] 1/2/a/d/(a+b*exp(d*x+c))^2+1/a^2/d/(a+b*exp(d*x+c))+x/a^3-ln(a+b*exp(d*x+c)
)/a^3/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2320, 46}

$$\int \frac{1}{(a+be^{c+dx})^3} dx = -\frac{\log(a+be^{c+dx})}{a^3d} + \frac{x}{a^3} + \frac{1}{a^2d(a+be^{c+dx})} + \frac{1}{2ad(a+be^{c+dx})^2}$$

```
[In] Int[(a + b*E^(c + d*x))^(-3),x]
```

```
[Out] 1/(2*a*d*(a + b*E^(c + d*x))^2) + 1/(a^2*d*(a + b*E^(c + d*x))) + x/a^3 - L
og[a + b*E^(c + d*x)]/(a^3*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, e^{c+dx}\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right)\,dx, x, e^{c+dx}\right)}{d}\\ &= \frac{1}{2ad\left(a+be^{c+dx}\right)^2} + \frac{1}{a^2d\left(a+be^{c+dx}\right)} + \frac{x}{a^3} - \frac{\log\left(a+be^{c+dx}\right)}{a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{\frac{a\left(3a+2be^{c+dx}\right)}{\left(a+be^{c+dx}\right)^2} + 2\log\left(e^{c+dx}\right) - 2\log\left(a+be^{c+dx}\right)}{2a^3d}$$

```
[In] Integrate[(a + b*E^(c + d*x))^(-3),x]
```

```
[Out] ((a*(3*a + 2*b*E^(c + d*x)))/(a + b*E^(c + d*x))^2 + 2*Log[E^(c + d*x)] - 2
*Log[a + b*E^(c + d*x)])/(2*a^3*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^{3}}+\frac{1}{a^{2}\left(a+b\,e^{dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{dx+c}\right)^{2}}+\frac{\ln\left(e^{dx+c}\right)}{a^{3}}}{d}$
default	$\frac{-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^{3}}+\frac{1}{a^{2}\left(a+b\,e^{dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{dx+c}\right)^{2}}+\frac{\ln\left(e^{dx+c}\right)}{a^{3}}}{d}$
risch	$\frac{x}{a^{3}}+\frac{c}{a^{3}d}+\frac{2b\,e^{dx+c}+3a}{2a^{2}d\left(a+b\,e^{dx+c}\right)^{2}}-\frac{\ln\left(e^{dx+c}+\frac{a}{b}\right)}{a^{3}d}$
norman	$\frac{\frac{x}{a}+\frac{b^{2}x\,e^{2dx+2c}}{a^{3}}+\frac{2bx\,e^{dx+c}}{a^{2}}-\frac{2b\,e^{dx+c}}{a^{2}d}-\frac{3b^{2}e^{2dx+2c}}{2a^{3}d}}{\left(a+b\,e^{dx+c}\right)^{2}}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^{3}d}$
parallelrisch	$-\frac{-2\,e^{2dx+2c}b^{2}dx+2\ln\left(a+b\,e^{dx+c}\right)e^{2dx+2c}b^{2}-4\,e^{dx+c}abdx+4\ln\left(a+b\,e^{dx+c}\right)e^{dx+c}ab-2a^{2}dx+3b^{2}e^{2dx+2c}+2\ln\left(a+b\,e^{d\cdots\right.}}{2a^{3}d\left(a+b\,e^{dx+c}\right)^{2}}$

```
[In] int(1/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^3*ln(a+b*exp(d*x+c))+1/a^2/(a+b*exp(d*x+c))+1/2/a/(a+b*exp(d*x+c)
)^2+1/a^3*ln(exp(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.84

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{2\,b^2dxe^{(2\,dx+2\,c)}+2\,a^2dx+3\,a^2+2\,(2\,abdx+ab)e^{(dx+c)}-2\left(b^2e^{(2\,dx+2\,c)}+2\,abe^{(dx+c)}+a^2\right)\log\left(be^{(dx+c)}+\cdots\right.}{2\left(a^3b^2de^{(2\,dx+2\,c)}+2\,a^4bde^{(dx+c)}+a^5d\right)}$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d*x*e^(2*d*x + 2*c) + 2*a^2*d*x + 3*a^2 + 2*(2*a*b*d*x + a*b)*e^
(d*x + c) - 2*(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) + a^2)*log(b*e^(d*x
+ c) + a))/(a^3*b^2*d*e^(2*d*x + 2*c) + 2*a^4*b*d*e^(d*x + c) + a^5*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{3a+2be^{c+dx}}{2a^4d+4a^3bde^{c+dx}+2a^2b^2de^{2c+2dx}}+\frac{x}{a^3}-\frac{\log\left(\frac{a}{b}+e^{c+dx}\right)}{a^3d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))**3,x)
```

```
[Out] (3*a + 2*b*exp(c + d*x))/(2*a**4*d + 4*a**3*b*d*exp(c + d*x) + 2*a**2*b**2*
d*exp(2*c + 2*d*x)) + x/a**3 - log(a/b + exp(c + d*x))/(a**3*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+be^{c+dx})^3}\,dx = \frac{2\,be^{(dx+c)}+3\,a}{2\,(a^2b^2e^{(2\,dx+2\,c)}+2\,a^3be^{(dx+c)}+a^4)d}+\frac{dx+c}{a^3d}-\frac{\log\left(be^{(dx+c)}+a\right)}{a^3d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(2*b*e^(d*x + c) + 3*a)/((a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c)
 + a^4)*d) + (d*x + c)/(a^3*d) - log(b*e^(d*x + c) + a)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+be^{c+dx})^3}\,dx = \frac{\frac{2\,(dx+c)}{a^3}-\frac{2\,\log\left(\left|be^{(dx+c)}+a\right|\right)}{a^3}+\frac{2\,abe^{(dx+c)}+3\,a^2}{\left(be^{(dx+c)}+a\right)^2a^3}}{2\,d}$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3,x, algorithm="giac")
[Out] 1/2*(2*(d*x + c)/a^3 - 2*log(abs(b*e^(d*x + c) + a))/a^3 + (2*a*b*e^(d*x +
c) + 3*a^2)/((b*e^(d*x + c) + a)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{1}{(a+be^{c+dx})^3}\,dx = \frac{\frac{x}{a}+\frac{b^2\,x\,\mathrm{e}^{2\,c+2\,d\,x}}{a^3}+\frac{2\,b\,x\,\mathrm{e}^{c+d\,x}}{a^2}-\frac{3\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{2\,a^3\,d}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2\,d}}{a^2+2\,\mathrm{e}^{c+d\,x}\,a\,b+\mathrm{e}^{2\,c+2\,d\,x}\,b^2}-\frac{\ln\left(a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a^3\,d}$$

```
[In] int(1/(a + b*exp(c + d*x))^3,x)
[Out] (x/a + (b^2*x*exp(2*c + 2*d*x))/a^3 + (2*b*x*exp(c + d*x))/a^2 - (3*b^2*exp
(2*c + 2*d*x))/(2*a^3*d) - (2*b*exp(c + d*x))/(a^2*d))/(a^2 + b^2*exp(2*c +
 2*d*x) + 2*a*b*exp(c + d*x)) - log(a + b*exp(d*x)*exp(c))/(a^3*d)
```

3.21 $\int \frac{1}{\left(a+be^{c+dx}\right)^3x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^3 x},x\right)$$

```
[Out] Unintegrable(1/(a+b*exp(d*x+c))^3/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx$$

```
[In] Int[1/((a + b*E^(c + d*x))^3*x),x]
```

```
[Out] Defer[Int][1/((a + b*E^(c + d*x))^3*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{c+dx})^3\,x}\,dx=\int \frac{1}{(a+be^{c+dx})^3\,x}\,dx$$

```
[In] Integrate[1/((a + b*E^(c + d*x))^3*x),x]
[Out] Integrate[1/((a + b*E^(c + d*x))^3*x), x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)^3 x}dx$$

```
[In] int(1/(a+b*exp(d*x+c))^3/x,x)
[Out] int(1/(a+b*exp(d*x+c))^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.12

$$\int \frac{1}{(a+be^{c+dx})^3\,x}\,dx=\int \frac{1}{\left(be^{(dx+c)}+a\right)^3x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3/x,x, algorithm="fricas")
[Out] integral(1/(b^3*x*e^(3*d*x + 3*c) + 3*a*b^2*x*e^(2*d*x + 2*c) + 3*a^2*b*x*e
^(d*x + c) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.71

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \frac{3adx+a+(2bdx+b)\,e^{c+dx}}{2a^4d^2x^2+4a^3bd^2x^2e^{c+dx}+2a^2b^2d^2x^2e^{2c+2dx}} + \frac{\int \frac{3dx}{ax^3+bx^3e^ce^{dx}}\,dx+\int \frac{2d^2x^2}{ax^3+bx^3e^ce^{dx}}\,dx+\int \frac{2}{ax^3+bx^3e^ce^{dx}}\,dx}{2a^2d^2}$$

```
[In] integrate(1/(a+b*exp(d*x+c))**3/x,x)
```

```
[Out] (3*a*d*x + a + (2*b*d*x + b)*exp(c + d*x))/(2*a**4*d**2*x**2 + 4*a**3*b*d**
2*x**2*exp(c + d*x) + 2*a**2*b**2*d**2*x**2*exp(2*c + 2*d*x)) + (Integral(3
*d*x/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x) + Integral(2*d**2*x**2/(a*x**3 +
 b*x**3*exp(c)*exp(d*x)), x) + Integral(2/(a*x**3 + b*x**3*exp(c)*exp(d*x))
, x))/(2*a**2*d**2)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*d*x + (2*b*d*x*e^c + b*e^c)*e^(d*x) + a)/(a^2*b^2*d^2*x^2*e^(2*d*x
 + 2*c) + 2*a^3*b*d^2*x^2*e^(d*x + c) + a^4*d^2*x^2) + integrate(1/2*(2*d^2
*x^2 + 3*d*x + 2)/(a^2*b*d^2*x^3*e^(d*x + c) + a^3*d^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3/x,x, algorithm="giac")
```

```
[Out] integrate(1/((b*e^(d*x + c) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})^3 x} \, dx = \int \frac{1}{x \left(a+b\,\mathrm{e}^{c+d\,x}\right)^3} \, dx$$

```
[In] int(1/(x*(a + b*exp(c + d*x))^3),x)
[Out] int(1/(x*(a + b*exp(c + d*x))^3), x)
```

3.22 $\int \frac{1}{\left(a+be^{c+dx}\right)^3x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx=\operatorname{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^3 x^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*exp(d*x+c))^3/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx=\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx$$

```
[In] Int[1/((a + b*E^(c + d*x))^3*x^2),x]
```

```
[Out] Defer[Int][1/((a + b*E^(c + d*x))^3*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx$$

```
[In] Integrate[1/((a + b*E^(c + d*x))^3*x^2),x]
[Out] Integrate[1/((a + b*E^(c + d*x))^3*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)^3 x^2}dx$$

```
[In] int(1/(a+b*exp(d*x+c))^3/x^2,x)
[Out] int(1/(a+b*exp(d*x+c))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.59

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x^2}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3/x^2,x, algorithm="fricas")
[Out] integral(1/(b^3*x^2*e^(3*d*x + 3*c) + 3*a*b^2*x^2*e^(2*d*x + 2*c) + 3*a^2*b
*x^2*e^(d*x + c) + a^3*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.71

$$\int \frac{1}{(a+be^{c+dx})^3 x^2}\,dx = \frac{3adx+2a+(2bdx+2b)\,e^{c+dx}}{2a^4d^2x^3+4a^3bd^2x^3e^{c+dx}+2a^2b^2d^2x^3e^{2c+2dx}} + \frac{\int \frac{3dx}{ax^4+bx^4e^ce^{dx}}\,dx+\int \frac{d^2x^2}{ax^4+bx^4e^ce^{dx}}\,dx+\int \frac{3}{ax^4+bx^4e^ce^{dx}}\,dx}{a^2d^2}$$

```
[In] integrate(1/(a+b*exp(d*x+c))**3/x**2,x)
```

```
[Out] (3*a*d*x + 2*a + (2*b*d*x + 2*b)*exp(c + d*x))/(2*a**4*d**2*x**3 + 4*a**3*b
*d**2*x**3*exp(c + d*x) + 2*a**2*b**2*d**2*x**3*exp(2*c + 2*d*x)) + (Integr
al(3*d*x/(a*x**4 + b*x**4*exp(c)*exp(d*x)), x) + Integral(d**2*x**2/(a*x**4
 + b*x**4*exp(c)*exp(d*x)), x) + Integral(3/(a*x**4 + b*x**4*exp(c)*exp(d*x
)), x))/(a**2*d**2)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{1}{(a+be^{c+dx})^3 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x^2}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*d*x + 2*(b*d*x*e^c + b*e^c)*e^(d*x) + 2*a)/(a^2*b^2*d^2*x^3*e^(2*d
*x + 2*c) + 2*a^3*b*d^2*x^3*e^(d*x + c) + a^4*d^2*x^3) + integrate((d^2*x^2
 + 3*d*x + 3)/(a^2*b*d^2*x^4*e^(d*x + c) + a^3*d^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})^3 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x^2}\,dx$$

```
[In] integrate(1/(a+b*exp(d*x+c))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*e^(d*x + c) + a)^3*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})^3 x^2} \, dx = \int \frac{1}{x^2 \left(a+b\,\mathrm{e}^{c+d\,x}\right)^3} \,dx$$

```
[In] int(1/(x^2*(a + b*exp(c + d*x))^3),x)
[Out] int(1/(x^2*(a + b*exp(c + d*x))^3), x)
```

3.23 $\int \frac{1}{(a+be^{c-dx})^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{1}{(a+be^{c-dx})^3} dx = -\frac{1}{2ad(a+be^{c-dx})^2} - \frac{1}{a^2d(a+be^{c-dx})} + \frac{x}{a^3} + \frac{\log(a+be^{c-dx})}{a^3d}$$

```
[Out] -1/2/a/d/(a+b*exp(-d*x+c))^2-1/a^2/d/(a+b*exp(-d*x+c))+x/a^3+ln(a+b*exp(-d*
x+c))/a^3/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2320, 46}

$$\int \frac{1}{(a+be^{c-dx})^3} dx = \frac{\log(a+be^{c-dx})}{a^3d} + \frac{x}{a^3} - \frac{1}{a^2d(a+be^{c-dx})} - \frac{1}{2ad(a+be^{c-dx})^2}$$

```
[In] Int[(a + b*E^(c - d*x))^(-3),x]
```

```
[Out] -1/2*1/(a*d*(a + b*E^(c - d*x))^2) - 1/(a^2*d*(a + b*E^(c - d*x))) + x/a^3
+ Log[a + b*E^(c - d*x)]/(a^3*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, e^{c-dx}\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)\,dx, x, e^{c-dx}\right)}{d}\\ &= -\frac{1}{2ad\left(a+be^{c-dx}\right)^2}-\frac{1}{a^2d\left(a+be^{c-dx}\right)}+\frac{x}{a^3}+\frac{\log\left(a+be^{c-dx}\right)}{a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = \frac{\frac{be^c\left(3be^c+4ae^{dx}\right)}{\left(be^c+ae^{dx}\right)^2}+2\log\left(a^2d\left(be^c+ae^{dx}\right)\right)}{2a^3d}$$

```
[In] Integrate[(a + b*E^(c - d*x))^(-3),x]
[Out] ((b*E^c*(3*b*E^c + 4*a*E^(d*x)))/(b*E^c + a*E^(d*x))^2 + 2*Log[a^2*d*(b*E^c
 + a*E^(d*x))])/(2*a^3*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^3}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,e^{-dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{-dx+c}\right)^2}}{d}$
default	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^3}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,e^{-dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{-dx+c}\right)^2}}{d}$
risch	$\frac{x}{a^3}-\frac{c}{a^3d}-\frac{2b\,e^{-dx+c}+3a}{2a^2d\left(a+b\,e^{-dx+c}\right)^2}+\frac{\ln\left(e^{-dx+c}+\frac{a}{b}\right)}{a^3d}$
norman	$\frac{\frac{x}{a}+\frac{b^2x\,e^{-2dx+2c}}{a^3}+\frac{2bx\,e^{-dx+c}}{a^2}+\frac{2b\,e^{-dx+c}}{a^2d}+\frac{3b^2\,e^{-2dx+2c}}{2a^3d}}{\left(a+b\,e^{-dx+c}\right)^2}+\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^3d}$
parallelrisch	$\frac{2\,e^{-2dx+2c}b^2dx+2\ln\left(a+b\,e^{-dx+c}\right)e^{-2dx+2c}b^2+4\,e^{-dx+c}abdx+4\ln\left(a+b\,e^{-dx+c}\right)e^{-dx+c}ab+2a^2dx+3b^2\,e^{-2dx+2c}+2\ln(}{2a^3d\left(a+b\,e^{-dx+c}\right)^2}$

```
[In] int(1/(a+b*exp(-d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/a^3*ln(exp(-d*x+c))-1/a^3*ln(a+b*exp(-d*x+c))+1/a^2/(a+b*exp(-d*x+c
))+1/2/a/(a+b*exp(-d*x+c))^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = \frac{2\,b^2dxe^{(-2\,dx+2\,c)}+2\,a^2dx-3\,a^2+2\,(2\,abdx-ab)e^{(-dx+c)}+2\,\left(2\,abe^{(-dx+c)}+b^2e^{(-2\,dx+2\,c)}+a^2\right)\log\left(be^{(-dx}\right.}{2\,\left(2\,a^4bde^{(-dx+c)}+a^3b^2de^{(-2\,dx+2\,c)}+a^5d\right)}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d*x*e^(-2*d*x + 2*c) + 2*a^2*d*x - 3*a^2 + 2*(2*a*b*d*x - a*b)*e
^(-d*x + c) + 2*(2*a*b*e^(-d*x + c) + b^2*e^(-2*d*x + 2*c) + a^2)*log(b*e^(
-d*x + c) + a))/(2*a^4*b*d*e^(-d*x + c) + a^3*b^2*d*e^(-2*d*x + 2*c) + a^5*
d)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+be^{c-dx})^3}\,dx = \frac{-3a-2be^{c-dx}}{2a^4d+4a^3bde^{c-dx}+2a^2b^2de^{2c-2dx}} + \frac{x}{a^3} + \frac{\log\left(\frac{a}{b}+e^{c-dx}\right)}{a^3d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))**3,x)
[Out] (-3*a - 2*b*exp(c - d*x))/(2*a**4*d + 4*a**3*b*d*exp(c - d*x) + 2*a**2*b**2
*d*exp(2*c - 2*d*x)) + x/a**3 + log(a/b + exp(c - d*x))/(a**3*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+be^{c-dx})^3}\,dx = -\frac{2\,be^{(-dx+c)}+3\,a}{2\,(2\,a^3be^{(-dx+c)}+a^2b^2e^{(-2\,dx+2\,c)}+a^4)d} + \frac{dx-c}{a^3d} + \frac{\log\left(be^{(-dx+c)}+a\right)}{a^3d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(2*b*e^(-d*x + c) + 3*a)/((2*a^3*b*e^(-d*x + c) + a^2*b^2*e^(-2*d*x +
2*c) + a^4)*d) + (d*x - c)/(a^3*d) + log(b*e^(-d*x + c) + a)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+be^{c-dx})^3}\,dx = \frac{\frac{2\,(dx-c)}{a^3} + \frac{2\,\log\left(\left|be^{(-dx+c)}+a\right|\right)}{a^3} - \frac{2\,abe^{(-dx+c)}+3\,a^2}{\left(be^{(-dx+c)}+a\right)^2a^3}}{2\,d}$$

```
[In] integrate(1/(a+b*exp(-d*x+c))^3,x, algorithm="giac")
[Out] 1/2*(2*(d*x - c)/a^3 + 2*log(abs(b*e^(-d*x + c) + a))/a^3 - (2*a*b*e^(-d*x
+ c) + 3*a^2)/((b*e^(-d*x + c) + a)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.72

$$\int \frac{1}{(a+be^{c-dx})^3}\,dx = \frac{\frac{x}{a}+\frac{b^2\,x\,\mathrm{e}^{2\,c-2\,d\,x}}{a^3}+\frac{2\,b\,x\,\mathrm{e}^{c-d\,x}}{a^2}+\frac{3\,b^2\,\mathrm{e}^{2\,c-2\,d\,x}}{2\,a^3\,d}+\frac{2\,b\,\mathrm{e}^{c-d\,x}}{a^2\,d}}{a^2+2\,\mathrm{e}^{c-d\,x}\,a\,b+\mathrm{e}^{2\,c-2\,d\,x}\,b^2}+\frac{\ln\left(a+b\,\mathrm{e}^{-d\,x}\,\mathrm{e}^{c}\right)}{a^3\,d}$$

```
[In] int(1/(a + b*exp(c - d*x))^3,x)
```

```
[Out] (x/a + (b^2*x*exp(2*c - 2*d*x))/a^3 + (2*b*x*exp(c - d*x))/a^2 + (3*b^2*exp
(2*c - 2*d*x))/(2*a^3*d) + (2*b*exp(c - d*x))/(a^2*d))/(a^2 + b^2*exp(2*c -
 2*d*x) + 2*a*b*exp(c - d*x)) + log(a + b*exp(-d*x)*exp(c))/(a^3*d)
```

3.24 $\int \frac{1}{\left(a+be^{-c-dx}\right)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3} dx = -\frac{1}{2ad\left(a+be^{-c-dx}\right)^2} - \frac{1}{a^2d\left(a+be^{-c-dx}\right)} + \frac{x}{a^3} + \frac{\log\left(a+be^{-c-dx}\right)}{a^3d}$$

```
[Out] -1/2/a/d/(a+b*exp(-d*x-c))^2-1/a^2/d/(a+b*exp(-d*x-c))+x/a^3+ln(a+b*exp(-d*
x-c))/a^3/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2320, 46}

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3} dx = \frac{\log\left(a+be^{-c-dx}\right)}{a^3d} + \frac{x}{a^3} - \frac{1}{a^2d\left(a+be^{-c-dx}\right)} - \frac{1}{2ad\left(a+be^{-c-dx}\right)^2}$$

```
[In] Int[(a + b*E^(-c - d*x))^(-3),x]
```

```
[Out] -1/2*1/(a*d*(a + b*E^(-c - d*x))^2) - 1/(a^2*d*(a + b*E^(-c - d*x))) + x/a^
3 + Log[a + b*E^(-c - d*x)]/(a^3*d)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, e^{-c-dx}\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)\,dx, x, e^{-c-dx}\right)}{d}\\ &= -\frac{1}{2ad\left(a+be^{-c-dx}\right)^2}-\frac{1}{a^2d\left(a+be^{-c-dx}\right)}+\frac{x}{a^3}+\frac{\log\left(a+be^{-c-dx}\right)}{a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = \frac{\frac{b\left(3b+4ae^{c+dx}\right)}{\left(b+ae^{c+dx}\right)^2}+2\log\left(b+ae^{c+dx}\right)}{2a^3d}$$

```
[In] Integrate[(a + b*E^(-c - d*x))^(-3),x]
[Out] ((b*(3*b + 4*a*E^(c + d*x)))/(b + a*E^(c + d*x))^2 + 2*Log[b + a*E^(c + d*x
)])/(2*a^3*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

method	result
risch	$\frac{x}{a^3}+\frac{c}{a^3 d}-\frac{2b\,\mathrm{e}^{-dx-c}+3a}{2a^2 d\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}+\frac{\ln\left(\mathrm{e}^{-dx-c}+\frac{a}{b}\right)}{a^3 d}$
derivativedivides	$-\frac{\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{a^3}-\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,\mathrm{e}^{-dx-c}\right)}+\frac{1}{2a\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}}{d}$
default	$-\frac{\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{a^3}-\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,\mathrm{e}^{-dx-c}\right)}+\frac{1}{2a\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}}{d}$
norman	$\frac{\frac{x}{a}+\frac{b^2 x\,\mathrm{e}^{-2dx-2c}}{a^3}+\frac{2bx\,\mathrm{e}^{-dx-c}}{a^2}+\frac{2b\,\mathrm{e}^{-dx-c}}{a^2 d}+\frac{3b^2\mathrm{e}^{-2dx-2c}}{2a^3 d}}{\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^3 d}$
parallelrisch	$\frac{2\,\mathrm{e}^{-2dx-2c}b^2dx+2\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)\mathrm{e}^{-2dx-2c}b^2+4\,\mathrm{e}^{-dx-c}abdx+4\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)\mathrm{e}^{-dx-c}ab+2a^2dx+3b^2\mathrm{e}^{-2dx-2c}+2\,\mathrm{l}}{2a^3 d\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}$

```
[In] int(1/(a+b*exp(-d*x-c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/a^3+1/a^3/d*c-1/2*(2*b*exp(-d*x-c)+3*a)/a^2/d/(a+b*exp(-d*x-c))^2+1/a^3/d
*ln(exp(-d*x-c)+a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx$$
$$=\frac{2\,b^2dxe^{(-2\,dx-2\,c)}+2\,a^2dx-3\,a^2+2\,(2\,abdx-ab)e^{(-dx-c)}+2\,\left(2\,abe^{(-dx-c)}+b^2e^{(-2\,dx-2\,c)}+a^2\right)\log\left(be^{(-\right.}}{2\,\left(2\,a^4bde^{(-dx-c)}+a^3b^2de^{(-2\,dx-2\,c)}+a^5d\right)}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d*x*e^(-2*d*x - 2*c) + 2*a^2*d*x - 3*a^2 + 2*(2*a*b*d*x - a*b)*e
^(-d*x - c) + 2*(2*a*b*e^(-d*x - c) + b^2*e^(-2*d*x - 2*c) + a^2)*log(b*e^(
-d*x - c) + a))/(2*a^4*b*d*e^(-d*x - c) + a^3*b^2*d*e^(-2*d*x - 2*c) + a^5*
d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+be^{-c-dx})^3}\,dx = \frac{-3a-2be^{-c-dx}}{2a^4d+4a^3bde^{-c-dx}+2a^2b^2de^{-2c-2dx}} + \frac{x}{a^3} + \frac{\log\left(\frac{a}{b}+e^{-c-dx}\right)}{a^3d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))**3,x)
```

```
[Out] (-3*a - 2*b*exp(-c - d*x))/(2*a**4*d + 4*a**3*b*d*exp(-c - d*x) + 2*a**2*b*
*2*d*exp(-2*c - 2*d*x)) + x/a**3 + log(a/b + exp(-c - d*x))/(a**3*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+be^{-c-dx})^3}\,dx = -\frac{2\,be^{(-dx-c)}+3\,a}{2\,(2\,a^3be^{(-dx-c)}+a^2b^2e^{(-2\,dx-2\,c)}+a^4)d} + \frac{dx+c}{a^3d} + \frac{\log\left(be^{(-dx-c)}+a\right)}{a^3d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*e^(-d*x - c) + 3*a)/((2*a^3*b*e^(-d*x - c) + a^2*b^2*e^(-2*d*x -
2*c) + a^4)*d) + (d*x + c)/(a^3*d) + log(b*e^(-d*x - c) + a)/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+be^{-c-dx})^3}\,dx = \frac{\frac{2\,(dx+c)}{a^3}+\frac{2\,\log\left(\left|be^{(-dx-c)}+a\right|\right)}{a^3}-\frac{2\,abe^{(-dx-c)}+3\,a^2}{\left(be^{(-dx-c)}+a\right)^2a^3}}{2\,d}$$

```
[In] integrate(1/(a+b*exp(-d*x-c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a^3 + 2*log(abs(b*e^(-d*x - c) + a))/a^3 - (2*a*b*e^(-d*x
- c) + 3*a^2)/((b*e^(-d*x - c) + a)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int \frac{1}{(a+be^{-c-dx})^3} dx = \frac{\frac{x}{a} + \frac{2\,b\,\mathrm{e}^{-c-d\,x}}{a^2\,d} + \frac{b^2\,x\,\mathrm{e}^{-2\,c-2\,d\,x}}{a^3} + \frac{3\,b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{2\,a^3\,d} + \frac{2\,b\,x\,\mathrm{e}^{-c-d\,x}}{a^2}}{a^2 + 2\,\mathrm{e}^{-c-d\,x}\,a\,b + \mathrm{e}^{-2\,c-2\,d\,x}\,b^2} + \frac{\ln\left(a + b\,\mathrm{e}^{-c}\,\mathrm{e}^{-d\,x}\right)}{a^3\,d}$$

```
[In] int(1/(a + b*exp(- c - d*x))^3,x)
```

```
[Out] (x/a + (2*b*exp(- c - d*x))/(a^2*d) + (b^2*x*exp(- 2*c - 2*d*x))/a^3 + (3*b
^2*exp(- 2*c - 2*d*x))/(2*a^3*d) + (2*b*x*exp(- c - d*x))/a^2)/(a^2 + b^2*e
xp(- 2*c - 2*d*x) + 2*a*b*exp(- c - d*x)) + log(a + b*exp(-c)*exp(-d*x))/(a
^3*d)
```

3.25 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx=\frac{a(c+dx)^4}{4d}-\frac{6bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}+\frac{6bd^2\left(F^{eg+fgx}\right)^n(c+dx)}{f^3g^3n^3\log^3(F)}-\frac{3bd\left(F^{eg+fgx}\right)^n(c+dx)^2}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^3}{fgn\log(F)}$$

```
[Out] 1/4*a*(d*x+c)^4/d-6*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/ln(F)^4+6*b*d^2*(F^
(f*g*x+e*g))^n*(d*x+c)/f^3/g^3/n^3/ln(F)^3-3*b*d*(F^(f*g*x+e*g))^n*(d*x+c)^
2/f^2/g^2/n^2/ln(F)^2+b*(F^(f*g*x+e*g))^n*(d*x+c)^3/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx=\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-\frac{3bd(c+dx)^2\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b(c+dx)^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{6bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^3,x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*b*d^3*(F^(e*g + f*g*x))^n)/(f^4*g^4*n^4*Log[F]^4
) + (6*b*d^2*(F^(e*g + f*g*x))^n*(c + d*x))/(f^3*g^3*n^3*Log[F]^3) - (3*b*d
```

```
*(F^(e*g + f*g*x))^n*(c + d*x)^2)/(f^2*g^2*n^2*Log[F]^2) + (b*(F^(e*g + f*g
*x))^n*(c + d*x)^3)/(f*g*n*Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(c+dx)^3+b\left(F^{eg+fgx}\right)^n (c+dx)^3\right)\,dx\\
&= \frac{a(c+dx)^4}{4d}+b\int \left(F^{eg+fgx}\right)^n (c+dx)^3\,dx\\
&= \frac{a(c+dx)^4}{4d}+\frac{b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)}-\frac{(3bd)\int \left(F^{eg+fgx}\right)^n (c+dx)^2\,dx}{fgn\log(F)}\\
&= \frac{a(c+dx)^4}{4d}-\frac{3bd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)}\\
&\quad+\frac{b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)}+\frac{\left(6bd^2\right)\int \left(F^{eg+fgx}\right)^n (c+dx)\,dx}{f^2g^2n^2\log^2(F)}\\
&= \frac{a(c+dx)^4}{4d}+\frac{6bd^2\left(F^{eg+fgx}\right)^n (c+dx)}{f^3g^3n^3\log^3(F)}-\frac{3bd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)}\\
&\quad+\frac{b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)}-\frac{\left(6bd^3\right)\int \left(F^{eg+fgx}\right)^n\,dx}{f^3g^3n^3\log^3(F)}\\
&= \frac{a(c+dx)^4}{4d}-\frac{6bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}+\frac{6bd^2\left(F^{eg+fgx}\right)^n (c+dx)}{f^3g^3n^3\log^3(F)}\\
&\quad-\frac{3bd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.85

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = ac^3x+\frac{3}{2}ac^2dx^2+acd^2x^3+\frac{1}{4}ad^3x^4 + \frac{b\left(F^{g(e+fx)}\right)^n\left(-6d^3+6d^2fgn(c+dx)\log(F)-3df^2g^2n^2(c+dx)^2\log^2(F)+f^3g^3n^3(c+dx)^3\log^3(F)\right)}{f^4g^4n^4\log^4(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^3,x]
```

```
[Out] a*c^3*x + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (a*d^3*x^4)/4 + (b*(F^(g*(e + f
*x)))^n*(-6*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 3*d*f^2*g^2*n^2*(c + d*x)^
2*Log[F]^2 + f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(f^4*g^4*n^4*Log[F]^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(151) = 302.

Time = 0.56 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.56

method	result
parallelrisch	$\frac{a\,d^3x^4n^4g^4f^4\ln(F)^4+4a\,d^2c\,x^3n^4g^4f^4\ln(F)^4+6ad\,c^2x^2n^4g^4f^4\ln(F)^4+4a\,c^3x\,n^4g^4f^4\ln(F)^4+4x^3\left(F^{g(fx+e)}\right)^n b\,d^3n^3g^3f^3\ln(}{}$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a*d^3*x^4*n^4*g^4*f^4*ln(F)^4+4*a*d^2*c*x^3*n^4*g^4*f^4*ln(F)^4+6*a*d*
c^2*x^2*n^4*g^4*f^4*ln(F)^4+4*a*c^3*x*n^4*g^4*f^4*ln(F)^4+4*x^3*(F^(g*(f*x+
e)))^n*b*d^3*n^3*g^3*f^3*ln(F)^3+12*ln(F)^3*x^2*(F^(g*(f*x+e)))^n*b*c*d^2*f
^3*g^3*n^3+12*ln(F)^3*x*(F^(g*(f*x+e)))^n*b*c^2*d*f^3*g^3*n^3+4*ln(F)^3*(F^
(g*(f*x+e)))^n*b*c^3*f^3*g^3*n^3-12*ln(F)^2*x^2*(F^(g*(f*x+e)))^n*b*d^3*f^2
*g^2*n^2-24*ln(F)^2*x*(F^(g*(f*x+e)))^n*b*c*d^2*f^2*g^2*n^2-12*ln(F)^2*(F^(
g*(f*x+e)))^n*b*c^2*d*f^2*g^2*n^2+24*ln(F)*x*(F^(g*(f*x+e)))^n*b*d^3*f*g*n+
24*ln(F)*(F^(g*(f*x+e)))^n*b*c*d^2*f*g*n-24*(F^(g*(f*x+e)))^n*b*d^3)/n^4/g^
4/f^4/ln(F)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.75

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$$
$$=\frac{\left(ad^3f^4g^4n^4x^4+4\,acd^2f^4g^4n^4x^3+6\,ac^2df^4g^4n^4x^2+4\,ac^3f^4g^4n^4x\right)\log\left(F\right)^4-4\left(6\,bd^3-\left(bd^3f^3g^3n^3x^3+3\right.\right.}{}$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x, algorithm="fricas")

[Out] 1/4*((a*d^3*f^4*g^4*n^4*x^4 + 4*a*c*d^2*f^4*g^4*n^4*x^3 + 6*a*c^2*d*f^4*g^4 *n^4*x^2 + 4*a*c^3*f^4*g^4*n^4*x)*log(F)^4 - 4*(6*b*d^3 - (b*d^3*f^3*g^3*n^ 3*x^3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + b*c^3*f^3*g^3 *n^3)*log(F)^3 + 3*(b*d^3*f^2*g^2*n^2*x^2 + 2*b*c*d^2*f^2*g^2*n^2*x + b*c^2 *d*f^2*g^2*n^2)*log(F)^2 - 6*(b*d^3*f*g*n*x + b*c*d^2*f*g*n)*log(F))*F^(f*g *n*x + e*g*n))/(f^4*g^4*n^4*log(F)^4)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(151) = 302.

Time = 1.77 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.24

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$$
$$=\begin{cases}(a+b)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\\ \left(a+b\left(F^{eg}\right)^n\right)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\\ (a+b)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\\ ac^3x+\frac{3ac^2dx^2}{2}+acd^2x^3+\frac{ad^3x^4}{4}+\frac{bc^3\left(F^{eg+fgx}\right)^n}{fgn\log\left(F\right)}+\frac{3bc^2dx\left(F^{eg+fgx}\right)^n}{fgn\log\left(F\right)}-\frac{3bc^2d\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log\left(F\right)^2}+\frac{3bcd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log\left(F\right)}-6b\end{cases}$$

[In] integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**3,x)

[Out] Piecewise(((a + b)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(f, 0)), ((a + b)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) | Eq(g, 0) | Eq(n , 0)), (a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + b*c* *3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 3*b*c**2*d*x*(F**(e*g + f*g*x))** n/(f*g*n*log(F)) - 3*b*c**2*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)*

```
*2) + 3*b*c*d**2*x**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*b*c*d**2*x*(
F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 6*b*c*d**2*(F**(e*g + f*g
*x))**n/(f**3*g**3*n**3*log(F)**3) + b*d**3*x**3*(F**(e*g + f*g*x))**n/(f*g
*n*log(F)) - 3*b*d**3*x**2*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2)
 + 6*b*d**3*x*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) - 6*b*d**3*(
F**(e*g + f*g*x))**n/(f**4*g**4*n**4*log(F)**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.84

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{F^{fgnx+egn}bc^3}{fgn\log\left(F\right)} + \frac{3\left(F^{egn}fgnx\log\left(F\right) - F^{egn}\right)F^{fgnx}bc^2d}{f^2g^2n^2\log\left(F\right)^2} + \frac{3\left(F^{egn}f^2g^2n^2x^2\log\left(F\right)^2 - 2\,F^{egn}fgnx\log\left(F\right) + 2\,F^{egn}\right)F^{fgnx}bcd^2}{f^3g^3n^3\log\left(F\right)^3} + \frac{\left(F^{egn}f^3g^3n^3x^3\log\left(F\right)^3 - 3\,F^{egn}f^2g^2n^2x^2\log\left(F\right)^2 + 6\,F^{egn}fgnx\log\left(F\right) - 6\,F^{egn}\right)F^{fgnx}bd^3}{f^4g^4n^4\log\left(F\right)^4}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + F^(f*g*n*x + e*g*
n)*b*c^3/(f*g*n*log(F)) + 3*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n
*x)*b*c^2*d/(f^2*g^2*n^2*log(F)^2) + 3*(F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2
- 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*b*c*d^2/(f^3*g^3*n^
3*log(F)^3) + (F^(e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 3*F^(e*g*n)*f^2*g^2*n^2
*x^2*log(F)^2 + 6*F^(e*g*n)*f*g*n*x*log(F) - 6*F^(e*g*n))*F^(f*g*n*x)*b*d^3
/(f^4*g^4*n^4*log(F)^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 5716, normalized size of antiderivative = 37.36

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (((3*pi^2*b*d^3*f
^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)
```

```
) + 2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*
log(abs(F))*sgn(F) - 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 6*b*c*d^2
*f^3*g^3*n^3*x^2*log(abs(F))^3 + 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*s
gn(F) - 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 6*b*c^2*d*f^3*g^3*n^3*x*
log(abs(F))^3 + 3*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*b*c^3*
f^3*g^3*n^3*log(abs(F)) + 2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*b*d^3*
f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*b*d^3*f^2*g^2*n^2*x^2 - 6*b*d^3*f^2*g^2*n^2
*x^2*log(abs(F))^2 - 6*pi^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*b*c*d^2*f
^2*g^2*n^2*x - 12*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b*c^2*d*f^2*
g^2*n^2*sgn(F) + 3*pi^2*b*c^2*d*f^2*g^2*n^2 - 6*b*c^2*d*f^2*g^2*n^2*log(abs
(F))^2 + 12*b*d^3*f*g*n*x*log(abs(F)) + 12*b*c*d^2*f*g*n*log(abs(F)) - 12*b
*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) -
pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs
(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F)
 - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(
abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log
(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F
))^3)^2) - 4*(pi^3*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*pi*b*d^3*f^3*g^3*n^3*x^
3*log(abs(F))^2*sgn(F) - pi^3*b*d^3*f^3*g^3*n^3*x^3 + 3*pi*b*d^3*f^3*g^3*n^
3*x^3*log(abs(F))^2 + 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*pi*b*c*d^2*
f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2 + 9*p
i*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 3*pi^3*b*c^2*d*f^3*g^3*n^3*x*sgn(
F) - 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*b*c^2*d*f^3*g
^3*n^3*x + 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + pi^3*b*c^3*f^3*g^3*n^
3*sgn(F) - 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*b*c^3*f^3*g^3
*n^3 + 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*b*d^3*f^2*g^2*n^2*x^2*lo
g(abs(F))*sgn(F) - 6*pi*b*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 12*pi*b*c*d^2*f
^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*b*c*d^2*f^2*g^2*n^2*x*log(abs(F)) +
 6*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*b*c^2*d*f^2*g^2*n^2*log
(abs(F)) - 6*pi*b*d^3*f*g*n*x*sgn(F) + 6*pi*b*d^3*f*g*n*x - 6*pi*b*c*d^2*f*
g*n*sgn(F) + 6*pi*b*c*d^2*f*g*n)*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*
f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^
4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs
(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4
*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^
4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*
n^4*log(abs(F))^3)^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi
*e*g*n*sgn(F) + 1/2*pi*e*g*n) - ((pi^3*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*pi*
b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - pi^3*b*d^3*f^3*g^3*n^3*x^3 + 3
*pi*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2*sg
n(F) - 9*pi*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*b*c*d^2*f
^3*g^3*n^3*x^2 + 9*pi*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 3*pi^3*b*c^2*
d*f^3*g^3*n^3*x*sgn(F) - 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) -
3*pi^3*b*c^2*d*f^3*g^3*n^3*x + 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + p
i^3*b*c^3*f^3*g^3*n^3*sgn(F) - 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F)
```

- pi^3*b*c^3*f^3*g^3*n^3 + 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*b*d^
3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*b*d^3*f^2*g^2*n^2*x^2*log(abs(F
)) + 12*pi*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*b*c*d^2*f^2*g^2
*n^2*x*log(abs(F)) + 6*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*b*c
^2*d*f^2*g^2*n^2*log(abs(F)) - 6*pi*b*d^3*f*g*n*x*sgn(F) + 6*pi*b*d^3*f*g*n
*x - 6*pi*b*c*d^2*f*g*n*sgn(F) + 6*pi*b*c*d^2*f*g*n)*(pi^4*f^4*g^4*n^4*sgn(
F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^
4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4*g^4*n^4*s
gn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2
*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*
g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4
*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) + 4*(3*pi^2*b*d^3*f
^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)
) + 2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*
log(abs(F))*sgn(F) - 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 6*b*c*d^2
*f^3*g^3*n^3*x^2*log(abs(F))^3 + 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*s
gn(F) - 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 6*b*c^2*d*f^3*g^3*n^3*x*
log(abs(F))^3 + 3*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*b*c^3*
f^3*g^3*n^3*log(abs(F)) + 2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*b*d^3*
f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*b*d^3*f^2*g^2*n^2*x^2 - 6*b*d^3*f^2*g^2*n^2
*x^2*log(abs(F))^2 - 6*pi^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*b*c*d^2*f
^2*g^2*n^2*x - 12*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b*c^2*d*f^2*
g^2*n^2*sgn(F) + 3*pi^2*b*c^2*d*f^2*g^2*n^2 - 6*b*c^2*d*f^2*g^2*n^2*log(abs
(F))^2 + 12*b*d^3*f*g*n*x*log(abs(F)) + 12*b*c*d^2*f*g*n*log(abs(F)) - 12*b
d^3)(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*s
gn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4
*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^
4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 +
16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn
(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*sin(
-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*
n))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F))) - 1/2*I*((pi^3*b*d^3*f^3*g^
3*n^3*x^3*sgn(F) + 3*I*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi
*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - pi^3*b*d^3*f^3*g^3*n^3*x^3 -
3*I*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 3*pi*b*d^3*f^3*g^3*n^3*x^3*log
(abs(F))^2 + 2*I*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 3*pi^3*b*c*d^2*f^3*g
^3*n^3*x^2*sgn(F) + 9*I*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9
*pi*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*b*c*d^2*f^3*g^3*n
^3*x^2 - 9*I*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 9*pi*b*c*d^2*f^3*g^
3*n^3*x^2*log(abs(F))^2 + 6*I*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 3*pi^
3*b*c^2*d*f^3*g^3*n^3*x*sgn(F) + 9*I*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))
*sgn(F) - 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*b*c^2*d*
f^3*g^3*n^3*x - 9*I*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 9*pi*b*c^2*d*f
^3*g^3*n^3*x*log(abs(F))^2 + 6*I*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + pi^3
*b*c^3*f^3*g^3*n^3*sgn(F) + 3*I*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) -

3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*b*c^3*f^3*g^3*n^3 - 3*I
*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F)) + 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2
+ 2*I*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*I*pi^2*b*d^3*f^2*g^2*n^2*x^2*sgn(
F) + 6*pi*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 3*I*pi^2*b*d^3*f^2*g^2
*n^2*x^2 - 6*pi*b*d^3*f^2*g^2*n^2*x^2*log(abs(F)) - 6*I*b*d^3*f^2*g^2*n^2*x
^2*log(abs(F))^2 - 6*I*pi^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 12*pi*b*c*d^2*f^
2*g^2*n^2*x*log(abs(F))*sgn(F) + 6*I*pi^2*b*c*d^2*f^2*g^2*n^2*x - 12*pi*b*c
*d^2*f^2*g^2*n^2*x*log(abs(F)) - 12*I*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 -
3*I*pi^2*b*c^2*d*f^2*g^2*n^2*sgn(F) + 6*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F))
*sgn(F) + 3*I*pi^2*b*c^2*d*f^2*g^2*n^2 - 6*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F
)) - 6*I*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 6*pi*b*d^3*f*g*n*x*sgn(F) + 6*
pi*b*d^3*f*g*n*x + 12*I*b*d^3*f*g*n*x*log(abs(F)) - 6*pi*b*c*d^2*f*g*n*sgn(
F) + 6*pi*b*c*d^2*f*g*n + 12*I*b*c*d^2*f*g*n*log(abs(F)) - 12*I*b*d^3)*e^(1
/2*I*pi*f*g*n*x*sgn(F) - 1/2*I*pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2*I*p
i*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) + 4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F)
- 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - 4*I*pi*f^4*g^4*n^4*log(abs(F))
^3*sgn(F) - pi^4*f^4*g^4*n^4 - 4*I*pi^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*f^
4*g^4*n^4*log(abs(F))^2 + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^4*
log(abs(F))^4) + (pi^3*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*I*pi^2*b*d^3*f^3*g^
3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn
(F) - pi^3*b*d^3*f^3*g^3*n^3*x^3 + 3*I*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F
)) + 3*pi*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 - 2*I*b*d^3*f^3*g^3*n^3*x^3*l
og(abs(F))^3 + 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*I*pi^2*b*c*d^2*f^3
*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^
2*sgn(F) - 3*pi^3*b*c*d^2*f^3*g^3*n^3*x^2 + 9*I*pi^2*b*c*d^2*f^3*g^3*n^3*x^
2*log(abs(F)) + 9*pi*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 - 6*I*b*c*d^2*f^
3*g^3*n^3*x^2*log(abs(F))^3 + 3*pi^3*b*c^2*d*f^3*g^3*n^3*x*sgn(F) - 9*I*pi^
2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 9*pi*b*c^2*d*f^3*g^3*n^3*x*log
(abs(F))^2*sgn(F) - 3*pi^3*b*c^2*d*f^3*g^3*n^3*x + 9*I*pi^2*b*c^2*d*f^3*g^3
*n^3*x*log(abs(F)) + 9*pi*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 - 6*I*b*c^2*d
*f^3*g^3*n^3*x*log(abs(F))^3 + pi^3*b*c^3*f^3*g^3*n^3*sgn(F) - 3*I*pi^2*b*c
^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sg
n(F) - pi^3*b*c^3*f^3*g^3*n^3 + 3*I*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F)) + 3*
pi*b*c^3*f^3*g^3*n^3*log(abs(F))^2 - 2*I*b*c^3*f^3*g^3*n^3*log(abs(F))^3 +
3*I*pi^2*b*d^3*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*b*d^3*f^2*g^2*n^2*x^2*log(abs(
F))*sgn(F) - 3*I*pi^2*b*d^3*f^2*g^2*n^2*x^2 - 6*pi*b*d^3*f^2*g^2*n^2*x^2*lo
g(abs(F)) + 6*I*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 + 6*I*pi^2*b*c*d^2*f^2*
g^2*n^2*x*sgn(F) + 12*pi*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 6*I*pi^
2*b*c*d^2*f^2*g^2*n^2*x - 12*pi*b*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 12*I*b*
c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 + 3*I*pi^2*b*c^2*d*f^2*g^2*n^2*sgn(F) + 6
*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 3*I*pi^2*b*c^2*d*f^2*g^2*n^2 -
6*pi*b*c^2*d*f^2*g^2*n^2*log(abs(F)) + 6*I*b*c^2*d*f^2*g^2*n^2*log(abs(F))
^2 - 6*pi*b*d^3*f*g*n*x*sgn(F) + 6*pi*b*d^3*f*g*n*x - 12*I*b*d^3*f*g*n*x*lo
g(abs(F)) - 6*pi*b*c*d^2*f*g*n*sgn(F) + 6*pi*b*c*d^2*f*g*n - 12*I*b*c*d^2*f
*g*n*log(abs(F)) + 12*I*b*d^3)*e^(-1/2*I*pi*f*g*n*x*sgn(F) + 1/2*I*pi*f*g*n

```
*x - 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) - 4*I
*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn
(F) + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 + 4*I*pi^3
*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 4*I*pi*f^4*g^
4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^4*log(abs(F))^4))*e^(f*g*n*x*log(abs(F))
+ e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.47

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx=\frac{a\,d^3\,x^4}{4}-\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\left(\frac{b\left(-c^3\,f^3\,g^3\,n^3\ln\left(F\right)^3+3\,c^2\,d\,f^2\,g^2\,n^2\ln\left(F\right)^2-6\,c\,d^2\,f\,g\,n\,\ln\left(F\right)+6\,d^3\right)}{f^4\,g^4\,n^4\ln\left(F\right)^4}-\frac{b\,d^3\,x^3}{f\,g\,n\,\ln\left(F\right)}-\frac{3\,b\,d\,x\left(c^2\,f^2\,g^2\,n^2\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{f^3\,g^3\,n^3\ln\left(F\right)^3}+\frac{3\,b\,d^2\,x^2\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\ln\left(F\right)^2}\right)+a\,c^3\,x+\frac{3\,a\,c^2\,d\,x^2}{2}+a\,c\,d^2\,x^3$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^3,x)
```

```
[Out] (a*d^3*x^4)/4 - (F^(f*g*x)*F^(e*g))^n*((b*(6*d^3 - c^3*f^3*g^3*n^3*log(F)^3
 - 6*c*d^2*f*g*n*log(F) + 3*c^2*d*f^2*g^2*n^2*log(F)^2))/(f^4*g^4*n^4*log(F
)^4) - (b*d^3*x^3)/(f*g*n*log(F)) - (3*b*d*x*(2*d^2 + c^2*f^2*g^2*n^2*log(F
)^2 - 2*c*d*f*g*n*log(F)))/(f^3*g^3*n^3*log(F)^3) + (3*b*d^2*x^2*(d - c*f*g
*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + a*c^3*x + (3*a*c^2*d*x^2)/2 + a*c*d^2
*x^3
```

3.26 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx=\frac{a(c+dx)^3}{3d}+\frac{2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-\frac{2bd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/ln(F)^3-2*b*d*(F^(f
*g*x+e*g))^n*(d*x+c)/f^2/g^2/n^2/ln(F)^2+b*(F^(f*g*x+e*g))^n*(d*x+c)^2/f/g/
n/ln(F)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx=\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b(c+dx)^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2,x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*(F^(e*g + f*g*x))^n)/(f^3*g^3*n^3*Log[F]^3
) - (2*b*d*(F^(e*g + f*g*x))^n*(c + d*x))/(f^2*g^2*n^2*Log[F]^2) + (b*(F^(e
*g + f*g*x))^n*(c + d*x)^2)/(f*g*n*Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a(c+dx)^2+b\left(F^{eg+fgx}\right)^n(c+dx)^2\right)\,dx\\
&= \frac{a(c+dx)^3}{3d}+b\int \left(F^{eg+fgx}\right)^n(c+dx)^2\,dx\\
&= \frac{a(c+dx)^3}{3d}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}-\frac{(2bd)\int \left(F^{eg+fgx}\right)^n(c+dx)\,dx}{fgn\log(F)}\\
&= \frac{a(c+dx)^3}{3d}-\frac{2bd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}+\frac{(2bd^2)\int \left(F^{eg+fgx}\right)^n\,dx}{f^2g^2n^2\log^2(F)}\\
&= \frac{a(c+dx)^3}{3d}+\frac{2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-\frac{2bd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$
\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx\\
&= ac^2x+acdx^2+\frac{1}{3}ad^2x^3\\
&\quad+\frac{b\left(F^{g(e+fx)}\right)^n\left(2d^2-2dfgn(c+dx)\log(F)+f^2g^2n^2(c+dx)^2\log^2(F)\right)}{f^3g^3n^3\log^3(F)}
\end{aligned}
$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2,x]
```

```
[Out] a*c^2*x + a*c*d*x^2 + (a*d^2*x^3)/3 + (b*(F^(g*(e + f*x)))^n*(2*d^2 - 2*d*f
*g*n*(c + d*x)*Log[F] + f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(f^3*g^3*n^3*Log
[F]^3)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

method	result
norman	$a\,c^2x + acd\,x^2 + \frac{b\left(\ln(F)^2c^2f^2g^2n^2-2\ln(F)cdfgn+2d^2\right)\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{\ln(F)^3f^3g^3n^3} + \frac{b\,d^2x^2\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)} + \frac{a\,d^2x}{3}$
parallelrisch	$\frac{a\,d^2x^3\ln(F)^3f^3g^3n^3+3acd\,x^2\ln(F)^3f^3g^3n^3+3a\,c^2x\ln(F)^3f^3g^3n^3+3x^2\left(F^{g(fx+e)}\right)^nb\,d^2\ln(F)^2f^2g^2n^2+6\ln(F)^2x\left(F^{g(fx+e}\right.}{3\ln(F}$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*c^2*x+a*c*d*x^2+b*(ln(F)^2*c^2*f^2*g^2*n^2-2*ln(F)*c*d*f*g*n+2*d^2)/ln(F)
^3/f^3/g^3/n^3*exp(n*ln(exp(g*(f*x+e)*ln(F))))+1/n/g/f/ln(F)*b*d^2*x^2*exp(
n*ln(exp(g*(f*x+e)*ln(F))))+1/3*a*d^2*x^3+2*b*d*(ln(F)*c*f*g*n-d)/ln(F)^2/f
^2/g^2/n^2*x*exp(n*ln(exp(g*(f*x+e)*ln(F))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$= \frac{(ad^2f^3g^3n^3x^3+3\,acdf^3g^3n^3x^2+3\,ac^2f^3g^3n^3x)\log(F)^3+3\left(2\,bd^2+(bd^2f^2g^2n^2x^2+2\,bcdf^2g^2n^2x+bc^2f\right.}{3\,f^3g^3n^3\log(F)^3}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a*d^2*f^3*g^3*n^3*x^3 + 3*a*c*d*f^3*g^3*n^3*x^2 + 3*a*c^2*f^3*g^3*n^3
*x)*log(F)^3 + 3*(2*b*d^2 + (b*d^2*f^2*g^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*x
+ b*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))*F^(
f*g*n*x + e*g*n))/(f^3*g^3*n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(110) = 220.

Time = 0.74 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.56

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx = \begin{cases} (a+b)\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ \left(a+b(F^{eg})^n\right)\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ (a+b)\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ ac^2x+acdx^2+\frac{ad^2x^3}{3}+\frac{bc^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{2bcdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{2bcd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{bd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{2bd^2x\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+ \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**2,x)
```

```
[Out] Piecewise(((a + b)*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) & Eq(f, 0) &
 Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)*(c**2*x + c*d*x**2 + d**2*x**
3/3), Eq(f, 0)), ((a + b)*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) | Eq(
g, 0) | Eq(n, 0)), (a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*(F**(e*g
 + f*g*x))**n/(f*g*n*log(F)) + 2*b*c*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log(F
)) - 2*b*c*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + b*d**2*x**2
*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 2*b*d**2*x*(F**(e*g + f*g*x))**n/(f
**2*g**2*n**2*log(F)**2) + 2*b*d**2*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*l
og(F)**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx = \frac{1}{3}ad^2x^3+acdx^2+ac^2x+\frac{F^{fgnx+egn}bc^2}{fgn\log(F)}+\frac{2\left(F^{egn}fgnx\log(F)-F^{egn}\right)F^{fgnx}bcd}{f^2g^2n^2\log(F)^2} + \frac{\left(F^{egn}f^2g^2n^2x^2\log(F)^2-2F^{egn}fgnx\log(F)+2F^{egn}\right)F^{fgnx}bd^2}{f^3g^3n^3\log(F)^3}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + F^(f*g*n*x + e*g*n)*b*c^2/(f*g*n*log(
F)) + 2*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*b*c*d/(f^2*g^2*n
^2*log(F)^2) + (F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(e*g*n)*f*g*n*x*lo
g(F) + 2*F^(e*g*n))*F^(f*g*n*x)*b*d^2/(f^3*g^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 2716, normalized size of antiderivative = 23.62

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - ((2*(pi*b*d^2*f^2*g^2*n^2*x^2*log(abs
(F))*sgn(F) - pi*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*b*c*d*f^2*g^2*n^2
*x*log(abs(F))*sgn(F) - 2*pi*b*c*d*f^2*g^2*n^2*x*log(abs(F)) + pi*b*c^2*f^2
*g^2*n^2*log(abs(F))*sgn(F) - pi*b*c^2*f^2*g^2*n^2*log(abs(F)) - pi*b*d^2*f
*g*n*x*sgn(F) + pi*b*d^2*f*g*n*x - pi*b*c*d*f*g*n*sgn(F) + pi*b*c*d*f*g*n)*
(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3
*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi
*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log
(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3
*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) - (pi^2*b*d^2*f^2*g^2*n^2*x^
2*sgn(F) - pi^2*b*d^2*f^2*g^2*n^2*x^2 + 2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))
^2 + 2*pi^2*b*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*b*c*d*f^2*g^2*n^2*x + 4*b*c
*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*b*c^2*f^2*g^2*n^2*sgn(F) - pi^2*b*c^2
*f^2*g^2*n^2 + 2*b*c^2*f^2*g^2*n^2*log(abs(F))^2 - 4*b*d^2*f*g*n*x*log(abs(
F)) - 4*b*c*d*f*g*n*log(abs(F)) + 4*b*d^2)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*
sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi
^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^
3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))
*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))
*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi
*e*g*n) - ((pi^2*b*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*b*d^2*f^2*g^2*n^2*x^2
+ 2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*b*c*d*f^2*g^2*n^2*x*sgn(F)
 - 2*pi^2*b*c*d*f^2*g^2*n^2*x + 4*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*
b*c^2*f^2*g^2*n^2*sgn(F) - pi^2*b*c^2*f^2*g^2*n^2 + 2*b*c^2*f^2*g^2*n^2*log
(abs(F))^2 - 4*b*d^2*f*g*n*x*log(abs(F)) - 4*b*c*d*f*g*n*log(abs(F)) + 4*b*
d^2)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^
3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) -
 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^
3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^
3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) + 2*(pi*b*d^2*f^2*g^2*n
^2*x^2*log(abs(F))*sgn(F) - pi*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*b*c
*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 2*pi*b*c*d*f^2*g^2*n^2*x*log(abs(F))
+ pi*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*b*c^2*f^2*g^2*n^2*log(abs(F)
) - pi*b*d^2*f*g*n*x*sgn(F) + pi*b*d^2*f*g*n*x - pi*b*c*d*f*g*n*sgn(F) + pi
*b*c*d*f*g*n)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*l
```

```
og(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*
f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(
abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*
log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))*sin(-1/2*pi*f*g*n*x*sgn(F) +
 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*e^(f*g*n*x*log(abs(F
)) + e*g*n*log(abs(F))) - 2*I*((-I*pi^2*b*d^2*f^2*g^2*n^2*x^2*sgn(F) + 2*pi
*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + I*pi^2*b*d^2*f^2*g^2*n^2*x^2 -
2*pi*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) - 2*I*b*d^2*f^2*g^2*n^2*x^2*log(abs(
F))^2 - 2*I*pi^2*b*c*d*f^2*g^2*n^2*x*sgn(F) + 4*pi*b*c*d*f^2*g^2*n^2*x*log(
abs(F))*sgn(F) + 2*I*pi^2*b*c*d*f^2*g^2*n^2*x - 4*pi*b*c*d*f^2*g^2*n^2*x*lo
g(abs(F)) - 4*I*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*b*c^2*f^2*g^2*n^
2*sgn(F) + 2*pi*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*b*c^2*f^2*g^2
*n^2 - 2*pi*b*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*b*c^2*f^2*g^2*n^2*log(abs(F
))^2 - 2*pi*b*d^2*f*g*n*x*sgn(F) + 2*pi*b*d^2*f*g*n*x + 4*I*b*d^2*f*g*n*x*l
og(abs(F)) - 2*pi*b*c*d*f*g*n*sgn(F) + 2*pi*b*c*d*f*g*n + 4*I*b*c*d*f*g*n*l
og(abs(F)) - 4*I*b*d^2)*e^(1/2*I*pi*f*g*n*x*sgn(F) - 1/2*I*pi*f*g*n*x + 1/2
*I*pi*e*g*n*sgn(F) - 1/2*I*pi*e*g*n)/(-4*I*pi^3*f^3*g^3*n^3*sgn(F) + 12*pi^
2*f^3*g^3*n^3*log(abs(F))*sgn(F) + 12*I*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F)
 + 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)) - 12*I*pi*f^3*g^3
*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3) - (-I*pi^2*b*d^2*f^2*g^2*
n^2*x^2*sgn(F) - 2*pi*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + I*pi^2*b*d
^2*f^2*g^2*n^2*x^2 + 2*pi*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) - 2*I*b*d^2*f^2
*g^2*n^2*x^2*log(abs(F))^2 - 2*I*pi^2*b*c*d*f^2*g^2*n^2*x*sgn(F) - 4*pi*b*c
*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 2*I*pi^2*b*c*d*f^2*g^2*n^2*x + 4*pi*b
*c*d*f^2*g^2*n^2*x*log(abs(F)) - 4*I*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*
pi^2*b*c^2*f^2*g^2*n^2*sgn(F) - 2*pi*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) +
 I*pi^2*b*c^2*f^2*g^2*n^2 + 2*pi*b*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*b*c^2*
f^2*g^2*n^2*log(abs(F))^2 + 2*pi*b*d^2*f*g*n*x*sgn(F) - 2*pi*b*d^2*f*g*n*x
+ 4*I*b*d^2*f*g*n*x*log(abs(F)) + 2*pi*b*c*d*f*g*n*sgn(F) - 2*pi*b*c*d*f*g*
n + 4*I*b*c*d*f*g*n*log(abs(F)) - 4*I*b*d^2)*e^(-1/2*I*pi*f*g*n*x*sgn(F) +
1/2*I*pi*f*g*n*x - 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(4*I*pi^3*f^3*g^
3*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 12*I*pi*f^3*g^3*n^3
*log(abs(F))^2*sgn(F) - 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(
F)) + 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3))*e^(
f*g*n*x*log(abs(F)) + e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$=\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\left(\frac{b\left(c^2\,f^2\,g^2\,n^2\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{f^3\,g^3\,n^3\ln\left(F\right)^3}+\frac{b\,d^2\,x^2}{f\,g\,n\,\ln\left(F\right)}-\frac{2\,b\,d\,x\,\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\ln\left(F\right)^2}\right)+\frac{a\,d^2\,x^3}{3}+a\,c^2\,x+a\,c\,d\,x^2$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2,x)
```

```
[Out] (F^(f*g*x)*F^(e*g))^n*((b*(2*d^2 + c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*l
og(F)))/(f^3*g^3*n^3*log(F)^3) + (b*d^2*x^2)/(f*g*n*log(F)) - (2*b*d*x*(d -
 c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + (a*d^2*x^3)/3 + a*c^2*x + a*c*d
*x^2
```

3.27 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx=\frac{a(c+dx)^2}{2d}-\frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}$$

```
[Out] 1/2*a*(d*x+c)^2/d-b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/ln(F)^2+b*(F^(f*g*x+e*g
))^n*(d*x+c)/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx=\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*Log[F]^2) +
(b*(F^(e*g + f*g*x))^n*(c + d*x))/(f*g*n*Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(c+dx)+b\left(F^{eg+fgx}\right)^n(c+dx)\right)\,dx\\
&= \frac{a(c+dx)^2}{2d}+b\int \left(F^{eg+fgx}\right)^n(c+dx)\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}-\frac{(bd)\int \left(F^{eg+fgx}\right)^n\,dx}{fgn\log(F)}\\
&= \frac{a(c+dx)^2}{2d}-\frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx=\frac{1}{2}ax(2c+dx)-\frac{bd\left(F^{g(e+fx)}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{g(e+fx)}\right)^n(c+dx)}{fgn\log(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x),x]
[Out] (a*x*(2*c + d*x))/2 - (b*d*(F^(g*(e + f*x)))^n)/(f^2*g^2*n^2*Log[F]^2) + (b
*(F^(g*(e + f*x)))^n*(c + d*x))/(f*g*n*Log[F])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

method	result	size
norman	$cax+\frac{b(\ln(F)cfgn-d)e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{n^2g^2f^2\ln(F)^2}+\frac{bdx\,e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)}+\frac{ad\,x^2}{2}$	84
parallelrisch	$\frac{ad\,x^2n^2g^2f^2\ln(F)^2+2cax\,n^2g^2f^2\ln(F)^2+2\left(F^{g(fx+e)}\right)^n\ln(F)bdfgnx+2\left(F^{g(fx+e)}\right)^n\ln(F)bcfgn-2\left(F^{g(fx+e)}\right)^nbd}{2n^2g^2f^2\ln(F)^2}$	110

```
[In] int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] c*a*x+b*(ln(F)*c*f*g*n-d)/n^2/g^2/f^2/ln(F)^2*exp(n*ln(exp(g*(f*x+e)*ln(F))
))+1/n/g/f/ln(F)*b*d*x*exp(n*ln(exp(g*(f*x+e)*ln(F))))+1/2*a*d*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = \frac{(adf^2g^2n^2x^2+2\,acf^2g^2n^2x)\log(F)^2-2\,(bd-(bdfgnx+bcfgn)\log(F))F^{fgnx+egn}}{2\,f^2g^2n^2\log(F)^2}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((a*d*f^2*g^2*n^2*x^2 + 2*a*c*f^2*g^2*n^2*x)*log(F)^2 - 2*(b*d - (b*d*f
*g*n*x + b*c*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^2*g^2*n^2*log(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(68) = 136.

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.87

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = \begin{cases} (a+b)\left(cx+\frac{dx^2}{2}\right) & \text{for}\ F=1\wedge f=0\wedge g=0\wedge n=0 \\ \left(a+b(F^{eg})^n\right)\left(cx+\frac{dx^2}{2}\right) & \text{for}\ f=0 \\ (a+b)\left(cx+\frac{dx^2}{2}\right) & \text{for}\ F=1\vee g=0\vee n=0 \\ acx+\frac{adx^2}{2}+\frac{bc\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{bdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c),x)
```

```
[Out] Piecewise(((a + b)*(c*x + d*x**2/2), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n,
 0)), ((a + b*(F**(e*g))**n)*(c*x + d*x**2/2), Eq(f, 0)), ((a + b)*(c*x + d
*x**2/2), Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), (a*c*x + a*d*x**2/2 + b*c*(F**(e
*g + f*g*x))**n/(f*g*n*log(F)) + b*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log(F))
 - b*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx=\frac{1}{2}\,adx^2+acx+\frac{F^{fgnx+egn}bc}{fgn\log\left(F\right)}+\frac{\left(F^{egn}fgnx\log\left(F\right)-F^{egn}\right)F^{fgnx}bd}{f^2g^2n^2\log\left(F\right)^2}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + F^(f*g*n*x + e*g*n)*b*c/(f*g*n*log(F)) + (F^(e*g*n)*f
*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*b*d/(f^2*g^2*n^2*log(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 1101, normalized size of antiderivative = 14.30

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + (2*((pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n
^2*log(abs(F)))*(pi*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x + pi*b*c*f*g*n*sgn(
F) - pi*b*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2
*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n
^2*log(abs(F)))^2) + (pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^
2*n^2*log(abs(F))^2)*(b*d*f*g*n*x*log(abs(F)) + b*c*f*g*n*log(abs(F)) - b*d
)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^
2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2
))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*
pi*e*g*n) + ((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*lo
g(abs(F))^2)*(pi*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x + pi*b*c*f*g*n*sgn(F)
```

```
- pi*b*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^
2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*
log(abs(F)))^2) - 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log
(abs(F)))*(b*d*f*g*n*x*log(abs(F)) + b*c*f*g*n*log(abs(F)) - b*d)/((pi^2*f^
2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(p
i*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*sin(-1/2
*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*
e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F))) - 1/2*I*((pi*b*d*f*g*n*x*sgn(F)
 - pi*b*d*f*g*n*x - 2*I*b*d*f*g*n*x*log(abs(F)) + pi*b*c*f*g*n*sgn(F) - pi*
b*c*f*g*n - 2*I*b*c*f*g*n*log(abs(F)) + 2*I*b*d)*e^(1/2*I*pi*f*g*n*x*sgn(F)
 - 1/2*I*pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2*I*pi*e*g*n)/(pi^2*f^2*g^2
*n^2*sgn(F) + 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^2*f^2*g^2*n^2 - 2*
I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^2) + (pi*b*d*f*g*n
*x*sgn(F) - pi*b*d*f*g*n*x + 2*I*b*d*f*g*n*x*log(abs(F)) + pi*b*c*f*g*n*sgn
(F) - pi*b*c*f*g*n + 2*I*b*c*f*g*n*log(abs(F)) - 2*I*b*d)*e^(-1/2*I*pi*f*g*
n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(pi
^2*f^2*g^2*n^2*sgn(F) - 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^2*f^2*g^
2*n^2 + 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^2))*e^(f
*g*n*x*log(abs(F)) + e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = a\,c\,x - \left(\frac{b\,(d-c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\,\ln(F)^2} - \frac{b\,d\,x}{f\,g\,n\,\ln(F)}\right)\left(F^{f\,g\,x}\,F^{e\,g}\right)^n + \frac{a\,d\,x^2}{2}$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x),x)
```

```
[Out] a*c*x - ((b*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2) - (b*d*x)/(f*g*n*l
og(F)))*(F^(f*g*x)*F^(e*g))^n + (a*d*x^2)/2
```

3.28 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx = ax+\frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

```
[Out] a*x+b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx = ax+\frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

```
[In] Int[a + b*(F^(g*(e + f*x)))^n,x]
[Out] a*x + (b*(F^(g*(e + f*x)))^n)/(f*g*n*Log[F])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + 
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= ax+b\int \left(F^{g(e+fx)}\right)^n\,dx \\ &= ax+\frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx=ax+\frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

```
[In] Integrate[a + b*(F^(g*(e + f*x)))^n,x]
[Out] a*x + (b*(F^(g*(e + f*x)))^n)/(f*g*n*Log[F])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$ax+\frac{b\left(F^{g(fx+e)}\right)^n}{fgn\ln(F)}$	31
parallelrisch	$ax+\frac{b\left(F^{g(fx+e)}\right)^n}{fgn\ln(F)}$	31
parts	$ax+\frac{b\left(F^{g(fx+e)}\right)^n}{fgn\ln(F)}$	31
norman	$ax+\frac{b\,e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)}$	34
derivativedivides	$\frac{b\left(F^{g(fx+e)}\right)^n+a\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{gf\ln(F)n}$	43

```
[In] int(a+b*(F^(g*(f*x+e)))^n,x,method=_RETURNVERBOSE)
[Out] a*x+b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx=\frac{afgnx\log\left(F\right)+F^{fgnx+egn}b}{fgn\log\left(F\right)}$$

```
[In] integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="fricas")
[Out] (a*f*g*n*x*log(F) + F^(f*g*n*x + e*g*n)*b)/(f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx=ax+b\left(\begin{cases} x & \text{for } F=1\wedge f=0\wedge g=0\wedge n=0\\ x\left(F^{eg}\right)^n & \text{for } f=0\\ x & \text{for } F=1\vee g=0\vee n=0\\ \frac{\left(F^{eg+fgx}\right)^n}{fgn\log\left(F\right)} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(a+b*(F**(g*(f*x+e)))**n,x)
[Out] a*x + b*Piecewise((x, Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), (x*(F**(e
*g))**n, Eq(f, 0)), (x, Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), ((F**(e*g + f*g*x)
)**n/(f*g*n*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx=ax+\frac{F^{(fx+e)gn}b}{fgn\log\left(F\right)}$$

```
[In] integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="maxima")
[Out] a*x + F^((f*x + e)*g*n)*b/(f*g*n*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx=ax+\frac{F^{fgnx+egn}b}{fgn\log\left(F\right)}$$

```
[In] integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="giac")
[Out] a*x + F^(f*g*n*x + e*g*n)*b/(f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right) dx = a\,x + \frac{b\,\left(F^{e\,g+f\,g\,x}\right)^n}{f\,g\,n\,\ln\left(F\right)}$$

```
[In] int(a + b*(F^(g*(e + f*x)))^n,x)
[Out] a*x + (b*(F^(e*g + f*g*x))^n)/(f*g*n*log(F))
```

3.29 $\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx=\frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n\operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

```
[Out] b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*Ei(f*g*n*(d*x+c)*ln(F)/d)
/d+a*ln(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2214, 2213, 2209}

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx=\frac{a\log(c+dx)}{d}+\frac{b\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x),x]
```

```
[Out] (b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*ExpIntegralEi[
(f*g*n*(c + d*x)*Log[F])/d])/d + (a*Log[c + d*x])/d
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{c+dx}+\frac{b\left(F^{eg+fgx}\right)^n}{c+dx}\right) dx\\
&= \frac{a\log(c+dx)}{d}+b\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx\\
&= \frac{a\log(c+dx)}{d}+\left(bF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx\\
&= \frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}+\frac{a\log(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \frac{bF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)+a\log(c+dx)}{d}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x),x]
[Out] ((b*(F^(g*(e + f*x)))^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/F^((f*g*
n*(c + d*x))/d) + a*Log[c + d*x])/d
```

Maple [F]

$$\int \frac{a+b\left(F^{g(fx+e)}\right)^{n}}{dx+c}dx$$

[In] int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)

[Out] int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^{n}}{c+dx}\,dx=\frac{F^{\frac{(de-cf)gn}{d}}b\mathrm{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)+a\log\left(dx+c\right)}{d}$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="fricas")

[Out] (F^((d*e - c*f)*g*n/d)*b*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d) + a*log(d*x + c
))/d

Sympy [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^{n}}{c+dx}\,dx=\int \frac{a+b\left(F^{eg+fgx}\right)^{n}}{c+dx}\,dx$$

[In] integrate((a+b*(F**(g*(f*x+e)))**n)/(d*x+c),x)

[Out] Integral((a + b*(F**(e*g + f*g*x))**n)/(c + d*x), x)

Maxima [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^{n}}{c+dx}\,dx=\int \frac{\left(F^{(fx+e)g}\right)^{n}b+a}{dx+c}\,dx$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="maxima")

[Out] F^(e*g*n)*b*integrate(F^(f*g*n*x)/(d*x + c), x) + a*log(d*x + c)/d

Giac [F]

$$\int \frac{a + b\left(F^{g(e+fx)}\right)^n}{c + dx}\, dx = \int \frac{\left(F^{(fx+e)g}\right)^n b + a}{dx + c}\, dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\left(F^{g(e+fx)}\right)^n}{c + dx}\, dx = \int \frac{a + b\,\left(F^{g\,(e+f\,x)}\right)^n}{c + d\,x}\, dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x),x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x), x)
```

3.30 $\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)}-\frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)} + \frac{bfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

```
[Out] -a/d/(d*x+c)-b*(F^(f*g*x+e*g))^n/d/(d*x+c)+b*f*F^((e-c*f/d)*g*n-g*n*(f*x+e)
)*(F^(f*g*x+e*g))^n*g*n*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2214, 2208, 2213, 2209}

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)} + \frac{bfgn\log(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) + (b*f*F^((e - (
c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(
c + d*x)*Log[F])/d]*Log[F])/d^2
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\right)\,dx\\
&= -\frac{a}{d(c+dx)}+b\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx\\
&= -\frac{a}{d(c+dx)}-\frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)}+\frac{(bfgn\log(F))\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx}{d}\\
&= -\frac{a}{d(c+dx)}-\frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)}+\frac{\left(bfF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n gn\log(F)\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx}{d}
\end{aligned}$$

$$= -\frac{a}{d(c+dx)} - \frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)} + \frac{bfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\mathrm{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \frac{-\frac{d\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{c+dx} + bfF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n gn\,\mathrm{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2,x]
```

```
[Out] (-((d*(a + b*(F^(g*(e + f*x)))^n))/(c + d*x)) + (b*f*(F^(g*(e + f*x)))^n*g*
n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((f*g*n*(c + d*x))/d)
)/d^2
```

Maple [F]

$$\int \frac{a+b\left(F^{g(fx+e)}\right)^n}{\left(dx+c\right)^2}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

```
[Out] int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \frac{(bdfgnx+bcfgn)F^{\frac{(de-cf)gn}{d}}\mathrm{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)\log\left(F\right) - F^{fgnx+egn}bd - ad}{d^3x+cd^2}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((b*d*f*g*n*x + b*c*f*g*n)*F^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*l
og(F)/d)*log(F) - F^(f*g*n*x + e*g*n)*b*d - a*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{a+b\left(F^{eg+fgx}\right)^n}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)/(d*x+c)**2,x)
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="maxima")
[Out] F^(e*g*n)*b*integrate(F^(f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) - a/(d^2*x
+ c*d)
```

Giac [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{a+b\,\left(F^{g\,(e+f\,x)}\right)^n}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2, x)
```

3.31 $\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}-\frac{bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} + \frac{bf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}$$

```
[Out] -1/2*a/d/(d*x+c)^2-1/2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-1/2*b*f*(F^(f*g*x+e*
g))^n*g*n*ln(F)/d^2/(d*x+c)+1/2*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g
*x+e*g))^n*g^2*n^2*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2214, 2208, 2213, 2209}

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2} + \frac{bf^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{2d^3} - \frac{bfgn\log(F)\left(F^{eg+fgx}\right)^n}{2d^2(c+dx)}-\frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - (b*(F^(e*g + f*g*x))^n)/(2*d*(c + d*x)^2) - (b*f*(
F^(e*g + f*g*x))^n*g*n*Log[F])/(2*d^2*(c + d*x)) + (b*f^2*F^((e - (c*f)/d)*
g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g^2*n^2*ExpIntegralEi[(f*g*n*(c +
d*x)*Log[F])/d]*Log[F]^2)/(2*d^3)
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{(c+dx)^3}+\frac{b\left(F^{eg+fgx}\right)^n}{(c+dx)^3}\right)dx\\
&= -\frac{a}{2d(c+dx)^2}+b\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}+\frac{(bfgn\log(F))\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}-\frac{bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)}+\frac{\left(bf^2g^2n^2\log^2(F)\right)\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx}{2d^2}
\end{aligned}$$

$$= -\frac{a}{2d(c+dx)^2} - \frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} + \frac{\left(bf^2F^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n g^2n^2\log^2(F)\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx}{2d^2}$$

$$= -\frac{a}{2d(c+dx)^2} - \frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} + \frac{bf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = -\frac{ad^2 - bf^2F^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n g^2n^2(c+dx)^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F) + bd\left(F^{g(e+fx)}\right)^n}{2d^3(c+dx)^2}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x]
[Out] -1/2*(a*d^2 - (b*f^2*(F^(g*(e + f*x)))^n*g^2*n^2*(c + d*x)^2*ExpIntegralEi[
(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((f*g*n*(c + d*x))/d) + b*d*(F^(g*(
e + f*x)))^n*(d + f*g*n*(c + d*x)*Log[F]))/(d^3*(c + d*x)^2)
```

Maple [F]

$$\int \frac{a+b\left(F^{g(fx+e)}\right)^n}{(dx+c)^3}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x)
[Out] int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = \frac{(bd^2f^2g^2n^2x^2 + 2\,bcdf^2g^2n^2x + bc^2f^2g^2n^2)F^{\frac{(de-cf)gn}{d}}\text{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)\log(F)^2 - ad^2 - (bd^2 + (bd^2fg}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b*d^2*f^2*g^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*x + b*c^2*f^2*g^2*n^2)*F^
((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)^2 - a*d^2 - (
b*d^2 + (b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(d^5*x^2
 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx=\int \frac{a+b\left(F^{eg+fgx}\right)^n}{\left(c+dx\right)^3}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx=\int \frac{\left(F^{(fx+e)g}\right)^n b+a}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] F^(e*g*n)*b*integrate(F^(f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
, x) - 1/2*a/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx=\int \frac{\left(F^{(fx+e)g}\right)^n b+a}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = \int \frac{a+b\,\left(F^{g\,(e+f\,x)}\right)^n}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3, x)
```

3.32 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 322

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx &= \frac{a^2(c+dx)^4}{4d}-\frac{12abd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} \\ &\quad -\frac{3b^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)}+\frac{12abd^2\left(F^{eg+fgx}\right)^n(c+dx)}{f^3g^3n^3\log^3(F)} \\ &\quad +\frac{3b^2d^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{4f^3g^3n^3\log^3(F)} \\ &\quad -\frac{6abd\left(F^{eg+fgx}\right)^n(c+dx)^2}{f^2g^2n^2\log^2(F)} \\ &\quad -\frac{3b^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)} \\ &\quad +\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^3}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)}\end{aligned}$$

```
[Out] 1/4*a^2*(d*x+c)^4/d-12*a*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/ln(F)^4-3/8*b^
2*d^3*(F^(f*g*x+e*g))^(2*n)/f^4/g^4/n^4/ln(F)^4+12*a*b*d^2*(F^(f*g*x+e*g))^
n*(d*x+c)/f^3/g^3/n^3/ln(F)^3+3/4*b^2*d^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^3
/g^3/n^3/ln(F)^3-6*a*b*d*(F^(f*g*x+e*g))^n*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2-3/
4*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2+2*a*b*(F^(f*g*x
+e*g))^n*(d*x+c)^3/f/g/n/ln(F)+1/2*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^3/f/g/
n/ln(F)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx=\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-\frac{6abd(c+dx)^2\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{2ab(c+dx)^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{12abd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}+\frac{3b^2d^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}-\frac{3b^2d(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}+\frac{b^2(c+dx)^3\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{3b^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^3,x]
```

```
[Out] (a^2*(c + d*x)^4)/(4*d) - (12*a*b*d^3*(F^(e*g + f*g*x))^n)/(f^4*g^4*n^4*Log
[F]^4) - (3*b^2*d^3*(F^(e*g + f*g*x))^(2*n))/(8*f^4*g^4*n^4*Log[F]^4) + (12
*a*b*d^2*(F^(e*g + f*g*x))^n*(c + d*x))/(f^3*g^3*n^3*Log[F]^3) + (3*b^2*d^2
*(F^(e*g + f*g*x))^(2*n)*(c + d*x))/(4*f^3*g^3*n^3*Log[F]^3) - (6*a*b*d*(F^
(e*g + f*g*x))^n*(c + d*x)^2)/(f^2*g^2*n^2*Log[F]^2) - (3*b^2*d*(F^(e*g + f
*g*x))^(2*n)*(c + d*x)^2)/(4*f^2*g^2*n^2*Log[F]^2) + (2*a*b*(F^(e*g + f*g*x
))^n*(c + d*x)^3)/(f*g*n*Log[F]) + (b^2*(F^(e*g + f*g*x))^(2*n)*(c + d*x)^3
)/(2*f*g*n*Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2(c+dx)^3 + 2ab\left(F^{eg+fgx}\right)^n (c+dx)^3 + b^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3\right) dx \\
&= \frac{a^2(c+dx)^4}{4d} + (2ab)\int \left(F^{eg+fgx}\right)^n (c+dx)^3\,dx + b^2\int \left(F^{eg+fgx}\right)^{2n} (c+dx)^3\,dx \\
&= \frac{a^2(c+dx)^4}{4d} + \frac{2ab\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3}{2fgn\log(F)} \\
&\quad - \frac{(6abd)\int \left(F^{eg+fgx}\right)^n (c+dx)^2\,dx}{fgn\log(F)} - \frac{(3b^2d)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)^2\,dx}{2fgn\log(F)} \\
&= \frac{a^2(c+dx)^4}{4d} - \frac{6abd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)} - \frac{3b^2d\left(F^{eg+fgx}\right)^{2n} (c+dx)^2}{4f^2g^2n^2\log^2(F)} \\
&\quad + \frac{2ab\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3}{2fgn\log(F)} \\
&\quad + \frac{(12abd^2)\int \left(F^{eg+fgx}\right)^n (c+dx)\,dx}{f^2g^2n^2\log^2(F)} + \frac{(3b^2d^2)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)\,dx}{2f^2g^2n^2\log^2(F)} \\
&= \frac{a^2(c+dx)^4}{4d} + \frac{12abd^2\left(F^{eg+fgx}\right)^n (c+dx)}{f^3g^3n^3\log^3(F)} + \frac{3b^2d^2\left(F^{eg+fgx}\right)^{2n} (c+dx)}{4f^3g^3n^3\log^3(F)} \\
&\quad - \frac{6abd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)} - \frac{3b^2d\left(F^{eg+fgx}\right)^{2n} (c+dx)^2}{4f^2g^2n^2\log^2(F)} \\
&\quad + \frac{2ab\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3}{2fgn\log(F)} \\
&\quad - \frac{(12abd^3)\int \left(F^{eg+fgx}\right)^n\,dx}{f^3g^3n^3\log^3(F)} - \frac{(3b^2d^3)\int \left(F^{eg+fgx}\right)^{2n}\,dx}{4f^3g^3n^3\log^3(F)} \\
&= \frac{a^2(c+dx)^4}{4d} - \frac{12abd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} - \frac{3b^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)} + \frac{12abd^2\left(F^{eg+fgx}\right)^n (c+dx)}{f^3g^3n^3\log^3(F)} \\
&\quad + \frac{3b^2d^2\left(F^{eg+fgx}\right)^{2n} (c+dx)}{4f^3g^3n^3\log^3(F)} - \frac{6abd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)} \\
&\quad - \frac{3b^2d\left(F^{eg+fgx}\right)^{2n} (c+dx)^2}{4f^2g^2n^2\log^2(F)} + \frac{2ab\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3}{2fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.74

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = a^2c^3x+\frac{3}{2}a^2c^2dx^2+a^2cd^2x^3+\frac{1}{4}a^2d^3x^4$$
$$+\frac{2ab\left(F^{g(e+fx)}\right)^n\left(-6d^3+6d^2fgn(c+dx)\log(F)-3df^2g^2n^2(c+dx)^2\log^2(F)+f^3g^3n^3(c+dx)^3\log^3(F\right.}{f^4g^4n^4\log^4(F)}$$
$$+\frac{b^2\left(F^{g(e+fx)}\right)^{2n}\left(-3d^3+6d^2fgn(c+dx)\log(F)-6df^2g^2n^2(c+dx)^2\log^2(F)+4f^3g^3n^3(c+dx)^3\log^3(F\right.}{8f^4g^4n^4\log^4(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^3,x]
```

```
[Out] a^2*c^3*x + (3*a^2*c^2*d*x^2)/2 + a^2*c*d^2*x^3 + (a^2*d^3*x^4)/4 + (2*a*b*
(F^(g*(e + f*x)))^n*(-6*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 3*d*f^2*g^2*n^
2*(c + d*x)^2*Log[F]^2 + f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(f^4*g^4*n^4*Lo
g[F]^4) + (b^2*(F^(g*(e + f*x)))^(2*n)*(-3*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[
F] - 6*d*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2 + 4*f^3*g^3*n^3*(c + d*x)^3*Log[F
]^3))/(8*f^4*g^4*n^4*Log[F]^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(312) = 624.

Time = 1.09 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.30

method	result
parallelrisch	$\frac{12\ln(F)^3x^2\left(F^{g(fx+e)}\right)^{2n}b^2cd^2f^3g^3n^3+12\ln(F)^3x\left(F^{g(fx+e)}\right)^{2n}b^2c^2df^3g^3n^3-48\ln(F)^2x^2\left(F^{g(fx+e)}\right)^{n}abd^3f^2g^2n^2-12\ln}{}$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(12*ln(F)^3*x^2*((F^(g*(f*x+e)))^n)^2*b^2*c*d^2*f^3*g^3*n^3+12*ln(F)^3*
x*((F^(g*(f*x+e)))^n)^2*b^2*c^2*d*f^3*g^3*n^3-48*ln(F)^2*x^2*(F^(g*(f*x+e))
)^n*a*b*d^3*f^2*g^2*n^2-12*ln(F)^2*x*((F^(g*(f*x+e)))^n)^2*b^2*c*d^2*f^2*g^
2*n^2-48*ln(F)^2*(F^(g*(f*x+e)))^n*a*b*c^2*d*f^2*g^2*n^2+96*ln(F)*x*(F^(g*(
f*x+e)))^n*a*b*d^3*f*g*n+96*ln(F)*(F^(g*(f*x+e)))^n*a*b*c*d^2*f*g*n+16*a*b*
d^3*(F^(g*(f*x+e)))^n*x^3*n^3*g^3*f^3*ln(F)^3-3*((F^(g*(f*x+e)))^n)^2*b^2*d
^3-96*(F^(g*(f*x+e)))^n*a*b*d^3+6*ln(F)*((F^(g*(f*x+e)))^n)^2*b^2*c*d^2*f*g
*n-96*ln(F)^2*x*(F^(g*(f*x+e)))^n*a*b*c*d^2*f^2*g^2*n^2+4*ln(F)^3*((F^(g*(f
*x+e)))^n)^2*b^2*c^3*f^3*g^3*n^3+48*ln(F)^3*x^2*(F^(g*(f*x+e)))^n*a*b*c*d^2
*f^3*g^3*n^3+48*ln(F)^3*x*(F^(g*(f*x+e)))^n*a*b*c^2*d*f^3*g^3*n^3+8*a^2*d^2
*c*x^3*n^4*g^4*f^4*ln(F)^4+12*a^2*d*c^2*x^2*n^4*g^4*f^4*ln(F)^4+4*b^2*d^3*(
(F^(g*(f*x+e)))^n)^2*x^3*n^3*g^3*f^3*ln(F)^3+16*ln(F)^3*(F^(g*(f*x+e)))^n*a
*b*c^3*f^3*g^3*n^3-6*ln(F)^2*x^2*((F^(g*(f*x+e)))^n)^2*b^2*d^3*f^2*g^2*n^2-
```

```
6*ln(F)^2*((F^(g*(f*x+e)))^n)^2*b^2*c^2*d*f^2*g^2*n^2+6*ln(F)*x*((F^(g*(f*x
+e)))^n)^2*b^2*d^3*f*g*n+2*a^2*d^3*x^4*n^4*g^4*f^4*ln(F)^4+8*a^2*c^3*x*n^4*
g^4*f^4*ln(F)^4)/n^4/g^4/f^4/ln(F)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.50

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx$$
$$= \frac{2\,(a^2d^3f^4g^4n^4x^4+4\,a^2cd^2f^4g^4n^4x^3+6\,a^2c^2df^4g^4n^4x^2+4\,a^2c^3f^4g^4n^4x)\log(F)^4-\left(3\,b^2d^3-4\,(b^2d^3f^3g^3n^3\right.}{}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(a^2*d^3*f^4*g^4*n^4*x^4 + 4*a^2*c*d^2*f^4*g^4*n^4*x^3 + 6*a^2*c^2*d
*f^4*g^4*n^4*x^2 + 4*a^2*c^3*f^4*g^4*n^4*x)*log(F)^4 - (3*b^2*d^3 - 4*(b^2*
d^3*f^3*g^3*n^3*x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3
*x + b^2*c^3*f^3*g^3*n^3)*log(F)^3 + 6*(b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2*c*d
^2*f^2*g^2*n^2*x + b^2*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(b^2*d^3*f*g*n*x + b
^2*c*d^2*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) - 16*(6*a*b*d^3 - (a*b*d^3*
f^3*g^3*n^3*x^3 + 3*a*b*c*d^2*f^3*g^3*n^3*x^2 + 3*a*b*c^2*d*f^3*g^3*n^3*x +
 a*b*c^3*f^3*g^3*n^3)*log(F)^3 + 3*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f
^2*g^2*n^2*x + a*b*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(a*b*d^3*f*g*n*x + a*b*c
*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^4*g^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(323) = 646.

Time = 4.99 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.84

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx$$
$$= \begin{cases} (a+b)^2\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \\ \left(a+b\left(F^{eg}\right)^n\right)^2\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \\ (a+b)^2\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \\ a^2c^3x+\frac{3a^2c^2dx^2}{2}+a^2cd^2x^3+\frac{a^2d^3x^4}{4}+\frac{2abc^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{6abc^2dx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{6abc^2d\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{6abcd^2x^2\left(F^{eg+fgx}\right.}{fgn\log(F)} \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**3,x)
```

```
[Out] Piecewise(((a + b)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**2*(c
**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(f, 0)), ((a + b)**
2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) | Eq(g,
0) | Eq(n, 0)), (a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a*
*2*d**3*x**4/4 + 2*a*b*c**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 6*a*b*c*
*2*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*a*b*c**2*d*(F**(e*g + f*g*x
))**n/(f**2*g**2*n**2*log(F)**2) + 6*a*b*c*d**2*x**2*(F**(e*g + f*g*x))**n/
(f*g*n*log(F)) - 12*a*b*c*d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(
F)**2) + 12*a*b*c*d**2*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) + 2
*a*b*d**3*x**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*a*b*d**3*x**2*(F**(
e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 12*a*b*d**3*x*(F**(e*g + f*g*
x))**n/(f**3*g**3*n**3*log(F)**3) - 12*a*b*d**3*(F**(e*g + f*g*x))**n/(f**4
*g**4*n**4*log(F)**4) + b**2*c**3*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)
) + 3*b**2*c**2*d*x*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 3*b**2*c**
2*d*(F**(e*g + f*g*x))**(2*n)/(4*f**2*g**2*n**2*log(F)**2) + 3*b**2*c*d**2*
x**2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 3*b**2*c*d**2*x*(F**(e*g
+ f*g*x))**(2*n)/(2*f**2*g**2*n**2*log(F)**2) + 3*b**2*c*d**2*(F**(e*g + f*
g*x))**(2*n)/(4*f**3*g**3*n**3*log(F)**3) + b**2*d**3*x**3*(F**(e*g + f*g*x
))**(2*n)/(2*f*g*n*log(F)) - 3*b**2*d**3*x**2*(F**(e*g + f*g*x))**(2*n)/(4*
f**2*g**2*n**2*log(F)**2) + 3*b**2*d**3*x*(F**(e*g + f*g*x))**(2*n)/(4*f**3
*g**3*n**3*log(F)**3) - 3*b**2*d**3*(F**(e*g + f*g*x))**(2*n)/(8*f**4*g**4*
n**4*log(F)**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.76

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^3\,dx \\
&= \frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + a^2c^3x + \frac{2F^{fgnx+egn}abc^3}{fgn\log(F)} + \frac{F^{2fgnx+2egn}b^2c^3}{2fgn\log(F)} \\
&\quad + \frac{6\left(F^{egn}fgnx\log(F) - F^{egn}\right)F^{fgnx}abc^2d}{f^2g^2n^2\log(F)^2} + \frac{3\left(2F^{2egn}fgnx\log(F) - F^{2egn}\right)F^{2fgnx}b^2c^2d}{4f^2g^2n^2\log(F)^2} \\
&\quad + \frac{6\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2F^{egn}fgnx\log(F) + 2F^{egn}\right)F^{fgnx}abcd^2}{f^3g^3n^3\log(F)^3} \\
&\quad + \frac{3\left(2F^{2egn}f^2g^2n^2x^2\log(F)^2 - 2F^{2egn}fgnx\log(F) + F^{2egn}\right)F^{2fgnx}b^2cd^2}{4f^3g^3n^3\log(F)^3} \\
&\quad + \frac{2\left(F^{egn}f^3g^3n^3x^3\log(F)^3 - 3F^{egn}f^2g^2n^2x^2\log(F)^2 + 6F^{egn}fgnx\log(F) - 6F^{egn}\right)F^{fgnx}abd^3}{f^4g^4n^4\log(F)^4} \\
&\quad + \frac{\left(4F^{2egn}f^3g^3n^3x^3\log(F)^3 - 6F^{2egn}f^2g^2n^2x^2\log(F)^2 + 6F^{2egn}fgnx\log(F) - 3F^{2egn}\right)F^{2fgnx}b^2d^3}{8f^4g^4n^4\log(F)^4}
\end{aligned}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x + 2*F^(f*g*
n*x + e*g*n)*a*b*c^3/(f*g*n*log(F)) + 1/2*F^(2*f*g*n*x + 2*e*g*n)*b^2*c^3/(
f*g*n*log(F)) + 6*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*a*b*c^
2*d/(f^2*g^2*n^2*log(F)^2) + 3/4*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n
))*F^(2*f*g*n*x)*b^2*c^2*d/(f^2*g^2*n^2*log(F)^2) + 6*(F^(e*g*n)*f^2*g^2*n^
2*x^2*log(F)^2 - 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*a*b*
c*d^2/(f^3*g^3*n^3*log(F)^3) + 3/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2
- 2*F^(2*e*g*n)*f*g*n*x*log(F) + F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*c*d^2/(f^3*
g^3*n^3*log(F)^3) + 2*(F^(e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 3*F^(e*g*n)*f^2
*g^2*n^2*x^2*log(F)^2 + 6*F^(e*g*n)*f*g*n*x*log(F) - 6*F^(e*g*n))*F^(f*g*n*
x)*a*b*d^3/(f^4*g^4*n^4*log(F)^4) + 1/8*(4*F^(2*e*g*n)*f^3*g^3*n^3*x^3*log(
F)^3 - 6*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(2*e*g*n)*f*g*n*x*log(F
) - 3*F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*d^3/(f^4*g^4*n^4*log(F)^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 12013, normalized size of antiderivative = 37.31

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x - 1/4*(((6*
pi^2*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi^2*b^2*d^3*f^3*g^3*n^
3*x^3*log(abs(F)) + 4*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 18*pi^2*b^2*c
*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi^2*b^2*c*d^2*f^3*g^3*n^3*x^2
*log(abs(F)) + 12*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 18*pi^2*b^2*c^2
*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi^2*b^2*c^2*d*f^3*g^3*n^3*x*log(a
bs(F)) + 12*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 6*pi^2*b^2*c^3*f^3*g^3*
n^3*log(abs(F))*sgn(F) - 6*pi^2*b^2*c^3*f^3*g^3*n^3*log(abs(F)) + 4*b^2*c^3
*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*b^2*d^3*f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2
*b^2*d^3*f^2*g^2*n^2*x^2 - 6*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2
*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*b^2*c*d^2*f^2*g^2*n^2*x - 12*b^2*c
*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b^2*c^2*d*f^2*g^2*n^2*sgn(F) + 3*
pi^2*b^2*c^2*d*f^2*g^2*n^2 - 6*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 6*b^2*
d^3*f*g*n*x*log(abs(F)) + 6*b^2*c*d^2*f*g*n*log(abs(F)) - 3*b^2*d^3)*(pi^4*
f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4
*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi
^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*
g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2
 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*s
```

gn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) - 4
*(2*pi^3*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) - 6*pi*b^2*d^3*f^3*g^3*n^3*x^3*log(
abs(F))^2*sgn(F) - 2*pi^3*b^2*d^3*f^3*g^3*n^3*x^3 + 6*pi*b^2*d^3*f^3*g^3*n^
3*x^3*log(abs(F))^2 + 6*pi^3*b^2*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 18*pi*b^2*c
*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*b^2*c*d^2*f^3*g^3*n^3*x^
2 + 18*pi*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 6*pi^3*b^2*c^2*d*f^3*g^
3*n^3*x*sgn(F) - 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 6*pi^
3*b^2*c^2*d*f^3*g^3*n^3*x + 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 2
*pi^3*b^2*c^3*f^3*g^3*n^3*sgn(F) - 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2*s
gn(F) - 2*pi^3*b^2*c^3*f^3*g^3*n^3 + 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2
 + 6*pi*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*b^2*d^3*f^2*g^2*n
^2*x^2*log(abs(F)) + 12*pi*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*
pi*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*b^2*c^2*d*f^2*g^2*n^2*log(abs
(F))*sgn(F) - 6*pi*b^2*c^2*d*f^2*g^2*n^2*log(abs(F)) - 3*pi*b^2*d^3*f*g*n*x
*sgn(F) + 3*pi*b^2*d^3*f*g*n*x - 3*pi*b^2*c*d^2*f*g*n*sgn(F) + 3*pi*b^2*c*d
^2*f*g*n)*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))
^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((
pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^
4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)
^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3
*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*
cos(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n) - ((2*pi^
3*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) - 6*pi*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))
^2*sgn(F) - 2*pi^3*b^2*d^3*f^3*g^3*n^3*x^3 + 6*pi*b^2*d^3*f^3*g^3*n^3*x^3*l
og(abs(F))^2 + 6*pi^3*b^2*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 18*pi*b^2*c*d^2*f^
3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 18*
pi*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 6*pi^3*b^2*c^2*d*f^3*g^3*n^3*x
*sgn(F) - 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 6*pi^3*b^2*c
^2*d*f^3*g^3*n^3*x + 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 2*pi^3*b
^2*c^3*f^3*g^3*n^3*sgn(F) - 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) -
 2*pi^3*b^2*c^3*f^3*g^3*n^3 + 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi
*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*b^2*d^3*f^2*g^2*n^2*x^2*
log(abs(F)) + 12*pi*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*b^2*
c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))*sg
n(F) - 6*pi*b^2*c^2*d*f^2*g^2*n^2*log(abs(F)) - 3*pi*b^2*d^3*f*g*n*x*sgn(F)
 + 3*pi*b^2*d^3*f*g*n*x - 3*pi*b^2*c*d^2*f*g*n*sgn(F) + 3*pi*b^2*c*d^2*f*g*
n)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^
4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F)
)^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) -
pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs
(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(ab
s(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^
3)^2) + 4*(6*pi^2*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi^2*b^2*d
^3*f^3*g^3*n^3*x^3*log(abs(F)) + 4*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 +
18*pi^2*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi^2*b^2*c*d^2*f^

3*g^3*n^3*x^2*log(abs(F)) + 12*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 18
*pi^2*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi^2*b^2*c^2*d*f^3*g^
3*n^3*x*log(abs(F)) + 12*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 6*pi^2*b^2
*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi^2*b^2*c^3*f^3*g^3*n^3*log(abs(F)
) + 4*b^2*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*b^2*d^3*f^2*g^2*n^2*x^2*sg
n(F) + 3*pi^2*b^2*d^3*f^2*g^2*n^2*x^2 - 6*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F
))^2 - 6*pi^2*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*b^2*c*d^2*f^2*g^2*n^2
*x - 12*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b^2*c^2*d*f^2*g^2*n^
2*sgn(F) + 3*pi^2*b^2*c^2*d*f^2*g^2*n^2 - 6*b^2*c^2*d*f^2*g^2*n^2*log(abs(F
))^2 + 6*b^2*d^3*f*g*n*x*log(abs(F)) + 6*b^2*c*d^2*f*g*n*log(abs(F)) - 3*b^
2*d^3)*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*
sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^
4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g
^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2
+ 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sg
n(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*sin
(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n))*e^(2*f*g*n*
x*log(abs(F)) + 2*e*g*n*log(abs(F))) - 1/8*I*((2*pi^3*b^2*d^3*f^3*g^3*n^3*x
^3*sgn(F) + 6*I*pi^2*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi*b^2*
d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - 2*pi^3*b^2*d^3*f^3*g^3*n^3*x^3 -
6*I*pi^2*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 6*pi*b^2*d^3*f^3*g^3*n^3*x^
3*log(abs(F))^2 + 4*I*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 6*pi^3*b^2*c*
d^2*f^3*g^3*n^3*x^2*sgn(F) + 18*I*pi^2*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)
)*sgn(F) - 18*pi*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*b^
2*c*d^2*f^3*g^3*n^3*x^2 - 18*I*pi^2*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) +
18*pi*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 12*I*b^2*c*d^2*f^3*g^3*n^3
*x^2*log(abs(F))^3 + 6*pi^3*b^2*c^2*d*f^3*g^3*n^3*x*sgn(F) + 18*I*pi^2*b^2*
c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(
abs(F))^2*sgn(F) - 6*pi^3*b^2*c^2*d*f^3*g^3*n^3*x - 18*I*pi^2*b^2*c^2*d*f^3
*g^3*n^3*x*log(abs(F)) + 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 12*I
*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 2*pi^3*b^2*c^3*f^3*g^3*n^3*sgn(F)
+ 6*I*pi^2*b^2*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi*b^2*c^3*f^3*g^3*n^
3*log(abs(F))^2*sgn(F) - 2*pi^3*b^2*c^3*f^3*g^3*n^3 - 6*I*pi^2*b^2*c^3*f^3*
g^3*n^3*log(abs(F)) + 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2 + 4*I*b^2*c^3*
f^3*g^3*n^3*log(abs(F))^3 - 3*I*pi^2*b^2*d^3*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*
b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 3*I*pi^2*b^2*d^3*f^2*g^2*n^2*x
^2 - 6*pi*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F)) - 6*I*b^2*d^3*f^2*g^2*n^2*x^2
*log(abs(F))^2 - 6*I*pi^2*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 12*pi*b^2*c*d^2*
f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 6*I*pi^2*b^2*c*d^2*f^2*g^2*n^2*x - 12*pi
*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F)) - 12*I*b^2*c*d^2*f^2*g^2*n^2*x*log(abs
(F))^2 - 3*I*pi^2*b^2*c^2*d*f^2*g^2*n^2*sgn(F) + 6*pi*b^2*c^2*d*f^2*g^2*n^2
*log(abs(F))*sgn(F) + 3*I*pi^2*b^2*c^2*d*f^2*g^2*n^2 - 6*pi*b^2*c^2*d*f^2*g
^2*n^2*log(abs(F)) - 6*I*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 3*pi*b^2*d^3
*f*g*n*x*sgn(F) + 3*pi*b^2*d^3*f*g*n*x + 6*I*b^2*d^3*f*g*n*x*log(abs(F)) -
3*pi*b^2*c*d^2*f*g*n*sgn(F) + 3*pi*b^2*c*d^2*f*g*n + 6*I*b^2*c*d^2*f*g*n*lo

g(abs(F)) - 3*I*b^2*d^3)*e^(I*pi*f*g*n*x*sgn(F) - I*pi*f*g*n*x + I*pi*e*g*n
*sgn(F) - I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) + 4*I*pi^3*f^4*g^4*n^4*log(a
bs(F))*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - 4*I*pi*f^4*g^4*n^
4*log(abs(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 - 4*I*pi^3*f^4*g^4*n^4*log(abs(F)
) + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2
*f^4*g^4*n^4*log(abs(F))^4) + (2*pi^3*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) - 6*I*
pi^2*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi*b^2*d^3*f^3*g^3*n^3*
x^3*log(abs(F))^2*sgn(F) - 2*pi^3*b^2*d^3*f^3*g^3*n^3*x^3 + 6*I*pi^2*b^2*d^
3*f^3*g^3*n^3*x^3*log(abs(F)) + 6*pi*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2
- 4*I*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 6*pi^3*b^2*c*d^2*f^3*g^3*n^3*
x^2*sgn(F) - 18*I*pi^2*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi
*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*b^2*c*d^2*f^3*g^3*
n^3*x^2 + 18*I*pi^2*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 18*pi*b^2*c*d^2
*f^3*g^3*n^3*x^2*log(abs(F))^2 - 12*I*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))
^3 + 6*pi^3*b^2*c^2*d*f^3*g^3*n^3*x*sgn(F) - 18*I*pi^2*b^2*c^2*d*f^3*g^3*n^
3*x*log(abs(F))*sgn(F) - 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F)
- 6*pi^3*b^2*c^2*d*f^3*g^3*n^3*x + 18*I*pi^2*b^2*c^2*d*f^3*g^3*n^3*x*log(a
bs(F)) + 18*pi*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 - 12*I*b^2*c^2*d*f^3*g
^3*n^3*x*log(abs(F))^3 + 2*pi^3*b^2*c^3*f^3*g^3*n^3*sgn(F) - 6*I*pi^2*b^2*c
^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2*
sgn(F) - 2*pi^3*b^2*c^3*f^3*g^3*n^3 + 6*I*pi^2*b^2*c^3*f^3*g^3*n^3*log(abs(
F)) + 6*pi*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2 - 4*I*b^2*c^3*f^3*g^3*n^3*log(
abs(F))^3 + 3*I*pi^2*b^2*d^3*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*b^2*d^3*f^2*g^2*
n^2*x^2*log(abs(F))*sgn(F) - 3*I*pi^2*b^2*d^3*f^2*g^2*n^2*x^2 - 6*pi*b^2*d^
3*f^2*g^2*n^2*x^2*log(abs(F)) + 6*I*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 +
6*I*pi^2*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 12*pi*b^2*c*d^2*f^2*g^2*n^2*x*lo
g(abs(F))*sgn(F) - 6*I*pi^2*b^2*c*d^2*f^2*g^2*n^2*x - 12*pi*b^2*c*d^2*f^2*g
^2*n^2*x*log(abs(F)) + 12*I*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 + 3*I*pi^
2*b^2*c^2*d*f^2*g^2*n^2*sgn(F) + 6*pi*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn
(F) - 3*I*pi^2*b^2*c^2*d*f^2*g^2*n^2 - 6*pi*b^2*c^2*d*f^2*g^2*n^2*log(abs(F
)) + 6*I*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 3*pi*b^2*d^3*f*g*n*x*sgn(F)
+ 3*pi*b^2*d^3*f*g*n*x - 6*I*b^2*d^3*f*g*n*x*log(abs(F)) - 3*pi*b^2*c*d^2*f
*g*n*sgn(F) + 3*pi*b^2*c*d^2*f*g*n - 6*I*b^2*c*d^2*f*g*n*log(abs(F)) + 3*I*
b^2*d^3)*e^(-I*pi*f*g*n*x*sgn(F) + I*pi*f*g*n*x - I*pi*e*g*n*sgn(F) + I*pi*
e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) - 4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) -
6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3
*sgn(F) - pi^4*f^4*g^4*n^4 + 4*I*pi^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*f^4*
g^4*n^4*log(abs(F))^2 - 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^4*lo
g(abs(F))^4))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F))) - 2*(((3*pi^2
*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*a*b*d^3*f^3*g^3*n^3*x^
3*log(abs(F)) + 2*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*a*b*c*d^2*
f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi^2*a*b*c*d^2*f^3*g^3*n^3*x^2*log(a
bs(F)) + 6*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 9*pi^2*a*b*c^2*d*f^3*g
^3*n^3*x*log(abs(F))*sgn(F) - 9*pi^2*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) +
6*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 3*pi^2*a*b*c^3*f^3*g^3*n^3*log(ab

s(F))*sgn(F) - 3*pi^2*a*b*c^3*f^3*g^3*n^3*log(abs(F)) + 2*a*b*c^3*f^3*g^3*n
^3*log(abs(F))^3 - 3*pi^2*a*b*d^3*f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*a*b*d^3*f
^2*g^2*n^2*x^2 - 6*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2*a*b*c*d^2
*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*a*b*c*d^2*f^2*g^2*n^2*x - 12*a*b*c*d^2*f^2*g
^2*n^2*x*log(abs(F))^2 - 3*pi^2*a*b*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*a*b*c
^2*d*f^2*g^2*n^2 - 6*a*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 12*a*b*d^3*f*g*n
*x*log(abs(F)) + 12*a*b*c*d^2*f*g*n*log(abs(F)) - 12*a*b*d^3)*(pi^4*f^4*g^4
*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 +
6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4*
g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4
 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(
pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) -
 pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) - 4*(pi^3*
a*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2
*sgn(F) - pi^3*a*b*d^3*f^3*g^3*n^3*x^3 + 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(a
bs(F))^2 + 3*pi^3*a*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*pi*a*b*c*d^2*f^3*g^3
*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c*d^2*f^3*g^3*n^3*x^2 + 9*pi*a*b
*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 3*pi^3*a*b*c^2*d*f^3*g^3*n^3*x*sgn(F
) - 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c^2*d*f^
3*g^3*n^3*x + 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + pi^3*a*b*c^3*f^3
*g^3*n^3*sgn(F) - 3*pi*a*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*a*b*
c^3*f^3*g^3*n^3 + 3*pi*a*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*a*b*d^3*f^2
*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))
+ 12*pi*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*a*b*c*d^2*f^2*g^
2*n^2*x*log(abs(F)) + 6*pi*a*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*
a*b*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*pi*a*b*d^3*f*g*n*x*sgn(F) + 6*pi*a*b*
d^3*f*g*n*x - 6*pi*a*b*c*d^2*f*g*n*sgn(F) + 6*pi*a*b*c*d^2*f*g*n)*(pi^3*f^4
*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^
4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sg
n(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*
f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g
^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*
g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*cos(-1/2*pi*f*g*n*x
*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n) - ((pi^3*a*b
*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sg
n(F) - pi^3*a*b*d^3*f^3*g^3*n^3*x^3 + 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(
F))^2 + 3*pi^3*a*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*pi*a*b*c*d^2*f^3*g^3*n^
3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c*d^2*f^3*g^3*n^3*x^2 + 9*pi*a*b*c*
d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 3*pi^3*a*b*c^2*d*f^3*g^3*n^3*x*sgn(F) -
 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c^2*d*f^3*g
^3*n^3*x + 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + pi^3*a*b*c^3*f^3*g^
3*n^3*sgn(F) - 3*pi*a*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*a*b*c^3
*f^3*g^3*n^3 + 3*pi*a*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*a*b*d^3*f^2*g^
2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 1
2*pi*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*a*b*c*d^2*f^2*g^2*n

^2*x*log(abs(F)) + 6*pi*a*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*a*b
*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*pi*a*b*d^3*f*g*n*x*sgn(F) + 6*pi*a*b*d^3
*f*g*n*x - 6*pi*a*b*c*d^2*f*g*n*sgn(F) + 6*pi*a*b*c*d^2*f*g*n)*(pi^4*f^4*g^
4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 +
 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4
*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^
4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*
(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F)
- pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) + 4*(3*pi
^2*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*a*b*d^3*f^3*g^3*n^3*
x^3*log(abs(F)) + 2*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*a*b*c*d^
2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi^2*a*b*c*d^2*f^3*g^3*n^3*x^2*log
(abs(F)) + 6*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 9*pi^2*a*b*c^2*d*f^3
*g^3*n^3*x*log(abs(F))*sgn(F) - 9*pi^2*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))
+ 6*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 3*pi^2*a*b*c^3*f^3*g^3*n^3*log(
abs(F))*sgn(F) - 3*pi^2*a*b*c^3*f^3*g^3*n^3*log(abs(F)) + 2*a*b*c^3*f^3*g^3
*n^3*log(abs(F))^3 - 3*pi^2*a*b*d^3*f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*a*b*d^3
*f^2*g^2*n^2*x^2 - 6*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2*a*b*c*d
^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*a*b*c*d^2*f^2*g^2*n^2*x - 12*a*b*c*d^2*f^2
*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*a*b*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*a*b
*c^2*d*f^2*g^2*n^2 - 6*a*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 12*a*b*d^3*f*g
*n*x*log(abs(F)) + 12*a*b*c*d^2*f*g*n*log(abs(F)) - 12*a*b*d^3)*(pi^3*f^4*g
^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*
g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(
F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^
4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4
*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^
4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*sin(-1/2*pi*f*g*n*x*s
gn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*e^(f*g*n*x*lo
g(abs(F)) + e*g*n*log(abs(F))) - I*((pi^3*a*b*d^3*f^3*g^3*n^3*x^3*sgn(F) +
3*I*pi^2*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi*a*b*d^3*f^3*g^3*
n^3*x^3*log(abs(F))^2*sgn(F) - pi^3*a*b*d^3*f^3*g^3*n^3*x^3 - 3*I*pi^2*a*b*
d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^
2 + 2*I*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 3*pi^3*a*b*c*d^2*f^3*g^3*n^
3*x^2*sgn(F) + 9*I*pi^2*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi
*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c*d^2*f^3*g^3*
n^3*x^2 - 9*I*pi^2*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 9*pi*a*b*c*d^2*f
^3*g^3*n^3*x^2*log(abs(F))^2 + 6*I*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3
+ 3*pi^3*a*b*c^2*d*f^3*g^3*n^3*x*sgn(F) + 9*I*pi^2*a*b*c^2*d*f^3*g^3*n^3*x*
log(abs(F))*sgn(F) - 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*
pi^3*a*b*c^2*d*f^3*g^3*n^3*x - 9*I*pi^2*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))
 + 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 6*I*a*b*c^2*d*f^3*g^3*n^3*x
*log(abs(F))^3 + pi^3*a*b*c^3*f^3*g^3*n^3*sgn(F) + 3*I*pi^2*a*b*c^3*f^3*g^3
*n^3*log(abs(F))*sgn(F) - 3*pi*a*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - p
i^3*a*b*c^3*f^3*g^3*n^3 - 3*I*pi^2*a*b*c^3*f^3*g^3*n^3*log(abs(F)) + 3*pi*a

```
*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 2*I*a*b*c^3*f^3*g^3*n^3*log(abs(F))^3 -
3*I*pi^2*a*b*d^3*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*a*b*d^3*f^2*g^2*n^2*x^2*log(
abs(F))*sgn(F) + 3*I*pi^2*a*b*d^3*f^2*g^2*n^2*x^2 - 6*pi*a*b*d^3*f^2*g^2*n^
2*x^2*log(abs(F)) - 6*I*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*I*pi^2*a*
b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 12*pi*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sg
n(F) + 6*I*pi^2*a*b*c*d^2*f^2*g^2*n^2*x - 12*pi*a*b*c*d^2*f^2*g^2*n^2*x*log
(abs(F)) - 12*I*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*I*pi^2*a*b*c^2*d*
f^2*g^2*n^2*sgn(F) + 6*pi*a*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) + 3*I*pi
^2*a*b*c^2*d*f^2*g^2*n^2 - 6*pi*a*b*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*I*a*b
*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 6*pi*a*b*d^3*f*g*n*x*sgn(F) + 6*pi*a*b*d
^3*f*g*n*x + 12*I*a*b*d^3*f*g*n*x*log(abs(F)) - 6*pi*a*b*c*d^2*f*g*n*sgn(F)
 + 6*pi*a*b*c*d^2*f*g*n + 12*I*a*b*c*d^2*f*g*n*log(abs(F)) - 12*I*a*b*d^3)*
e^(1/2*I*pi*f*g*n*x*sgn(F) - 1/2*I*pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2
*I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) + 4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sg
n(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - 4*I*pi*f^4*g^4*n^4*log(abs
(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 - 4*I*pi^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^
2*f^4*g^4*n^4*log(abs(F))^2 + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*
n^4*log(abs(F))^4) + (pi^3*a*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*I*pi^2*a*b*d^
3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(abs
(F))^2*sgn(F) - pi^3*a*b*d^3*f^3*g^3*n^3*x^3 + 3*I*pi^2*a*b*d^3*f^3*g^3*n^3
*x^3*log(abs(F)) + 3*pi*a*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 - 2*I*a*b*d^3
*f^3*g^3*n^3*x^3*log(abs(F))^3 + 3*pi^3*a*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) -
9*I*pi^2*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi*a*b*c*d^2*f^3*
g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c*d^2*f^3*g^3*n^3*x^2 + 9*I*p
i^2*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 9*pi*a*b*c*d^2*f^3*g^3*n^3*x^2*
log(abs(F))^2 - 6*I*a*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 3*pi^3*a*b*c^
2*d*f^3*g^3*n^3*x*sgn(F) - 9*I*pi^2*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn
(F) - 9*pi*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*a*b*c^2*d*
f^3*g^3*n^3*x + 9*I*pi^2*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 9*pi*a*b*c^2
*d*f^3*g^3*n^3*x*log(abs(F))^2 - 6*I*a*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3
+ pi^3*a*b*c^3*f^3*g^3*n^3*sgn(F) - 3*I*pi^2*a*b*c^3*f^3*g^3*n^3*log(abs(F)
)*sgn(F) - 3*pi*a*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*a*b*c^3*f^3
*g^3*n^3 + 3*I*pi^2*a*b*c^3*f^3*g^3*n^3*log(abs(F)) + 3*pi*a*b*c^3*f^3*g^3*
n^3*log(abs(F))^2 - 2*I*a*b*c^3*f^3*g^3*n^3*log(abs(F))^3 + 3*I*pi^2*a*b*d^
3*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F)
- 3*I*pi^2*a*b*d^3*f^2*g^2*n^2*x^2 - 6*pi*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F
)) + 6*I*a*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 + 6*I*pi^2*a*b*c*d^2*f^2*g^2
*n^2*x*sgn(F) + 12*pi*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 6*I*pi^2
*a*b*c*d^2*f^2*g^2*n^2*x - 12*pi*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 12*I
*a*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 + 3*I*pi^2*a*b*c^2*d*f^2*g^2*n^2*sgn
(F) + 6*pi*a*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 3*I*pi^2*a*b*c^2*d*f^
2*g^2*n^2 - 6*pi*a*b*c^2*d*f^2*g^2*n^2*log(abs(F)) + 6*I*a*b*c^2*d*f^2*g^2*
n^2*log(abs(F))^2 - 6*pi*a*b*d^3*f*g*n*x*sgn(F) + 6*pi*a*b*d^3*f*g*n*x - 12
*I*a*b*d^3*f*g*n*x*log(abs(F)) - 6*pi*a*b*c*d^2*f*g*n*sgn(F) + 6*pi*a*b*c*d
^2*f*g*n - 12*I*a*b*c*d^2*f*g*n*log(abs(F)) + 12*I*a*b*d^3)*e^(-1/2*I*pi*f*
```

```
g*n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(
pi^4*f^4*g^4*n^4*sgn(F) - 4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - 6*pi^2*
f^4*g^4*n^4*log(abs(F))^2*sgn(F) + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F)
- pi^4*f^4*g^4*n^4 + 4*I*pi^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*f^4*g^4*n^4*
log(abs(F))^2 - 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^4*log(abs(F)
)^4))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.36

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = a^2\,c^3\,x - \left(F^{f\,g\,x}\,F^{e\,g}\right)^n \left(\frac{2\,a\,b\,\left(-c^3\,f^3\,g^3\,n^3\ln\left(F\right)^3+3\,c^2\,d\,f^2\,g^2\,n^2\ln\left(F\right)^2-6\,c\,d^2\,f\,g\,n\,\ln\left(F\right)+6\,d^3\right)}{f^4\,g^4\,n^4\ln\left(F\right)^4} - \frac{2\,a\,b\,d^3\,x^3}{f\,g\,n\,\ln\left(F\right)} - \frac{6\,a\,b\,d\,x\,\left(c^2\,f^2\,g^2\,n^2\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{f^3\,g^3\,n^3\ln\left(F\right)^3} + \frac{6\,a\,b\,d^2\,x^2\,\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\ln\left(F\right)^2}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n} \left(\frac{b^2\,\left(-4\,c^3\,f^3\,g^3\,n^3\ln\left(F\right)^3+6\,c^2\,d\,f^2\,g^2\,n^2\ln\left(F\right)^2-6\,c\,d^2\,f\,g\,n\,\ln\left(F\right)+3\,d^3\right)}{8\,f^4\,g^4\,n^4\ln\left(F\right)^4} - \frac{b^2\,d^3\,x^3}{2\,f\,g\,n\,\ln\left(F\right)} - \frac{3\,b^2\,d\,x\,\left(2\,c^2\,f^2\,g^2\,n^2\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+d^2\right)}{4\,f^3\,g^3\,n^3\ln\left(F\right)^3} + \frac{3\,b^2\,d^2\,x^2\,\left(d-2\,c\,f\,g\,n\,\ln\left(F\right)\right)}{4\,f^2\,g^2\,n^2\ln\left(F\right)^2}\right) + \frac{a^2\,d^3\,x^4}{4} + \frac{3\,a^2\,c^2\,d\,x^2}{2} + a^2\,c\,d^2\,x^3$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^3,x)
```

```
[Out] a^2*c^3*x - (F^(f*g*x)*F^(e*g))^n*((2*a*b*(6*d^3 - c^3*f^3*g^3*n^3*log(F)^3
 - 6*c*d^2*f*g*n*log(F) + 3*c^2*d*f^2*g^2*n^2*log(F)^2))/(f^4*g^4*n^4*log(F
)^4) - (2*a*b*d^3*x^3)/(f*g*n*log(F)) - (6*a*b*d*x*(2*d^2 + c^2*f^2*g^2*n^2
*log(F)^2 - 2*c*d*f*g*n*log(F)))/(f^3*g^3*n^3*log(F)^3) + (6*a*b*d^2*x^2*(d
 - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) - (F^(f*g*x)*F^(e*g))^(2*n)*((b
^2*(3*d^3 - 4*c^3*f^3*g^3*n^3*log(F)^3 - 6*c*d^2*f*g*n*log(F) + 6*c^2*d*f^2
*g^2*n^2*log(F)^2))/(8*f^4*g^4*n^4*log(F)^4) - (b^2*d^3*x^3)/(2*f*g*n*log(F
)) - (3*b^2*d*x*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/(4
*f^3*g^3*n^3*log(F)^3) + (3*b^2*d^2*x^2*(d - 2*c*f*g*n*log(F)))/(4*f^2*g^2*
n^2*log(F)^2)) + (a^2*d^3*x^4)/4 + (3*a^2*c^2*d*x^2)/2 + a^2*c*d^2*x^3
```

3.33 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2 \, dx = \frac{a^2(c+dx)^3}{3d} + \frac{4abd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} + \frac{b^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)} - \frac{4abd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)} - \frac{b^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2f^2g^2n^2\log^2(F)} + \frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)}$$

```
[Out] 1/3*a^2*(d*x+c)^3/d+4*a*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/ln(F)^3+1/4*b^2
*d^2*(F^(f*g*x+e*g))^(2*n)/f^3/g^3/n^3/ln(F)^3-4*a*b*d*(F^(f*g*x+e*g))^n*(d
*x+c)/f^2/g^2/n^2/ln(F)^2-1/2*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^2/g^2/n
^2/ln(F)^2+2*a*b*(F^(f*g*x+e*g))^n*(d*x+c)^2/f/g/n/ln(F)+1/2*b^2*(F^(f*g*x+
e*g))^(2*n)*(d*x+c)^2/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx = \frac{a^2(c+dx)^3}{3d} - \frac{4abd(c+dx)\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)} + \frac{4abd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} - \frac{b^2d(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2f^2g^2n^2\log^2(F)} + \frac{b^2(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)} + \frac{b^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2,x]
```

```
[Out] (a^2*(c + d*x)^3)/(3*d) + (4*a*b*d^2*(F^(e*g + f*g*x))^n)/(f^3*g^3*n^3*Log[
F]^3) + (b^2*d^2*(F^(e*g + f*g*x))^(2*n))/(4*f^3*g^3*n^3*Log[F]^3) - (4*a*b
*d*(F^(e*g + f*g*x))^n*(c + d*x))/(f^2*g^2*n^2*Log[F]^2) - (b^2*d*(F^(e*g +
 f*g*x))^(2*n)*(c + d*x))/(2*f^2*g^2*n^2*Log[F]^2) + (2*a*b*(F^(e*g + f*g*x
))^n*(c + d*x)^2)/(f*g*n*Log[F]) + (b^2*(F^(e*g + f*g*x))^(2*n)*(c + d*x)^2
)/(2*f*g*n*Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2(c+dx)^2+2ab\left(F^{eg+fgx}\right)^n(c+dx)^2+b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2\right)\,dx\\
&= \frac{a^2(c+dx)^3}{3d}+(2ab)\int\left(F^{eg+fgx}\right)^n(c+dx)^2\,dx+b^2\int\left(F^{eg+fgx}\right)^{2n}(c+dx)^2\,dx\\
&= \frac{a^2(c+dx)^3}{3d}+\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)}\\
&\quad-\frac{(4abd)\int\left(F^{eg+fgx}\right)^n(c+dx)\,dx}{fgn\log(F)}-\frac{(b^2d)\int\left(F^{eg+fgx}\right)^{2n}(c+dx)\,dx}{fgn\log(F)}\\
&= \frac{a^2(c+dx)^3}{3d}-\frac{4abd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}-\frac{b^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2f^2g^2n^2\log^2(F)}\\
&\quad+\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)}\\
&\quad+\frac{(4abd^2)\int\left(F^{eg+fgx}\right)^n\,dx}{f^2g^2n^2\log^2(F)}+\frac{(b^2d^2)\int\left(F^{eg+fgx}\right)^{2n}\,dx}{2f^2g^2n^2\log^2(F)}\\
&= \frac{a^2(c+dx)^3}{3d}+\frac{4abd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}+\frac{b^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}-\frac{4abd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}\\
&\quad-\frac{b^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2f^2g^2n^2\log^2(F)}+\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$
\begin{aligned}
&\int\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2\,dx\\
&= a^2c^2x+a^2cdx^2+\frac{1}{3}a^2d^2x^3\\
&\quad+\frac{2ab\left(F^{g(e+fx)}\right)^n\left(2d^2-2dfgn(c+dx)\log(F)+f^2g^2n^2(c+dx)^2\log^2(F)\right)}{f^3g^3n^3\log^3(F)}\\
&\quad+\frac{b^2\left(F^{g(e+fx)}\right)^{2n}\left(d^2-2dfgn(c+dx)\log(F)+2f^2g^2n^2(c+dx)^2\log^2(F)\right)}{4f^3g^3n^3\log^3(F)}
\end{aligned}
$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2,x]
```

```
[Out] a^2*c^2*x + a^2*c*d*x^2 + (a^2*d^2*x^3)/3 + (2*a*b*(F^(g*(e + f*x)))^n*(2*d
^2 - 2*d*f*g*n*(c + d*x)*Log[F] + f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(f^3*g
^3*n^3*Log[F]^3) + (b^2*(F^(g*(e + f*x)))^(2*n)*(d^2 - 2*d*f*g*n*(c + d*x)*
Log[F] + 2*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(4*f^3*g^3*n^3*Log[F]^3)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.77

method	result
parallelrisch	$\frac{4a^2d^2x^3n^3g^3f^3\ln(F)^3+12a^2dc\,x^2n^3g^3f^3\ln(F)^3+12a^2c^2x\,n^3g^3f^3\ln(F)^3+6x^2\left(F^{g(fx+e)}\right)^{2n}b^2d^2n^2g^2f^2\ln(F)^2+24x^2\left(F^{g(f}\right.}{}$

[In] int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/12*(4*a^2*d^2*x^3*n^3*g^3*f^3*ln(F)^3+12*a^2*d*c*x^2*n^3*g^3*f^3*ln(F)^3+12*a^2*c^2*x*n^3*g^3*f^3*ln(F)^3+6*x^2*((F^(g*(f*x+e)))^n)^2*b^2*d^2*n^2*g^2*f^2*ln(F)^2+24*x^2*(F^(g*(f*x+e)))^n*a*b*d^2*n^2*g^2*f^2*ln(F)^2+12*ln(F)^2*x*((F^(g*(f*x+e)))^n)^2*b^2*c*d*f^2*g^2*n^2+48*ln(F)^2*x*(F^(g*(f*x+e)))^n*a*b*c*d*f^2*g^2*n^2+6*ln(F)^2*((F^(g*(f*x+e)))^n)^2*b^2*c^2*f^2*g^2*n^2+24*ln(F)^2*(F^(g*(f*x+e)))^n*a*b*c^2*f^2*g^2*n^2-6*ln(F)*x*((F^(g*(f*x+e)))^n)^2*b^2*d^2*f*g*n-48*ln(F)*x*(F^(g*(f*x+e)))^n*a*b*d^2*f*g*n-6*ln(F)*((F^(g*(f*x+e)))^n)^2*b^2*c*d*f*g*n-48*ln(F)*(F^(g*(f*x+e)))^n*a*b*c*d*f*g*n+3*((F^(g*(f*x+e)))^n)^2*b^2*d^2+48*(F^(g*(f*x+e)))^n*a*b*d^2)/n^3/g^3/f^3/ln(F)^3

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.20

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2\,dx$$
$$=\frac{4\left(a^2d^2f^3g^3n^3x^3+3\,a^2cdf^3g^3n^3x^2+3\,a^2c^2f^3g^3n^3x\right)\log\left(F\right)^3+3\left(b^2d^2+2\left(b^2d^2f^2g^2n^2x^2+2\,b^2cdf^2g^2n^2x\right.\right.}{}$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x, algorithm="fricas")

[Out] 1/12*(4*(a^2*d^2*f^3*g^3*n^3*x^3 + 3*a^2*c*d*f^3*g^3*n^3*x^2 + 3*a^2*c^2*f^3*g^3*n^3*x)*log(F)^3 + 3*(b^2*d^2 + 2*(b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2*c*d*f^2*g^2*n^2*x + b^2*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(b^2*d^2*f*g*n*x + b^2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 24*(2*a*b*d^2 + (a*b*d^2*f^2*g^2*n^2*x^2 + 2*a*b*c*d*f^2*g^2*n^2*x + a*b*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^3*g^3*n^3*log(F)^3)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(231) = 462.

Time = 1.92 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.21

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2\,dx$$
$$=\begin{cases}(a+b)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\\(a+b(F^{eg})^n)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\\(a+b)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\\a^2c^2x+a^2cdx^2+\frac{a^2d^2x^3}{3}+\frac{2abc^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{4abcdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{4abcd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{2abd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{4abd^2x\left(F^{e\cdots}\right.}{f^2g^2n^2\log\cdots}\end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**2,x)
```

```
[Out] Piecewise(((a + b)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) & Eq(f, 0
) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**2*(c**2*x + c*d*x**2 + d*
*2*x**3/3), Eq(f, 0)), ((a + b)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F,
 1) | Eq(g, 0) | Eq(n, 0)), (a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3
 + 2*a*b*c**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 4*a*b*c*d*x*(F**(e*g +
 f*g*x))**n/(f*g*n*log(F)) - 4*a*b*c*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**
2*log(F)**2) + 2*a*b*d**2*x**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 4*a*b
*d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 4*a*b*d**2*(F**(
e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) + b**2*c**2*(F**(e*g + f*g*x))*
*(2*n)/(2*f*g*n*log(F)) + b**2*c*d*x*(F**(e*g + f*g*x))**(2*n)/(f*g*n*log(F
)) - b**2*c*d*(F**(e*g + f*g*x))**(2*n)/(2*f**2*g**2*n**2*log(F)**2) + b**2
*d**2*x**2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - b**2*d**2*x*(F**(e*
g + f*g*x))**(2*n)/(2*f**2*g**2*n**2*log(F)**2) + b**2*d**2*(F**(e*g + f*g*
x))**(2*n)/(4*f**3*g**3*n**3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.45

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2\,dx = \frac{1}{3}a^2d^2x^3 + a^2cdx^2 + a^2c^2x + \frac{2F^{fgnx+egn}abc^2}{fgn\log(F)} + \frac{F^{2fgnx+2egn}b^2c^2}{2fgn\log(F)} + \frac{4(F^{egn}fgnx\log(F) - F^{egn})F^{fgnx}abcd}{f^2g^2n^2\log(F)^2} + \frac{(2F^{2egn}fgnx\log(F) - F^{2egn})F^{2fgnx}b^2cd}{2f^2g^2n^2\log(F)^2} + \frac{2\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2F^{egn}fgnx\log(F) + 2F^{egn}\right)F^{fgnx}abd^2}{f^3g^3n^3\log(F)^3} + \frac{\left(2F^{2egn}f^2g^2n^2x^2\log(F)^2 - 2F^{2egn}fgnx\log(F) + F^{2egn}\right)F^{2fgnx}b^2d^2}{4f^3g^3n^3\log(F)^3}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + a^2*c^2*x + 2*F^(f*g*n*x + e*g*n)*a*b*c^2/(
f*g*n*log(F)) + 1/2*F^(2*f*g*n*x + 2*e*g*n)*b^2*c^2/(f*g*n*log(F)) + 4*(F^(
e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*a*b*c*d/(f^2*g^2*n^2*log(F)^
2) + 1/2*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*c*d
/(f^2*g^2*n^2*log(F)^2) + 2*(F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(e*g*
n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*a*b*d^2/(f^3*g^3*n^3*log(F)^3)
 + 1/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(2*e*g*n)*f*g*n*x*log(
F) + F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*d^2/(f^3*g^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 5675, normalized size of antiderivative = 23.74

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + a^2*c^2*x - 1/2*(((2*pi*b^2*d^2*f^2*g^2*n^2
*x^2*log(abs(F))*sgn(F) - 2*pi*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 4*pi*b
^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 4*pi*b^2*c*d*f^2*g^2*n^2*x*log(ab
s(F)) + 2*pi*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*f^2*g^2*
n^2*log(abs(F)) - pi*b^2*d^2*f*g*n*x*sgn(F) + pi*b^2*d^2*f*g*n*x - pi*b^2*c
*d*f*g*n*sgn(F) + pi*b^2*c*d*f*g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3
```

```
*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))
^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^
3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log
(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F)
)^3)^2) - (pi^2*b^2*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*b^2*d^2*f^2*g^2*n^2*x
^2 + 2*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*b^2*c*d*f^2*g^2*n^2*x
*sgn(F) - 2*pi^2*b^2*c*d*f^2*g^2*n^2*x + 4*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)
)^2 + pi^2*b^2*c^2*f^2*g^2*n^2*sgn(F) - pi^2*b^2*c^2*f^2*g^2*n^2 + 2*b^2*c^
2*f^2*g^2*n^2*log(abs(F))^2 - 2*b^2*d^2*f*g*n*x*log(abs(F)) - 2*b^2*c*d*f*g
*n*log(abs(F)) + b^2*d^2)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f
^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi^3*f^3*g^3*n^3*sg
n(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*
g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*
f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))*cos(-pi*f*g*n*x*
sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n) - ((pi^2*b^2*d^2*f^2*g^2*
n^2*x^2*sgn(F) - pi^2*b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2*d^2*f^2*g^2*n^2*x^2*l
og(abs(F))^2 + 2*pi^2*b^2*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*b^2*c*d*f^2*g^2
*n^2*x + 4*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*b^2*c^2*f^2*g^2*n^2*s
gn(F) - pi^2*b^2*c^2*f^2*g^2*n^2 + 2*b^2*c^2*f^2*g^2*n^2*log(abs(F))^2 - 2*
b^2*d^2*f*g*n*x*log(abs(F)) - 2*b^2*c*d*f*g*n*log(abs(F)) + b^2*d^2)*(pi^3*
f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n
^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g
^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F
))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(a
bs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) + (2*pi*b^2*d^2*f^2*g^2*n^2*x^2*lo
g(abs(F))*sgn(F) - 2*pi*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 4*pi*b^2*c*d*
f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 4*pi*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)) +
 2*pi*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*f^2*g^2*n^2*log
(abs(F)) - pi*b^2*d^2*f*g*n*x*sgn(F) + pi*b^2*d^2*f*g*n*x - pi*b^2*c*d*f*g*
n*sgn(F) + pi*b^2*c*d*f*g*n)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^
2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi^3*f^3*g^3*n^3
*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f
^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi
^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))*sin(-pi*f*g*n
*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n))*e^(2*f*g*n*x*log(abs(
F)) + 2*e*g*n*log(abs(F))) - I*((-I*pi^2*b^2*d^2*f^2*g^2*n^2*x^2*sgn(F) + 2
*pi*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + I*pi^2*b^2*d^2*f^2*g^2*n^2
*x^2 - 2*pi*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) - 2*I*b^2*d^2*f^2*g^2*n^2*x
^2*log(abs(F))^2 - 2*I*pi^2*b^2*c*d*f^2*g^2*n^2*x*sgn(F) + 4*pi*b^2*c*d*f^2
*g^2*n^2*x*log(abs(F))*sgn(F) + 2*I*pi^2*b^2*c*d*f^2*g^2*n^2*x - 4*pi*b^2*c
*d*f^2*g^2*n^2*x*log(abs(F)) - 4*I*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*
pi^2*b^2*c^2*f^2*g^2*n^2*sgn(F) + 2*pi*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(
F) + I*pi^2*b^2*c^2*f^2*g^2*n^2 - 2*pi*b^2*c^2*f^2*g^2*n^2*log(abs(F)) - 2*
I*b^2*c^2*f^2*g^2*n^2*log(abs(F))^2 - pi*b^2*d^2*f*g*n*x*sgn(F) + pi*b^2*d^
2*f*g*n*x + 2*I*b^2*d^2*f*g*n*x*log(abs(F)) - pi*b^2*c*d*f*g*n*sgn(F) + pi*
```

```
b^2*c*d*f*g*n + 2*I*b^2*c*d*f*g*n*log(abs(F)) - I*b^2*d^2)*e^(I*pi*f*g*n*x*
sgn(F) - I*pi*f*g*n*x + I*pi*e*g*n*sgn(F) - I*pi*e*g*n)/(-4*I*pi^3*f^3*g^3*
n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) + 12*I*pi*f^3*g^3*n^3*l
og(abs(F))^2*sgn(F) + 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)
) - 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3) - (-I*
pi^2*b^2*d^2*f^2*g^2*n^2*x^2*sgn(F) - 2*pi*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(
F))*sgn(F) + I*pi^2*b^2*d^2*f^2*g^2*n^2*x^2 + 2*pi*b^2*d^2*f^2*g^2*n^2*x^2*
log(abs(F)) - 2*I*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 - 2*I*pi^2*b^2*c*d*
f^2*g^2*n^2*x*sgn(F) - 4*pi*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 2*I*
pi^2*b^2*c*d*f^2*g^2*n^2*x + 4*pi*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)) - 4*I*b
^2*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*b^2*c^2*f^2*g^2*n^2*sgn(F) - 2*
pi*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*b^2*c^2*f^2*g^2*n^2 + 2*
pi*b^2*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*b^2*c^2*f^2*g^2*n^2*log(abs(F))^2
 + pi*b^2*d^2*f*g*n*x*sgn(F) - pi*b^2*d^2*f*g*n*x + 2*I*b^2*d^2*f*g*n*x*log(
abs(F)) + pi*b^2*c*d*f*g*n*sgn(F) - pi*b^2*c*d*f*g*n + 2*I*b^2*c*d*f*g*n*lo
g(abs(F)) - I*b^2*d^2)*e^(-I*pi*f*g*n*x*sgn(F) + I*pi*f*g*n*x - I*pi*e*g*n*
sgn(F) + I*pi*e*g*n)/(4*I*pi^3*f^3*g^3*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log
(abs(F))*sgn(F) - 12*I*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 4*I*pi^3*f^3*g
^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)) + 12*I*pi*f^3*g^3*n^3*log(abs(F))^
2 + 8*f^3*g^3*n^3*log(abs(F))^3))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(ab
s(F))) - 2*((2*(pi*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - pi*a*b*d^2*
f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F)
 - 2*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F)) + pi*a*b*c^2*f^2*g^2*n^2*log(abs(
F))*sgn(F) - pi*a*b*c^2*f^2*g^2*n^2*log(abs(F)) - pi*a*b*d^2*f*g*n*x*sgn(F)
 + pi*a*b*d^2*f*g*n*x - pi*a*b*c*d*f*g*n*sgn(F) + pi*a*b*c*d*f*g*n)*(pi^3*f
^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^
3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^
3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F)
)^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(ab
s(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) - (pi^2*a*b*d^2*f^2*g^2*n^2*x^2*sgn
(F) - pi^2*a*b*d^2*f^2*g^2*n^2*x^2 + 2*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^
2 + 2*pi^2*a*b*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*a*b*c*d*f^2*g^2*n^2*x + 4*
a*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*a*b*c^2*f^2*g^2*n^2*sgn(F) - pi^
2*a*b*c^2*f^2*g^2*n^2 + 2*a*b*c^2*f^2*g^2*n^2*log(abs(F))^2 - 4*a*b*d^2*f*g
*n*x*log(abs(F)) - 4*a*b*c*d*f*g*n*log(abs(F)) + 4*a*b*d^2)*(3*pi^2*f^3*g^3
*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*lo
g(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn
(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^
3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*l
og(abs(F))^3)^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*
n*sgn(F) + 1/2*pi*e*g*n) - ((pi^2*a*b*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*a*b
*d^2*f^2*g^2*n^2*x^2 + 2*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*a*b
*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*a*b*c*d*f^2*g^2*n^2*x + 4*a*b*c*d*f^2*g^
2*n^2*x*log(abs(F))^2 + pi^2*a*b*c^2*f^2*g^2*n^2*sgn(F) - pi^2*a*b*c^2*f^2*
g^2*n^2 + 2*a*b*c^2*f^2*g^2*n^2*log(abs(F))^2 - 4*a*b*d^2*f*g*n*x*log(abs(F
```

)) - 4*a*b*c*d*f*g*n*log(abs(F)) + 4*a*b*d^2)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*
pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*l
og(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sg
n(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g
^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*
log(abs(F))^3)^2) + 2*(pi*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - pi*a
*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))
*sgn(F) - 2*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F)) + pi*a*b*c^2*f^2*g^2*n^2*l
og(abs(F))*sgn(F) - pi*a*b*c^2*f^2*g^2*n^2*log(abs(F)) - pi*a*b*d^2*f*g*n*x
*sgn(F) + pi*a*b*d^2*f*g*n*x - pi*a*b*c*d*f*g*n*sgn(F) + pi*a*b*c*d*f*g*n)*
(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2
*f^3*g^3*n^3*log(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*lo
g(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 +
(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) +
2*f^3*g^3*n^3*log(abs(F))^3)^2))*sin(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*
x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*e^(f*g*n*x*log(abs(F)) + e*g*n*log
(abs(F))) - 4*I*((-I*pi^2*a*b*d^2*f^2*g^2*n^2*x^2*sgn(F) + 2*pi*a*b*d^2*f^2
*g^2*n^2*x^2*log(abs(F))*sgn(F) + I*pi^2*a*b*d^2*f^2*g^2*n^2*x^2 - 2*pi*a*b
*d^2*f^2*g^2*n^2*x^2*log(abs(F)) - 2*I*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^
2 - 2*I*pi^2*a*b*c*d*f^2*g^2*n^2*x*sgn(F) + 4*pi*a*b*c*d*f^2*g^2*n^2*x*log(
abs(F))*sgn(F) + 2*I*pi^2*a*b*c*d*f^2*g^2*n^2*x - 4*pi*a*b*c*d*f^2*g^2*n^2*
x*log(abs(F)) - 4*I*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*a*b*c^2*f^
2*g^2*n^2*sgn(F) + 2*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*a*b
*c^2*f^2*g^2*n^2 - 2*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*a*b*c^2*f^2*g
^2*n^2*log(abs(F))^2 - 2*pi*a*b*d^2*f*g*n*x*sgn(F) + 2*pi*a*b*d^2*f*g*n*x +
4*I*a*b*d^2*f*g*n*x*log(abs(F)) - 2*pi*a*b*c*d*f*g*n*sgn(F) + 2*pi*a*b*c*d
*f*g*n + 4*I*a*b*c*d*f*g*n*log(abs(F)) - 4*I*a*b*d^2)*e^(1/2*I*pi*f*g*n*x*s
gn(F) - 1/2*I*pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2*I*pi*e*g*n)/(-4*I*pi
^3*f^3*g^3*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) + 12*I*pi*f^
3*g^3*n^3*log(abs(F))^2*sgn(F) + 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3
*log(abs(F)) - 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F)
)^3) - (-I*pi^2*a*b*d^2*f^2*g^2*n^2*x^2*sgn(F) - 2*pi*a*b*d^2*f^2*g^2*n^2*x
^2*log(abs(F))*sgn(F) + I*pi^2*a*b*d^2*f^2*g^2*n^2*x^2 + 2*pi*a*b*d^2*f^2*g
^2*n^2*x^2*log(abs(F)) - 2*I*a*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 - 2*I*pi
^2*a*b*c*d*f^2*g^2*n^2*x*sgn(F) - 4*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))*sg
n(F) + 2*I*pi^2*a*b*c*d*f^2*g^2*n^2*x + 4*pi*a*b*c*d*f^2*g^2*n^2*x*log(abs(
F)) - 4*I*a*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*a*b*c^2*f^2*g^2*n^2*
sgn(F) - 2*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*a*b*c^2*f^2*g
^2*n^2 + 2*pi*a*b*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*a*b*c^2*f^2*g^2*n^2*log
(abs(F))^2 + 2*pi*a*b*d^2*f*g*n*x*sgn(F) - 2*pi*a*b*d^2*f*g*n*x + 4*I*a*b*d
^2*f*g*n*x*log(abs(F)) + 2*pi*a*b*c*d*f*g*n*sgn(F) - 2*pi*a*b*c*d*f*g*n + 4
*I*a*b*c*d*f*g*n*log(abs(F)) - 4*I*a*b*d^2)*e^(-1/2*I*pi*f*g*n*x*sgn(F) + 1
/2*I*pi*f*g*n*x - 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(4*I*pi^3*f^3*g^3
*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 12*I*pi*f^3*g^3*n^3*
log(abs(F))^2*sgn(F) - 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F

```
)) + 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3))*e^(f
*g*n*x*log(abs(F)) + e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.12

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx$$
$$= \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{b^2\,\left(2\,c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+d^2\right)}{4\,f^3\,g^3\,n^3\,\ln\left(F\right)^3}+\frac{b^2\,d^2\,x^2}{2\,f\,g\,n\,\ln\left(F\right)}-\frac{b^2\,d\,x\,\left(d-2\,c\,f\,g\,n\,\ln\left(F\right)\right)}{2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right)+\left(F^{f\,g\,x}\,F^{e\,g}\right)^{n}\left(\frac{2\,a\,b\,\left(c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{f^3\,g^3\,n^3\,\ln\left(F\right)^3}+\frac{2\,a\,b\,d^2\,x^2}{f\,g\,n\,\ln\left(F\right)}-\frac{4\,a\,b\,d\,x\,\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right)+a^2\,c^2\,x+\frac{a^2\,d^2\,x^3}{3}+a^2\,c\,d\,x^2$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2,x)
```

```
[Out] (F^(f*g*x)*F^(e*g))^(2*n)*((b^2*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f
*g*n*log(F)))/(4*f^3*g^3*n^3*log(F)^3) + (b^2*d^2*x^2)/(2*f*g*n*log(F)) - (
b^2*d*x*(d - 2*c*f*g*n*log(F)))/(2*f^2*g^2*n^2*log(F)^2)) + (F^(f*g*x)*F^(e
*g))^n*((2*a*b*(2*d^2 + c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/(f^
3*g^3*n^3*log(F)^3) + (2*a*b*d^2*x^2)/(f*g*n*log(F)) - (4*a*b*d*x*(d - c*f*
g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + a^2*c^2*x + (a^2*d^2*x^3)/3 + a^2*c*
d*x^2
```

3.34 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx=\frac{a^2(c+dx)^2}{2d}-\frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}-\frac{b^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}+\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)}$$

```
[Out] 1/2*a^2*(d*x+c)^2/d-2*a*b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/ln(F)^2-1/4*b^2*d
*(F^(f*g*x+e*g))^(2*n)/f^2/g^2/n^2/ln(F)^2+2*a*b*(F^(f*g*x+e*g))^n*(d*x+c)/
f/g/n/ln(F)+1/2*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx=\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{b^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x),x]
```

```
[Out] (a^2*(c + d*x)^2)/(2*d) - (2*a*b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*Log[F]
^2) - (b^2*d*(F^(e*g + f*g*x))^(2*n))/(4*f^2*g^2*n^2*Log[F]^2) + (2*a*b*(F^
```

```
(e*g + f*g*x))^n*(c + d*x))/(f*g*n*Log[F]) + (b^2*(F^(e*g + f*g*x))^(2*n)*(
c + d*x))/(2*f*g*n*Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2(c+dx)+2ab\left(F^{eg+fgx}\right)^n(c+dx)+b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)\right)dx \\
&= \frac{a^2(c+dx)^2}{2d}+(2ab)\int \left(F^{eg+fgx}\right)^n(c+dx)\,dx+b^2\int \left(F^{eg+fgx}\right)^{2n}(c+dx)\,dx \\
&= \frac{a^2(c+dx)^2}{2d}+\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)} \\
&\quad -\frac{(2abd)\int \left(F^{eg+fgx}\right)^n dx}{fgn\log(F)}-\frac{(b^2d)\int \left(F^{eg+fgx}\right)^{2n} dx}{2fgn\log(F)} \\
&= \frac{a^2(c+dx)^2}{2d}-\frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}-\frac{b^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} \\
&\quad +\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$$
$$= \frac{-bd\left(F^{g(e+fx)}\right)^n \left(8a+b\left(F^{g(e+fx)}\right)^n\right)+2bf\left(F^{g(e+fx)}\right)^n \left(4a+b\left(F^{g(e+fx)}\right)^n\right) gn(c+dx)\log(F)+2a^2f^2g^2n^2}{4f^2g^2n^2\log^2(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x),x]
```

```
[Out] (-(b*d*(F^(g*(e + f*x)))^n*(8*a + b*(F^(g*(e + f*x)))^n)) + 2*b*f*(F^(g*(e
+ f*x)))^n*(4*a + b*(F^(g*(e + f*x)))^n)*g*n*(c + d*x)*Log[F] + 2*a^2*f^2*g
^2*n^2*x*(2*c + d*x)*Log[F]^2)/(4*f^2*g^2*n^2*Log[F]^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12

method	result
norman	$a^2cx+\frac{a^2dx^2}{2}+\frac{b^2(2\ln(F)cfgn-d)e^{2n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{4n^2g^2f^2\ln(F)^2}+\frac{2ab(\ln(F)cfgn-d)e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{n^2g^2f^2\ln(F)^2}+\frac{b^2dx\,e^{2n\ln\left(e^{g(fx}\right.}}{2ngf\ln(}$
parallelrisch	$\frac{2a^2dx^2n^2g^2f^2\ln(F)^2+4a^2cx\,n^2g^2f^2\ln(F)^2+2x\left(F^{g(fx+e)}\right)^{2n}b^2dngf\ln(F)+8x\left(F^{g(fx+e)}\right)^n abdngf\ln(F)+2\ln(F)\left(F^{g(fx+e)}\right.}{4n^2g^2f^2\ln(F)^2}$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x+1/2*a^2*d*x^2+1/4*b^2*(2*ln(F)*c*f*g*n-d)/n^2/g^2/f^2/ln(F)^2*exp(n
*ln(exp(g*(f*x+e)*ln(F))))^2+2*a*b*(ln(F)*c*f*g*n-d)/n^2/g^2/f^2/ln(F)^2*ex
p(n*ln(exp(g*(f*x+e)*ln(F))))+1/2/n/g/f/ln(F)*b^2*d*x*exp(n*ln(exp(g*(f*x+e
)*ln(F))))^2+2/n/g/f/ln(F)*a*b*d*x*exp(n*ln(exp(g*(f*x+e)*ln(F))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$$
$$= \frac{2\left(a^2df^2g^2n^2x^2+2\,a^2cf^2g^2n^2x\right)\log\left(F\right)^2-\left(b^2d-2\left(b^2dfgnx+b^2cfgn\right)\log\left(F\right)\right)F^{2\,fgnx+2\,egn}-8\left(abd-(a\right.}{4\,f^2g^2n^2\log\left(F\right)^2}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*d*f^2*g^2*n^2*x^2 + 2*a^2*c*f^2*g^2*n^2*x)*log(F)^2 - (b^2*d -
2*(b^2*d*f*g*n*x + b^2*c*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) - 8*(a*b*d
- (a*b*d*f*g*n*x + a*b*c*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^2*g^2*n^2*l
og(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.65

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \begin{cases} (a+b)^2\left(cx+\frac{dx^2}{2}\right) \\ (a+b(F^{eg})^n)^2\left(cx+\frac{dx^2}{2}\right) \\ (a+b)^2\left(cx+\frac{dx^2}{2}\right) \\ a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{2abdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{b^2c\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}+\frac{b^2dx\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{b^2d\left(F^{eg+}\right.}{4f^2g^2n^2\,\mathrm{l}} \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c),x)
```

```
[Out] Piecewise(((a + b)**2*(c*x + d*x**2/2), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq
(n, 0)), ((a + b*(F**(e*g))**n)**2*(c*x + d*x**2/2), Eq(f, 0)), ((a + b)**2
*(c*x + d*x**2/2), Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), (a**2*c*x + a**2*d*x**2
/2 + 2*a*b*c*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 2*a*b*d*x*(F**(e*g + f*
g*x))**n/(f*g*n*log(F)) - 2*a*b*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log
(F)**2) + b**2*c*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + b**2*d*x*(F**
(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - b**2*d*(F**(e*g + f*g*x))**(2*n)/(
4*f**2*g**2*n**2*log(F)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \frac{1}{2}a^2dx^2+a^2cx+\frac{2F^{fgnx+egn}abc}{fgn\log(F)}+\frac{F^{2fgnx+2egn}b^2c}{2fgn\log(F)} + \frac{2\left(F^{egn}fgnx\log(F)-F^{egn}\right)F^{fgnx}abd}{f^2g^2n^2\log(F)^2} + \frac{\left(2F^{2egn}fgnx\log(F)-F^{2egn}\right)F^{2fgnx}b^2d}{4f^2g^2n^2\log(F)^2}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + a^2*c*x + 2*F^(f*g*n*x + e*g*n)*a*b*c/(f*g*n*log(F)) + 1/2*
F^(2*f*g*n*x + 2*e*g*n)*b^2*c/(f*g*n*log(F)) + 2*(F^(e*g*n)*f*g*n*x*log(F)
- F^(e*g*n))*F^(f*g*n*x)*a*b*d/(f^2*g^2*n^2*log(F)^2) + 1/4*(2*F^(2*e*g*n)*
f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*d/(f^2*g^2*n^2*log(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 2284, normalized size of antiderivative = 14.64

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 + a^2*c*x + 1/2*((2*(pi*b^2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*n
*x + pi*b^2*c*f*g*n*sgn(F) - pi*b^2*c*f*g*n)*(pi*f^2*g^2*n^2*log(abs(F))*sg
n(F) - pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2
*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F
) - pi*f^2*g^2*n^2*log(abs(F)))^2) + (pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^
2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(2*b^2*d*f*g*n*x*log(abs(F)) + 2*b^2*c
*f*g*n*log(abs(F)) - b^2*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 +
2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*
f^2*g^2*n^2*log(abs(F)))^2))*cos(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n
*sgn(F) + pi*e*g*n) + ((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*
g^2*n^2*log(abs(F))^2)*(pi*b^2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*n*x + pi*b^2
*c*f*g*n*sgn(F) - pi*b^2*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*
n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F)
 - pi*f^2*g^2*n^2*log(abs(F)))^2) - 2*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) -
pi*f^2*g^2*n^2*log(abs(F)))*(2*b^2*d*f*g*n*x*log(abs(F)) + 2*b^2*c*f*g*n*lo
g(abs(F)) - b^2*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2
*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n
^2*log(abs(F)))^2))*sin(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) +
 pi*e*g*n))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F))) - 1/4*I*((pi*b^
2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*n*x - 2*I*b^2*d*f*g*n*x*log(abs(F)) + pi*
b^2*c*f*g*n*sgn(F) - pi*b^2*c*f*g*n - 2*I*b^2*c*f*g*n*log(abs(F)) + I*b^2*d
)*e^(I*pi*f*g*n*x*sgn(F) - I*pi*f*g*n*x + I*pi*e*g*n*sgn(F) - I*pi*e*g*n)/(
pi^2*f^2*g^2*n^2*sgn(F) + 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^2*f^2*
g^2*n^2 - 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^2) + (
pi*b^2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*n*x + 2*I*b^2*d*f*g*n*x*log(abs(F))
+ pi*b^2*c*f*g*n*sgn(F) - pi*b^2*c*f*g*n + 2*I*b^2*c*f*g*n*log(abs(F)) - I*
b^2*d)*e^(-I*pi*f*g*n*x*sgn(F) + I*pi*f*g*n*x - I*pi*e*g*n*sgn(F) + I*pi*e*
g*n)/(pi^2*f^2*g^2*n^2*sgn(F) - 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^
2*f^2*g^2*n^2 + 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^
2))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F))) + 2*(2*((pi*a*b*d*f*g*n
```

```
*x*sgn(F) - pi*a*b*d*f*g*n*x + pi*a*b*c*f*g*n*sgn(F) - pi*a*b*c*f*g*n)*(pi*
f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2
*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2
*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) + (pi^2*f^2*g^
2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(a*b*d*f*g*n
*x*log(abs(F)) + a*b*c*f*g*n*log(abs(F)) - a*b*d)/((pi^2*f^2*g^2*n^2*sgn(F)
 - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*lo
g(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*cos(-1/2*pi*f*g*n*x*sgn(
F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n) + ((pi^2*f^2*g^2*
n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(pi*a*b*d*f*g*
n*x*sgn(F) - pi*a*b*d*f*g*n*x + pi*a*b*c*f*g*n*sgn(F) - pi*a*b*c*f*g*n)/((p
i^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2
+ 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) - 4
*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))*(a*b*d*f*
g*n*x*log(abs(F)) + a*b*c*f*g*n*log(abs(F)) - a*b*d)/((pi^2*f^2*g^2*n^2*sgn
(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2
*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*sin(-1/2*pi*f*g*n*x*s
gn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*e^(f*g*n*x*lo
g(abs(F)) + e*g*n*log(abs(F))) - I*((pi*a*b*d*f*g*n*x*sgn(F) - pi*a*b*d*f*g
*n*x - 2*I*a*b*d*f*g*n*x*log(abs(F)) + pi*a*b*c*f*g*n*sgn(F) - pi*a*b*c*f*g
*n - 2*I*a*b*c*f*g*n*log(abs(F)) + 2*I*a*b*d)*e^(1/2*I*pi*f*g*n*x*sgn(F) -
1/2*I*pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2*I*pi*e*g*n)/(pi^2*f^2*g^2*n^
2*sgn(F) + 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^2*f^2*g^2*n^2 - 2*I*p
i*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^2) + (pi*a*b*d*f*g*n*
x*sgn(F) - pi*a*b*d*f*g*n*x + 2*I*a*b*d*f*g*n*x*log(abs(F)) + pi*a*b*c*f*g*
n*sgn(F) - pi*a*b*c*f*g*n + 2*I*a*b*c*f*g*n*log(abs(F)) - 2*I*a*b*d)*e^(-1/
2*I*pi*f*g*n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi
*e*g*n)/(pi^2*f^2*g^2*n^2*sgn(F) - 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) -
pi^2*f^2*g^2*n^2 + 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F
))^2))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \frac{a^2\,d\,x^2}{2} - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{b^2\,\left(d-2\,c\,f\,g\,n\,\ln\left(F\right)\right)}{4\,f^2\,g^2\,n^2\,{\ln\left(F\right)}^2} - \frac{b^2\,d\,x}{2\,f\,g\,n\,\ln\left(F\right)}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^n\left(\frac{2\,a\,b\,\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\,{\ln\left(F\right)}^2} - \frac{2\,a\,b\,d\,x}{f\,g\,n\,\ln\left(F\right)}\right) + a^2\,c\,x$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x),x)
```

```
[Out] (a^2*d*x^2)/2 - (F^(f*g*x)*F^(e*g))^(2*n)*((b^2*(d - 2*c*f*g*n*log(F)))/(4*
f^2*g^2*n^2*log(F)^2) - (b^2*d*x)/(2*f*g*n*log(F))) - (F^(f*g*x)*F^(e*g))^n
*((2*a*b*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2) - (2*a*b*d*x)/(f*g*n*
log(F))) + a^2*c*x
```

3.35 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = a^2x+\frac{2ab\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{b^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}$$

```
[Out] a^2*x+2*a*b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)+1/2*b^2*(F^(g*(f*x+e)))^(2*n)/f/g
/n/ln(F)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2320, 272, 45}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = a^2x+\frac{2ab\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{b^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

```
[Out] a^2*x + (2*a*b*(F^(g*(e + f*x)))^n)/(f*g*n*Log[F]) + (b^2*(F^(g*(e + f*x)))
^(2*n))/(2*f*g*n*Log[F])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^2}{x}\,dx, x, F^{g(e+fx)}\right)}{fg\log(F)} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)} \\
&= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)} \\
&= a^2x+\frac{2ab\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{b^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2\,dx = \frac{b\left(F^{g(e+fx)}\right)^n\left(4a+b\left(F^{g(e+fx)}\right)^n\right)+2a^2\log\left(\left(F^{g(e+fx)}\right)^n\right)}{2fgn\log(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2,x]
[Out] (b*(F^(g*(e + f*x)))^n*(4*a + b*(F^(g*(e + f*x)))^n) + 2*a^2*Log[(F^(g*(e +
 f*x)))^n])/(2*f*g*n*Log[F])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{2a^{2}x\ln\left(F\right)fgn+b^{2}\left(F^{g\left(fx+e\right)}\right)^{2n}+4ab\left(F^{g\left(fx+e\right)}\right)^{n}}{2\ln\left(F\right)fgn}$	60
derivativedivides	$\frac{\frac{b^{2}\left(F^{g\left(fx+e\right)}\right)^{2n}}{2}+2ab\left(F^{g\left(fx+e\right)}\right)^{n}+a^{2}\ln\left(\left(F^{g\left(fx+e\right)}\right)^{n}\right)}{gf\ln\left(F\right)n}$	65
default	$\frac{\frac{b^{2}\left(F^{g\left(fx+e\right)}\right)^{2n}}{2}+2ab\left(F^{g\left(fx+e\right)}\right)^{n}+a^{2}\ln\left(\left(F^{g\left(fx+e\right)}\right)^{n}\right)}{gf\ln\left(F\right)n}$	65
norman	$a^{2}x+\frac{b^{2}{\mathrm e}^{2n\ln\left({\mathrm e}^{g\left(fx+e\right)\ln\left(F\right)}\right)}}{2ngf\ln\left(F\right)}+\frac{2ab\,{\mathrm e}^{n\ln\left({\mathrm e}^{g\left(fx+e\right)\ln\left(F\right)}\right)}}{ngf\ln\left(F\right)}$	72

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*a^2*x*ln(F)*f*g*n+b^2*((F^(g*(f*x+e)))^n)^2+4*a*b*(F^(g*(f*x+e)))^n)
/ln(F)/f/g/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2\,dx=\frac{2\,a^2fgnx\log\left(F\right)+4\,F^{fgnx+egn}ab+F^{2\,fgnx+2\,egn}b^2}{2\,fgn\log\left(F\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
[Out] 1/2*(2*a^2*f*g*n*x*log(F) + 4*F^(f*g*n*x + e*g*n)*a*b + F^(2*f*g*n*x + 2*e*
g*n)*b^2)/(f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2\,dx = \begin{cases} x(a+b)^2 & \text{for}\ F=1\wedge f=0\wedge g=0\wedge n=0\\ x\left(a+b(F^{eg})^n\right)^2 & \text{for}\ f=0\\ x(a+b)^2 & \text{for}\ F=1\vee g=0\vee n=0\\ a^2x+\frac{2ab\left(F^{eg+fgx}\right)^n}{fgn\log\left(F\right)}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log\left(F\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2,x)
```

```
[Out] Piecewise((x*(a + b)**2, Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), (x*(a
+ b*(F**(e*g))**n)**2, Eq(f, 0)), (x*(a + b)**2, Eq(F, 1) | Eq(g, 0) | Eq(n
, 0)), (a**2*x + 2*a*b*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + b**2*(F**(e*g
 + f*g*x))**(2*n)/(2*f*g*n*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2\,dx=a^2x+\frac{2\,F^{(fx+e)gn}ab}{fgn\log\left(F\right)}+\frac{F^{2\,(fx+e)gn}b^2}{2\,fgn\log\left(F\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*F^((f*x + e)*g*n)*a*b/(f*g*n*log(F)) + 1/2*F^(2*(f*x + e)*g*n)*b^
2/(f*g*n*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2\,dx=\frac{4\,F^{fgnx}F^{egn}ab+F^{2\,fgnx}F^{2\,egn}b^2+2\,a^2\log\left(|F|^{fgnx}|F|^{egn}\right)}{2\,fgn\log\left(F\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*F^(f*g*n*x)*F^(e*g*n)*a*b + F^(2*f*g*n*x)*F^(2*e*g*n)*b^2 + 2*a^2*lo
g(abs(F)^(f*g*n*x)*abs(F)^(e*g*n)))/(f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2\,dx=\frac{\frac{b^2\,{\left(F^{e\,g+f\,g\,x}\right)}^{2\,n}}{2}+2\,a\,b\,{\left(F^{e\,g+f\,g\,x}\right)}^n}{f\,g\,n\,\ln\left(F\right)}+\frac{a^2\,\ln\left(F^{g\,\left(e+f\,x\right)}\right)}{f\,g\,\ln\left(F\right)}$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2,x)
```

```
[Out] ((b^2*(F^(e*g + f*g*x))^(2*n))/2 + 2*a*b*(F^(e*g + f*g*x))^n)/(f*g*n*log(F)
) + (a^2*log(F^(g*(e + f*x))))/(f*g*log(F))
```

3.36 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n \text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} \text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{a^2\log(c+dx)}{d}$$

```
[Out] 2*a*b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*Ei(f*g*n*(d*x+c)*ln(F
)/d)/d+b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*Ei(2*f*g
*n*(d*x+c)*ln(F)/d)/d+a^2*ln(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2214, 2213, 2209}

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \frac{a^2\log(c+dx)}{d} + \frac{2ab\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{b^2\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)} \text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x),x]
```

```
[Out] (2*a*b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*ExpIntegra
lEi[(f*g*n*(c + d*x)*Log[F])/d])/d + (b^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e
 + f*x))*(F^(e*g + f*g*x))^(2*n)*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d
])/d + (a^2*Log[c + d*x])/d
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{c+dx}+\frac{2ab\left(F^{eg+fgx}\right)^n}{c+dx}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{c+dx}\right)dx\\
&= \frac{a^2\log(c+dx)}{d}+(2ab)\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx+b^2\int \frac{\left(F^{eg+fgx}\right)^{2n}}{c+dx}\,dx\\
&= \frac{a^2\log(c+dx)}{d}+\left(2abF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx\\
&\quad+\left(b^2F^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n}\right)\int \frac{F^{2n(eg+fgx)}}{c+dx}\,dx\\
&= \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n\operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}\\
&\quad+\frac{b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}\operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}+\frac{a^2\log(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \frac{2abF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n \text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)+b^2F^{-\frac{2fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^{2n}\text{ExpIntegralEi}\left(\frac{2f}{}\right.}{d}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x),x]
[Out] ((2*a*b*(F^(g*(e + f*x)))^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/F^((
f*g*n*(c + d*x))/d) + (b^2*(F^(g*(e + f*x)))^(2*n)*ExpIntegralEi[(2*f*g*n*(
c + d*x)*Log[F])/d])/F^((2*f*g*n*(c + d*x))/d) + a^2*Log[c + d*x])/d
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}{dx+c}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
[Out] int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \frac{F^{\frac{2(de-cf)gn}{d}}b^2\text{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right)+2F^{\frac{(de-cf)gn}{d}}ab\text{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)+a^2\log(dx+c)}{d}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="fricas")
[Out] (F^(2*(d*e - c*f)*g*n/d)*b^2*Ei(2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 2*F^((d
*e - c*f)*g*n/d)*a*b*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d) + a^2*log(d*x + c))
/d
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx=\int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}{c+dx}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c),x)
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**2/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{dx+c}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="maxima")
[Out] F^(2*e*g*n)*b^2*integrate(F^(2*f*g*n*x)/(d*x + c), x) + 2*F^(e*g*n)*a*b*int
egrate(F^(f*g*n*x)/(d*x + c), x) + a^2*log(d*x + c)/d
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{dx+c}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^2/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx=\int \frac{\left(a+b\,\left(F^{g\,(e+f\,x)}\right)^n\right)^2}{c+d\,x}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x),x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x), x)
```

3.37 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\begin{aligned}\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx &= -\frac{a^2}{d(c+dx)}-\frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)}-\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}\\ &\quad+\frac{2abfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}\\ &\quad+\frac{2b^2fF^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} gn\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}\end{aligned}$$

```
[Out] -a^2/d/(d*x+c)-2*a*b*(F^(f*g*x+e*g))^n/d/(d*x+c)-b^2*(F^(f*g*x+e*g))^(2*n)/
d/(d*x+c)+2*a*b*f*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g*n*Ei(f*
g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2+2*b^2*f*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F
^(f*g*x+e*g))^(2*n)*g*n*Ei(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{2214, 2208, 2213, 2209\}$

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$$
$$= -\frac{a^2}{d(c+dx)}$$
$$+ \frac{2abfgn\log(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2}$$
$$- \frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)}$$
$$+ \frac{2b^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^2}$$
$$- \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2,x]
```

```
[Out] -(a^2/(d*(c + d*x))) - (2*a*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) - (b^2*(F^
(e*g + f*g*x))^(2*n))/(d*(c + d*x)) + (2*a*b*f*F^((e - (c*f)/d)*g*n - g*n*(
e + f*x))*(F^(e*g + f*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]
*Log[F])/d^2 + (2*b^2*f*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g +
 f*g*x))^(2*n)*g*n*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{(c+dx)^2} + \frac{2ab\left(F^{eg+fgx}\right)^n}{(c+dx)^2} + \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}\right) dx \\
&= -\frac{a^2}{d(c+dx)} + (2ab)\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx + b^2\int \frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}\,dx \\
&= -\frac{a^2}{d(c+dx)} - \frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)} \\
&\quad + \frac{(2abfgn\log(F))\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx}{d} + \frac{(2b^2fgn\log(F))\int \frac{\left(F^{eg+fgx}\right)^{2n}}{c+dx}\,dx}{d} \\
&= -\frac{a^2}{d(c+dx)} - \frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)} \\
&\quad + \frac{\left(2abfF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n gn\log(F)\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx}{d} \\
&\quad + \frac{\left(2b^2fF^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n} gn\log(F)\right)\int \frac{F^{2n(eg+fgx)}}{c+dx}\,dx}{d} \\
&= -\frac{a^2}{d(c+dx)} - \frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)} \\
&\quad + \frac{2abfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2} \\
&\quad + \frac{2b^2fF^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} gn\text{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$$

$$= \frac{-\frac{d\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx} + 2abfF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F) + 2b^2fF^{-\frac{2fgn}{}}}{d^2}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2,x]
```

```
[Out] (-((d*(a + b*(F^(g*(e + f*x)))^n)^2)/(c + d*x)) + (2*a*b*f*(F^(g*(e + f*x))
)^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((f*g*n*(c + d*
x))/d) + (2*b^2*f*(F^(g*(e + f*x)))^(2*n)*g*n*ExpIntegralEi[(2*f*g*n*(c + d
*x)*Log[F])/d]*Log[F])/F^((2*f*g*n*(c + d*x))/d))/d^2
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^{2}}{\left(dx+c\right)^{2}}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

```
[Out] int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{2}}{(c+dx)^{2}}\,dx = -\frac{2\,F^{fgnx+egn}abd+F^{2\,fgnx+2\,egn}b^{2}d-2\,(b^{2}dfgnx+b^{2}cfgn)F^{\frac{2\,(de-cf)gn}{d}}\operatorname{Ei}\left(\frac{2\,(dfgnx+cfgn)\log(F)}{d}\right)\log\left(F\right)-2\,(\cdots}{d^{3}x+cd^{2}}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*F^(f*g*n*x + e*g*n)*a*b*d + F^(2*f*g*n*x + 2*e*g*n)*b^2*d - 2*(b^2*d*f*
g*n*x + b^2*c*f*g*n)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n)*log
(F)/d)*log(F) - 2*(a*b*d*f*g*n*x + a*b*c*f*g*n)*F^((d*e - c*f)*g*n/d)*Ei((d
*f*g*n*x + c*f*g*n)*log(F)/d)*log(F) + a^2*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{2}}{(c+dx)^{2}}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^{n}\right)^{2}}{\left(c+dx\right)^{2}}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**2/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{\left(dx+c\right)^2}\,dx$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="maxima")

[Out] F^(2*e*g*n)*b^2*integrate(F^(2*f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) + 2*F
^(e*g*n)*a*b*integrate(F^(f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) - a^2/(d^2
*x + c*d)

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{\left(dx+c\right)^2}\,dx$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate(((F^((f*x + e)*g))^n*b + a)^2/(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx=\int \frac{\left(a+b\,\left(F^{g\,(e+f\,x)}\right)^n\right)^2}{\left(c+d\,x\right)^2}\,dx$$

[In] int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2,x)

[Out] int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2, x)

3.38 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 286

$$\begin{aligned}
&\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3} dx \\
&= -\frac{a^2}{2d(c+dx)^2} - \frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} \\
&\quad - \frac{abf\left(F^{eg+fgx}\right)^n gn\log(F)}{d^2(c+dx)} - \frac{b^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} \\
&\quad + \frac{abf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2 \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3} \\
&\quad + \frac{2b^2f^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2 \operatorname{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3}
\end{aligned}$$

```
[Out] -1/2*a^2/d/(d*x+c)^2-a*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-1/2*b^2*(F^(f*g*x+e*
g))^(2*n)/d/(d*x+c)^2-a*b*f*(F^(f*g*x+e*g))^n*g*n*ln(F)/d^2/(d*x+c)-b^2*f*(
F^(f*g*x+e*g))^(2*n)*g*n*ln(F)/d^2/(d*x+c)+a*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*
x+e))*(F^(f*g*x+e*g))^n*g^2*n^2*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3+2*b^2
*f^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g^2*n^2*Ei(2*f
*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2214, 2208, 2213, 2209}

$$\begin{aligned}
&\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3} \, dx \\
&= -\frac{a^2}{2d(c+dx)^2} \\
&\quad+\frac{abf^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^3} \\
&\quad-\frac{abfgn\log(F)\left(F^{eg+fgx}\right)^n}{d^2(c+dx)}-\frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2} \\
&\quad+\frac{2b^2f^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^3} \\
&\quad-\frac{b^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n}}{d^2(c+dx)}-\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2}
\end{aligned}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3,x]
```

```
[Out] -1/2*a^2/(d*(c + d*x)^2) - (a*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)^2) - (b^2
*(F^(e*g + f*g*x))^(2*n))/(2*d*(c + d*x)^2) - (a*b*f*(F^(e*g + f*g*x))^n*g*
n*Log[F])/(d^2*(c + d*x)) - (b^2*f*(F^(e*g + f*g*x))^(2*n)*g*n*Log[F])/(d^2
*(c + d*x)) + (a*b*f^2*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*
x))^n*g^2*n^2*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/d^3 + (2*
b^2*f^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*g
^2*n^2*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/d^3
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\left(F^{eg+fgx}\right)^n}{(c+dx)^3}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3}\right)dx$$

$$= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^3}\,dx+b^2\int \frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3}\,dx$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2}-\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} + \frac{(abfgn\log(F))\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx}{d}+\frac{(b^2fgn\log(F))\int \frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}\,dx}{d}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2}-\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{abf\left(F^{eg+fgx}\right)^n gn\log(F)}{d^2(c+dx)}-\frac{b^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} + \frac{\left(abf^2g^2n^2\log^2(F)\right)\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx}{d^2}+\frac{\left(2b^2f^2g^2n^2\log^2(F)\right)\int \frac{\left(F^{eg+fgx}\right)^{2n}}{c+dx}\,dx}{d^2}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2}-\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{abf\left(F^{eg+fgx}\right)^n gn\log(F)}{d^2(c+dx)}-\frac{b^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} + \frac{\left(abf^2F^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n g^2n^2\log^2(F)\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx}{d^2} + \frac{\left(2b^2f^2F^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2\log^2(F)\right)\int \frac{F^{2n(eg+fgx)}}{c+dx}\,dx}{d^2}$$

$$= -\frac{a^2}{2d(c+dx)^2} - \frac{ab(F^{eg+fgx})^n}{d(c+dx)^2} - \frac{b^2(F^{eg+fgx})^{2n}}{2d(c+dx)^2} - \frac{abf(F^{eg+fgx})^n gn\log(F)}{d^2(c+dx)} - \frac{b^2f(F^{eg+fgx})^{2n} gn\log(F)}{d^2(c+dx)} + \frac{abf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}(F^{eg+fgx})^n g^2n^2\text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3} + \frac{2b^2f^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}(F^{eg+fgx})^{2n} g^2n^2\text{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = -\frac{a^2d^2 - 2abf^2F^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n g^2n^2(c+dx)^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F) - 4b^2f^2F^{-2\cdots}}{\cdots}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3,x]
```

```
[Out] -1/2*(a^2*d^2 - (2*a*b*f^2*(F^(g*(e + f*x)))^n*g^2*n^2*(c + d*x)^2*ExpInteg
ralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((f*g*n*(c + d*x))/d) - (4*b^
2*f^2*(F^(g*(e + f*x)))^(2*n)*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(2*f*g*n*(c
 + d*x)*Log[F])/d]*Log[F]^2)/F^((2*f*g*n*(c + d*x))/d) + 2*a*b*d*(F^(g*(e +
 f*x)))^n*(d + f*g*n*(c + d*x)*Log[F]) + b^2*d*(F^(g*(e + f*x)))^(2*n)*(d +
 2*f*g*n*(c + d*x)*Log[F]))/(d^3*(c + d*x)^2)
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}{(dx+c)^3}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x)
```

```
[Out] int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx$$
$$= \frac{4\left(b^2d^2f^2g^2n^2x^2+2\,b^2cdf^2g^2n^2x+b^2c^2f^2g^2n^2\right)F^{\frac{2\,(de-cf)gn}{d}}\,\mathrm{Ei}\left(\frac{2\,(dfgnx+cfgn)\log(F)}{d}\right)\log(F)^2+2\left(abd^2f^2g^2n^2x\right.}{}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2*c*d*f^2*g^2*n^2*x + b^2*c^2*f^2*g^2
*n^2)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)^2
 + 2*(a*b*d^2*f^2*g^2*n^2*x^2 + 2*a*b*c*d*f^2*g^2*n^2*x + a*b*c^2*f^2*g^2*n
^2)*F^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)^2 - a^2
*d^2 - (b^2*d^2 + 2*(b^2*d^2*f*g*n*x + b^2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x
+ 2*e*g*n) - 2*(a*b*d^2 + (a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*log(F))*F^(f*g*
n*x + e*g*n))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}{\left(c+dx\right)^3}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**2/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] F^(2*e*g*n)*b^2*integrate(F^(2*f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3), x) + 2*F^(e*g*n)*a*b*integrate(F^(f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3), x) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^2/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx=\int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3, x)
```

3.39 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 496

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = {} & \frac{a^3(c+dx)^4}{4d}-\frac{18a^2bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}-\frac{9ab^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)}\\ & -\frac{2b^3d^3\left(F^{eg+fgx}\right)^{3n}}{27f^4g^4n^4\log^4(F)}+\frac{18a^2bd^2\left(F^{eg+fgx}\right)^n(c+dx)}{f^3g^3n^3\log^3(F)}\\ & +\frac{9ab^2d^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{4f^3g^3n^3\log^3(F)}\\ & +\frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}(c+dx)}{9f^3g^3n^3\log^3(F)}\\ & -\frac{9a^2bd\left(F^{eg+fgx}\right)^n(c+dx)^2}{f^2g^2n^2\log^2(F)}\\ & -\frac{9ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)}\\ & -\frac{b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3f^2g^2n^2\log^2(F)}\\ & +\frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)^3}{fgn\log(F)}\\ & +\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)}\\ & +\frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^3}{3fgn\log(F)}\end{aligned}$$

```
[Out] 1/4*a^3*(d*x+c)^4/d-18*a^2*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/ln(F)^4-9/8*
a*b^2*d^3*(F^(f*g*x+e*g))^(2*n)/f^4/g^4/n^4/ln(F)^4-2/27*b^3*d^3*(F^(f*g*x+
```

```
e*g))^(3*n)/f^4/g^4/n^4/ln(F)^4+18*a^2*b*d^2*(F^(f*g*x+e*g))^n*(d*x+c)/f^3/
g^3/n^3/ln(F)^3+9/4*a*b^2*d^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^3/g^3/n^3/ln(
F)^3+2/9*b^3*d^2*(F^(f*g*x+e*g))^(3*n)*(d*x+c)/f^3/g^3/n^3/ln(F)^3-9*a^2*b*
d*(F^(f*g*x+e*g))^n*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2-9/4*a*b^2*d*(F^(f*g*x+e*g
))^(2*n)*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2-1/3*b^3*d*(F^(f*g*x+e*g))^(3*n)*(d*x
+c)^2/f^2/g^2/n^2/ln(F)^2+3*a^2*b*(F^(f*g*x+e*g))^n*(d*x+c)^3/f/g/n/ln(F)+3
/2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^3/f/g/n/ln(F)+1/3*b^3*(F^(f*g*x+e*g)
)^(3*n)*(d*x+c)^3/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2214, 2207, 2225}

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = {} & \frac{a^3(c+dx)^4}{4d}+\frac{18a^2bd^2(c+dx)\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} \\ & -\frac{9a^2bd(c+dx)^2\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} \\ & +\frac{3a^2b(c+dx)^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{18a^2bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} \\ & +\frac{9ab^2d^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)} \\ & -\frac{9ab^2d(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} \\ & +\frac{3ab^2(c+dx)^3\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{9ab^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)} \\ & +\frac{2b^3d^2(c+dx)\left(F^{eg+fgx}\right)^{3n}}{9f^3g^3n^3\log^3(F)} \\ & -\frac{b^3d(c+dx)^2\left(F^{eg+fgx}\right)^{3n}}{3f^2g^2n^2\log^2(F)} \\ & +\frac{b^3(c+dx)^3\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)}-\frac{2b^3d^3\left(F^{eg+fgx}\right)^{3n}}{27f^4g^4n^4\log^4(F)}\end{aligned}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^3,x]
```

```
[Out] (a^3*(c + d*x)^4)/(4*d) - (18*a^2*b*d^3*(F^(e*g + f*g*x))^n)/(f^4*g^4*n^4*L
og[F]^4) - (9*a*b^2*d^3*(F^(e*g + f*g*x))^(2*n))/(8*f^4*g^4*n^4*Log[F]^4) -
 (2*b^3*d^3*(F^(e*g + f*g*x))^(3*n))/(27*f^4*g^4*n^4*Log[F]^4) + (18*a^2*b*
d^2*(F^(e*g + f*g*x))^n*(c + d*x))/(f^3*g^3*n^3*Log[F]^3) + (9*a*b^2*d^2*(F
^(e*g + f*g*x))^(2*n)*(c + d*x))/(4*f^3*g^3*n^3*Log[F]^3) + (2*b^3*d^2*(F^(
```

```
e*g + f*g*x))^(3*n)*(c + d*x))/(9*f^3*g^3*n^3*Log[F]^3) - (9*a^2*b*d*(F^(e*
g + f*g*x))^n*(c + d*x)^2)/(f^2*g^2*n^2*Log[F]^2) - (9*a*b^2*d*(F^(e*g + f*
g*x))^(2*n)*(c + d*x)^2)/(4*f^2*g^2*n^2*Log[F]^2) - (b^3*d*(F^(e*g + f*g*x)
)^(3*n)*(c + d*x)^2)/(3*f^2*g^2*n^2*Log[F]^2) + (3*a^2*b*(F^(e*g + f*g*x))^
n*(c + d*x)^3)/(f*g*n*Log[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*(c + d*x)^
3)/(2*f*g*n*Log[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x)^3)/(3*f*g*n*Lo
g[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \int \left(a^3(c+dx)^3 + 3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^3 + 3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3 + b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^3\right)\,dx$$

$$= \frac{a^3(c+dx)^4}{4d} + \left(3a^2b\right)\int \left(F^{eg+fgx}\right)^n (c+dx)^3\,dx + \left(3ab^2\right)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)^3\,dx + b^3\int \left(F^{eg+fgx}\right)^{3n} (c+dx)^3\,dx$$

$$= \frac{a^3(c+dx)^4}{4d} + \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^3}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^3}{3fgn\log(F)} - \frac{\left(9a^2bd\right)\int \left(F^{eg+fgx}\right)^n (c+dx)^2\,dx}{fgn\log(F)} - \frac{\left(9ab^2d\right)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)^2\,dx}{2fgn\log(F)} - \frac{\left(b^3d\right)\int \left(F^{eg+fgx}\right)^{3n} (c+dx)^2\,dx}{fgn\log(F)}$$

$$
\begin{aligned}
&= \frac{a^3(c+dx)^4}{4d} - \frac{9a^2bd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)} \\
&- \frac{9ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)} - \frac{b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3f^2g^2n^2\log^2(F)} \\
&+ \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)} \\
&+ \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^3}{3fgn\log(F)} + \frac{(18a^2bd^2)\int \left(F^{eg+fgx}\right)^n (c+dx)\,dx}{f^2g^2n^2\log^2(F)} \\
&+ \frac{(9ab^2d^2)\int \left(F^{eg+fgx}\right)^{2n}(c+dx)\,dx}{2f^2g^2n^2\log^2(F)} + \frac{(2b^3d^2)\int \left(F^{eg+fgx}\right)^{3n}(c+dx)\,dx}{3f^2g^2n^2\log^2(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^3(c+dx)^4}{4d} + \frac{18a^2bd^2\left(F^{eg+fgx}\right)^n (c+dx)}{f^3g^3n^3\log^3(F)} + \frac{9ab^2d^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{4f^3g^3n^3\log^3(F)} \\
&+ \frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}(c+dx)}{9f^3g^3n^3\log^3(F)} - \frac{9a^2bd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)} \\
&- \frac{9ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)} - \frac{b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3f^2g^2n^2\log^2(F)} \\
&+ \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)} \\
&+ \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^3}{3fgn\log(F)} - \frac{(18a^2bd^3)\int \left(F^{eg+fgx}\right)^n \,dx}{f^3g^3n^3\log^3(F)} \\
&- \frac{(9ab^2d^3)\int \left(F^{eg+fgx}\right)^{2n}\,dx}{4f^3g^3n^3\log^3(F)} - \frac{(2b^3d^3)\int \left(F^{eg+fgx}\right)^{3n}\,dx}{9f^3g^3n^3\log^3(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^3(c+dx)^4}{4d} - \frac{18a^2bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} - \frac{9ab^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)} - \frac{2b^3d^3\left(F^{eg+fgx}\right)^{3n}}{27f^4g^4n^4\log^4(F)} \\
&+ \frac{18a^2bd^2\left(F^{eg+fgx}\right)^n (c+dx)}{f^3g^3n^3\log^3(F)} + \frac{9ab^2d^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{4f^3g^3n^3\log^3(F)} \\
&+ \frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}(c+dx)}{9f^3g^3n^3\log^3(F)} - \frac{9a^2bd\left(F^{eg+fgx}\right)^n (c+dx)^2}{f^2g^2n^2\log^2(F)} \\
&- \frac{9ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)} - \frac{b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3f^2g^2n^2\log^2(F)} \\
&+ \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^3}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^3}{3fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.69

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = a^3c^3x+\frac{3}{2}a^3c^2dx^2+a^3cd^2x^3+\frac{1}{4}a^3d^3x^4$$
$$+\frac{3a^2b\left(F^{g(e+fx)}\right)^n\left(-6d^3+6d^2fgn(c+dx)\log(F)-3df^2g^2n^2(c+dx)^2\log^2(F)+f^3g^3n^3(c+dx)^3\log^3(F\right.}{f^4g^4n^4\log^4(F)}$$
$$+\frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}\left(-3d^3+6d^2fgn(c+dx)\log(F)-6df^2g^2n^2(c+dx)^2\log^2(F)+4f^3g^3n^3(c+dx)^3\log^3(\right.}{8f^4g^4n^4\log^4(F)}$$
$$+\frac{b^3\left(F^{g(e+fx)}\right)^{3n}\left(-2d^3+6d^2fgn(c+dx)\log(F)-9df^2g^2n^2(c+dx)^2\log^2(F)+9f^3g^3n^3(c+dx)^3\log^3(F\right.}{27f^4g^4n^4\log^4(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^3*c^2*d*x^2)/2 + a^3*c*d^2*x^3 + (a^3*d^3*x^4)/4 + (3*a^2*
b*(F^(g*(e + f*x)))^n*(-6*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 3*d*f^2*g^2*
n^2*(c + d*x)^2*Log[F]^2 + f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(f^4*g^4*n^4*
Log[F]^4) + (3*a*b^2*(F^(g*(e + f*x)))^(2*n)*(-3*d^3 + 6*d^2*f*g*n*(c + d*x
)*Log[F] - 6*d*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2 + 4*f^3*g^3*n^3*(c + d*x)^3
*Log[F]^3))/(8*f^4*g^4*n^4*Log[F]^4) + (b^3*(F^(g*(e + f*x)))^(3*n)*(-2*d^3
 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 9*d*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2 + 9*
f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(27*f^4*g^4*n^4*Log[F]^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. 2(478) = 956.

Time = 2.32 (sec) , antiderivative size = 1097, normalized size of antiderivative = 2.21

method	result	size
parallelrisch	Expression too large to display	1097

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(-16*((F^(g*(f*x+e)))^n)^3*b^3*d^3+216*ln(F)^3*x*((F^(g*(f*x+e)))^n)^
3*b^3*c^2*d*f^3*g^3*n^3-486*ln(F)^2*x^2*((F^(g*(f*x+e)))^n)^2*a*b^2*d^3*f^2
*g^2*n^2-144*ln(F)^2*x*((F^(g*(f*x+e)))^n)^3*b^3*c*d^2*f^2*g^2*n^2-1944*ln(
F)^2*x^2*(F^(g*(f*x+e)))^n*a^2*b*d^3*f^2*g^2*n^2-486*ln(F)^2*((F^(g*(f*x+e)
))^n)^2*a*b^2*c^2*d*f^2*g^2*n^2-1944*ln(F)^2*(F^(g*(f*x+e)))^n*a^2*b*c^2*d*
f^2*g^2*n^2+486*ln(F)*x*((F^(g*(f*x+e)))^n)^2*a*b^2*d^3*f*g*n+3888*ln(F)*x*
(F^(g*(f*x+e)))^n*a^2*b*d^3*f*g*n+486*ln(F)*((F^(g*(f*x+e)))^n)^2*a*b^2*c*d
^2*f*g*n+3888*ln(F)*(F^(g*(f*x+e)))^n*a^2*b*c*d^2*f*g*n+324*a*b^2*d^3*((F^(
g*(f*x+e)))^n)^2*x^3*n^3*g^3*f^3*ln(F)^3+216*ln(F)^3*x^2*((F^(g*(f*x+e)))^n
```

```
)^3*b^3*c*d^2*f^3*g^3*n^3+648*a^2*b*d^3*(F^(g*(f*x+e)))^n*x^3*n^3*g^3*f^3*l
n(F)^3-972*ln(F)^2*x*((F^(g*(f*x+e)))^n)^2*a*b^2*c*d^2*f^2*g^2*n^2-3888*ln(
F)^2*x*(F^(g*(f*x+e)))^n*a^2*b*c*d^2*f^2*g^2*n^2+972*ln(F)^3*x^2*((F^(g*(f*
x+e)))^n)^2*a*b^2*c*d^2*f^3*g^3*n^3+54*a^3*d^3*x^4*n^4*g^4*f^4*ln(F)^4+216*
a^3*c^3*x*n^4*g^4*f^4*ln(F)^4+72*ln(F)^3*((F^(g*(f*x+e)))^n)^3*b^3*c^3*f^3*
g^3*n^3+1944*ln(F)^3*x^2*(F^(g*(f*x+e)))^n*a^2*b*c*d^2*f^3*g^3*n^3+972*ln(F
)^3*x*((F^(g*(f*x+e)))^n)^2*a*b^2*c^2*d*f^3*g^3*n^3+1944*ln(F)^3*x*(F^(g*(f
*x+e)))^n*a^2*b*c^2*d*f^3*g^3*n^3+216*a^3*d^2*c*x^3*n^4*g^4*f^4*ln(F)^4-388
8*(F^(g*(f*x+e)))^n*a^2*b*d^3-243*((F^(g*(f*x+e)))^n)^2*a*b^2*d^3+48*ln(F)*
x*((F^(g*(f*x+e)))^n)^3*b^3*d^3*f*g*n+48*ln(F)*((F^(g*(f*x+e)))^n)^3*b^3*c*
d^2*f*g*n+324*a^3*d*c^2*x^2*n^4*g^4*f^4*ln(F)^4+72*d^3*b^3*((F^(g*(f*x+e)))
^n)^3*x^3*n^3*g^3*f^3*ln(F)^3+324*ln(F)^3*((F^(g*(f*x+e)))^n)^2*a*b^2*c^3*f
^3*g^3*n^3+648*ln(F)^3*(F^(g*(f*x+e)))^n*a^2*b*c^3*f^3*g^3*n^3-72*ln(F)^2*x
^2*((F^(g*(f*x+e)))^n)^3*b^3*d^3*f^2*g^2*n^2-72*ln(F)^2*((F^(g*(f*x+e)))^n)
^3*b^3*c^2*d*f^2*g^2*n^2)/n^4/g^4/f^4/ln(F)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.43

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx$$
$$= \frac{54\left(a^3d^3f^4g^4n^4x^4+4\,a^3cd^2f^4g^4n^4x^3+6\,a^3c^2df^4g^4n^4x^2+4\,a^3c^3f^4g^4n^4x\right)\log\left(F\right)^4-8\left(2\,b^3d^3-9\left(b^3d^3f^3\right.\right.}{}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(54*(a^3*d^3*f^4*g^4*n^4*x^4 + 4*a^3*c*d^2*f^4*g^4*n^4*x^3 + 6*a^3*c^
2*d*f^4*g^4*n^4*x^2 + 4*a^3*c^3*f^4*g^4*n^4*x)*log(F)^4 - 8*(2*b^3*d^3 - 9*
(b^3*d^3*f^3*g^3*n^3*x^3 + 3*b^3*c*d^2*f^3*g^3*n^3*x^2 + 3*b^3*c^2*d*f^3*g^
3*n^3*x + b^3*c^3*f^3*g^3*n^3)*log(F)^3 + 9*(b^3*d^3*f^2*g^2*n^2*x^2 + 2*b^
3*c*d^2*f^2*g^2*n^2*x + b^3*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(b^3*d^3*f*g*n*
x + b^3*c*d^2*f*g*n)*log(F))*F^(3*f*g*n*x + 3*e*g*n) - 81*(3*a*b^2*d^3 - 4*
(a*b^2*d^3*f^3*g^3*n^3*x^3 + 3*a*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*a*b^2*c^2*d*
f^3*g^3*n^3*x + a*b^2*c^3*f^3*g^3*n^3)*log(F)^3 + 6*(a*b^2*d^3*f^2*g^2*n^2*
x^2 + 2*a*b^2*c*d^2*f^2*g^2*n^2*x + a*b^2*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(
a*b^2*d^3*f*g*n*x + a*b^2*c*d^2*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) - 64
8*(6*a^2*b*d^3 - (a^2*b*d^3*f^3*g^3*n^3*x^3 + 3*a^2*b*c*d^2*f^3*g^3*n^3*x^2
 + 3*a^2*b*c^2*d*f^3*g^3*n^3*x + a^2*b*c^3*f^3*g^3*n^3)*log(F)^3 + 3*(a^2*b
*d^3*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d^2*f^2*g^2*n^2*x + a^2*b*c^2*d*f^2*g^2*n^
2)*log(F)^2 - 6*(a^2*b*d^3*f*g*n*x + a^2*b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x
+ e*g*n))/(f^4*g^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(498) = 996$.

Time = 11.92 (sec) , antiderivative size = 1324, normalized size of antiderivative = 2.67

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c)**3,x)
```

```
[Out] Piecewise(((a + b)**3*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**3*(c
**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(f, 0)), ((a + b)**
3*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) | Eq(g,
0) | Eq(n, 0)), (a**3*c**3*x + 3*a**3*c**2*d*x**2/2 + a**3*c*d**2*x**3 + a*
*3*d**3*x**4/4 + 3*a**2*b*c**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 9*a**
2*b*c**2*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 9*a**2*b*c**2*d*(F**(e*
g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 9*a**2*b*c*d**2*x**2*(F**(e*g +
 f*g*x))**n/(f*g*n*log(F)) - 18*a**2*b*c*d**2*x*(F**(e*g + f*g*x))**n/(f**2
*g**2*n**2*log(F)**2) + 18*a**2*b*c*d**2*(F**(e*g + f*g*x))**n/(f**3*g**3*n
**3*log(F)**3) + 3*a**2*b*d**3*x**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) -
9*a**2*b*d**3*x**2*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 18*a*
*2*b*d**3*x*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) - 18*a**2*b*d*
*3*(F**(e*g + f*g*x))**n/(f**4*g**4*n**4*log(F)**4) + 3*a*b**2*c**3*(F**(e*
g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + 9*a*b**2*c**2*d*x*(F**(e*g + f*g*x))*
*(2*n)/(2*f*g*n*log(F)) - 9*a*b**2*c**2*d*(F**(e*g + f*g*x))**(2*n)/(4*f**2
*g**2*n**2*log(F)**2) + 9*a*b**2*c*d**2*x**2*(F**(e*g + f*g*x))**(2*n)/(2*f
*g*n*log(F)) - 9*a*b**2*c*d**2*x*(F**(e*g + f*g*x))**(2*n)/(2*f**2*g**2*n**
2*log(F)**2) + 9*a*b**2*c*d**2*(F**(e*g + f*g*x))**(2*n)/(4*f**3*g**3*n**3*
log(F)**3) + 3*a*b**2*d**3*x**3*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F))
- 9*a*b**2*d**3*x**2*(F**(e*g + f*g*x))**(2*n)/(4*f**2*g**2*n**2*log(F)**2)
 + 9*a*b**2*d**3*x*(F**(e*g + f*g*x))**(2*n)/(4*f**3*g**3*n**3*log(F)**3) -
 9*a*b**2*d**3*(F**(e*g + f*g*x))**(2*n)/(8*f**4*g**4*n**4*log(F)**4) + b**
3*c**3*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)) + b**3*c**2*d*x*(F**(e*g
+ f*g*x))**(3*n)/(f*g*n*log(F)) - b**3*c**2*d*(F**(e*g + f*g*x))**(3*n)/(3*
f**2*g**2*n**2*log(F)**2) + b**3*c*d**2*x**2*(F**(e*g + f*g*x))**(3*n)/(f*g
*n*log(F)) - 2*b**3*c*d**2*x*(F**(e*g + f*g*x))**(3*n)/(3*f**2*g**2*n**2*lo
g(F)**2) + 2*b**3*c*d**2*(F**(e*g + f*g*x))**(3*n)/(9*f**3*g**3*n**3*log(F)
**3) + b**3*d**3*x**3*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)) - b**3*d**
3*x**2*(F**(e*g + f*g*x))**(3*n)/(3*f**2*g**2*n**2*log(F)**2) + 2*b**3*d**3
*x*(F**(e*g + f*g*x))**(3*n)/(9*f**3*g**3*n**3*log(F)**3) - 2*b**3*d**3*(F*
*(e*g + f*g*x))**(3*n)/(27*f**4*g**4*n**4*log(F)**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.72

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx$$
$$= \frac{1}{4}a^3d^3x^4 + a^3cd^2x^3 + \frac{3}{2}a^3c^2dx^2 + a^3c^3x + \frac{3F^{fgnx+egn}a^2bc^3}{fgn\log(F)} + \frac{3F^{2fgnx+2egn}ab^2c^3}{2fgn\log(F)}$$
$$+ \frac{F^{3fgnx+3egn}b^3c^3}{3fgn\log(F)} + \frac{9\left(F^{egn}fgnx\log(F) - F^{egn}\right)F^{fgnx}a^2bc^2d}{f^2g^2n^2\log(F)^2}$$
$$+ \frac{9\left(2F^{2egn}fgnx\log(F) - F^{2egn}\right)F^{2fgnx}ab^2c^2d}{4f^2g^2n^2\log(F)^2}$$
$$+ \frac{\left(3F^{3egn}fgnx\log(F) - F^{3egn}\right)F^{3fgnx}b^3c^2d}{3f^2g^2n^2\log(F)^2}$$
$$+ \frac{9\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2F^{egn}fgnx\log(F) + 2F^{egn}\right)F^{fgnx}a^2bcd^2}{f^3g^3n^3\log(F)^3}$$
$$+ \frac{9\left(2F^{2egn}f^2g^2n^2x^2\log(F)^2 - 2F^{2egn}fgnx\log(F) + F^{2egn}\right)F^{2fgnx}ab^2cd^2}{4f^3g^3n^3\log(F)^3}$$
$$+ \frac{\left(9F^{3egn}f^2g^2n^2x^2\log(F)^2 - 6F^{3egn}fgnx\log(F) + 2F^{3egn}\right)F^{3fgnx}b^3cd^2}{9f^3g^3n^3\log(F)^3}$$
$$+ \frac{3\left(F^{egn}f^3g^3n^3x^3\log(F)^3 - 3F^{egn}f^2g^2n^2x^2\log(F)^2 + 6F^{egn}fgnx\log(F) - 6F^{egn}\right)F^{fgnx}a^2bd^3}{f^4g^4n^4\log(F)^4}$$
$$+ \frac{3\left(4F^{2egn}f^3g^3n^3x^3\log(F)^3 - 6F^{2egn}f^2g^2n^2x^2\log(F)^2 + 6F^{2egn}fgnx\log(F) - 3F^{2egn}\right)F^{2fgnx}ab^2d^3}{8f^4g^4n^4\log(F)^4}$$
$$+ \frac{\left(9F^{3egn}f^3g^3n^3x^3\log(F)^3 - 9F^{3egn}f^2g^2n^2x^2\log(F)^2 + 6F^{3egn}fgnx\log(F) - 2F^{3egn}\right)F^{3fgnx}b^3d^3}{27f^4g^4n^4\log(F)^4}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*x + 3*F^(f*g*
n*x + e*g*n)*a^2*b*c^3/(f*g*n*log(F)) + 3/2*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*c
^3/(f*g*n*log(F)) + 1/3*F^(3*f*g*n*x + 3*e*g*n)*b^3*c^3/(f*g*n*log(F)) + 9*
(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*a^2*b*c^2*d/(f^2*g^2*n^2
*log(F)^2) + 9/4*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)
*a*b^2*c^2*d/(f^2*g^2*n^2*log(F)^2) + 1/3*(3*F^(3*e*g*n)*f*g*n*x*log(F) - F
^(3*e*g*n))*F^(3*f*g*n*x)*b^3*c^2*d/(f^2*g^2*n^2*log(F)^2) + 9*(F^(e*g*n)*f
^2*g^2*n^2*x^2*log(F)^2 - 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*
n*x)*a^2*b*c*d^2/(f^3*g^3*n^3*log(F)^3) + 9/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^
2*log(F)^2 - 2*F^(2*e*g*n)*f*g*n*x*log(F) + F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^
2*c*d^2/(f^3*g^3*n^3*log(F)^3) + 1/9*(9*F^(3*e*g*n)*f^2*g^2*n^2*x^2*log(F)^
```

```
2 - 6*F^(3*e*g*n)*f*g*n*x*log(F) + 2*F^(3*e*g*n))*F^(3*f*g*n*x)*b^3*c*d^2/(
f^3*g^3*n^3*log(F)^3) + 3*(F^(e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 3*F^(e*g*n)
*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(e*g*n)*f*g*n*x*log(F) - 6*F^(e*g*n))*F^(f*
g*n*x)*a^2*b*d^3/(f^4*g^4*n^4*log(F)^4) + 3/8*(4*F^(2*e*g*n)*f^3*g^3*n^3*x^
3*log(F)^3 - 6*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(2*e*g*n)*f*g*n*x
*log(F) - 3*F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*d^3/(f^4*g^4*n^4*log(F)^4) + 1
/27*(9*F^(3*e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 9*F^(3*e*g*n)*f^2*g^2*n^2*x^2
*log(F)^2 + 6*F^(3*e*g*n)*f*g*n*x*log(F) - 2*F^(3*e*g*n))*F^(3*f*g*n*x)*b^3
*d^3/(f^4*g^4*n^4*log(F)^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 18707, normalized size of antiderivative = 37.72

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*x - 1/27*(((2
7*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 27*pi^2*b^3*d^3*f^3*g^3
*n^3*x^3*log(abs(F)) + 18*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 81*pi^2*b
^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 81*pi^2*b^3*c*d^2*f^3*g^3*n^3
*x^2*log(abs(F)) + 54*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 81*pi^2*b^3
*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 81*pi^2*b^3*c^2*d*f^3*g^3*n^3*x*l
og(abs(F)) + 54*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 27*pi^2*b^3*c^3*f^3
*g^3*n^3*log(abs(F))*sgn(F) - 27*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F)) + 18*
b^3*c^3*f^3*g^3*n^3*log(abs(F))^3 - 9*pi^2*b^3*d^3*f^2*g^2*n^2*x^2*sgn(F) +
 9*pi^2*b^3*d^3*f^2*g^2*n^2*x^2 - 18*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2
- 18*pi^2*b^3*c*d^2*f^2*g^2*n^2*x*sgn(F) + 18*pi^2*b^3*c*d^2*f^2*g^2*n^2*x
- 36*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 9*pi^2*b^3*c^2*d*f^2*g^2*n^2*s
gn(F) + 9*pi^2*b^3*c^2*d*f^2*g^2*n^2 - 18*b^3*c^2*d*f^2*g^2*n^2*log(abs(F))
^2 + 12*b^3*d^3*f*g*n*x*log(abs(F)) + 12*b^3*c*d^2*f*g*n*log(abs(F)) - 4*b^
3*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) -
 pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(ab
s(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F
) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log
(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*lo
g(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(
F))^3)^2) - 12*(3*pi^3*b^3*d^3*f^3*g^3*n^3*x^3*sgn(F) - 9*pi*b^3*d^3*f^3*g^
3*n^3*x^3*log(abs(F))^2*sgn(F) - 3*pi^3*b^3*d^3*f^3*g^3*n^3*x^3 + 9*pi*b^3*
d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 9*pi^3*b^3*c*d^2*f^3*g^3*n^3*x^2*sgn(F)
 - 27*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 9*pi^3*b^3*c*d^2*
f^3*g^3*n^3*x^2 + 27*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 9*pi^3*b^
```

```
3*c^2*d*f^3*g^3*n^3*x*sgn(F) - 27*pi*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*
sgn(F) - 9*pi^3*b^3*c^2*d*f^3*g^3*n^3*x + 27*pi*b^3*c^2*d*f^3*g^3*n^3*x*log
(abs(F))^2 + 3*pi^3*b^3*c^3*f^3*g^3*n^3*sgn(F) - 9*pi*b^3*c^3*f^3*g^3*n^3*l
og(abs(F))^2*sgn(F) - 3*pi^3*b^3*c^3*f^3*g^3*n^3 + 9*pi*b^3*c^3*f^3*g^3*n^3
*log(abs(F))^2 + 6*pi*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*b^3
*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 12*pi*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F)
)*sgn(F) - 12*pi*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*b^3*c^2*d*f^2*g
^2*n^2*log(abs(F))*sgn(F) - 6*pi*b^3*c^2*d*f^2*g^2*n^2*log(abs(F)) - 2*pi*b
^3*d^3*f*g*n*x*sgn(F) + 2*pi*b^3*d^3*f*g*n*x - 2*pi*b^3*c*d^2*f*g*n*sgn(F)
 + 2*pi*b^3*c*d^2*f*g*n)*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n
^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log
(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sg
n(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*
log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4
*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(a
bs(F))^3)^2))*cos(-3/2*pi*f*g*n*x*sgn(F) + 3/2*pi*f*g*n*x - 3/2*pi*e*g*n*sg
n(F) + 3/2*pi*e*g*n) - (3*(3*pi^3*b^3*d^3*f^3*g^3*n^3*x^3*sgn(F) - 9*pi*b^3
*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - 3*pi^3*b^3*d^3*f^3*g^3*n^3*x^3
 + 9*pi*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 9*pi^3*b^3*c*d^2*f^3*g^3*n^3
*x^2*sgn(F) - 27*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 9*pi^3
*b^3*c*d^2*f^3*g^3*n^3*x^2 + 27*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2
 + 9*pi^3*b^3*c^2*d*f^3*g^3*n^3*x*sgn(F) - 27*pi*b^3*c^2*d*f^3*g^3*n^3*x*log
(abs(F))^2*sgn(F) - 9*pi^3*b^3*c^2*d*f^3*g^3*n^3*x + 27*pi*b^3*c^2*d*f^3*g^
3*n^3*x*log(abs(F))^2 + 3*pi^3*b^3*c^3*f^3*g^3*n^3*sgn(F) - 9*pi*b^3*c^3*f^
3*g^3*n^3*log(abs(F))^2*sgn(F) - 3*pi^3*b^3*c^3*f^3*g^3*n^3 + 9*pi*b^3*c^3*
f^3*g^3*n^3*log(abs(F))^2 + 6*pi*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F)
 - 6*pi*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 12*pi*b^3*c*d^2*f^2*g^2*n^2*x
*log(abs(F))*sgn(F) - 12*pi*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*b^3*
c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*b^3*c^2*d*f^2*g^2*n^2*log(abs(F
)) - 2*pi*b^3*d^3*f*g*n*x*sgn(F) + 2*pi*b^3*d^3*f*g*n*x - 2*pi*b^3*c*d^2*f*
g*n*sgn(F) + 2*pi*b^3*c*d^2*f*g*n)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^
4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(
F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4
*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(a
bs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F)
)*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F
)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) + 4*(27*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*
log(abs(F))*sgn(F) - 27*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 18*b^3*d
^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 81*pi^2*b^3*c*d^2*f^3*g^3*n^3*x^2*log(ab
s(F))*sgn(F) - 81*pi^2*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 54*b^3*c*d^2
*f^3*g^3*n^3*x^2*log(abs(F))^3 + 81*pi^2*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F)
)*sgn(F) - 81*pi^2*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 54*b^3*c^2*d*f^3*g
^3*n^3*x*log(abs(F))^3 + 27*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 2
7*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F)) + 18*b^3*c^3*f^3*g^3*n^3*log(abs(F))
^3 - 9*pi^2*b^3*d^3*f^2*g^2*n^2*x^2*sgn(F) + 9*pi^2*b^3*d^3*f^2*g^2*n^2*x^2
```

- 18*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 18*pi^2*b^3*c*d^2*f^2*g^2*n^2
*x*sgn(F) + 18*pi^2*b^3*c*d^2*f^2*g^2*n^2*x - 36*b^3*c*d^2*f^2*g^2*n^2*x*lo
g(abs(F))^2 - 9*pi^2*b^3*c^2*d*f^2*g^2*n^2*sgn(F) + 9*pi^2*b^3*c^2*d*f^2*g^
2*n^2 - 18*b^3*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 12*b^3*d^3*f*g*n*x*log(abs
(F)) + 12*b^3*c*d^2*f*g*n*log(abs(F)) - 4*b^3*d^3)*(pi^3*f^4*g^4*n^4*log(ab
s(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(a
bs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f
^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log
(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(
F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs
(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*sin(-3/2*pi*f*g*n*x*sgn(F) + 3/2*p
i*f*g*n*x - 3/2*pi*e*g*n*sgn(F) + 3/2*pi*e*g*n))*e^(3*f*g*n*x*log(abs(F)) +
3*e*g*n*log(abs(F))) - 1/54*I*((9*pi^3*b^3*d^3*f^3*g^3*n^3*x^3*sgn(F) + 27
*I*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 27*pi*b^3*d^3*f^3*g^3*
n^3*x^3*log(abs(F))^2*sgn(F) - 9*pi^3*b^3*d^3*f^3*g^3*n^3*x^3 - 27*I*pi^2*b
^3*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 27*pi*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(
F))^2 + 18*I*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 27*pi^3*b^3*c*d^2*f^3*
g^3*n^3*x^2*sgn(F) + 81*I*pi^2*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F)
- 81*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 27*pi^3*b^3*c*d^2
*f^3*g^3*n^3*x^2 - 81*I*pi^2*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 81*pi*
b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 54*I*b^3*c*d^2*f^3*g^3*n^3*x^2*lo
g(abs(F))^3 + 27*pi^3*b^3*c^2*d*f^3*g^3*n^3*x*sgn(F) + 81*I*pi^2*b^3*c^2*d*
f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 81*pi*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F)
)^2*sgn(F) - 27*pi^3*b^3*c^2*d*f^3*g^3*n^3*x - 81*I*pi^2*b^3*c^2*d*f^3*g^3*
n^3*x*log(abs(F)) + 81*pi*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 54*I*b^3*
c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 9*pi^3*b^3*c^3*f^3*g^3*n^3*sgn(F) + 27*
I*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 27*pi*b^3*c^3*f^3*g^3*n^3*l
og(abs(F))^2*sgn(F) - 9*pi^3*b^3*c^3*f^3*g^3*n^3 - 27*I*pi^2*b^3*c^3*f^3*g^
3*n^3*log(abs(F)) + 27*pi*b^3*c^3*f^3*g^3*n^3*log(abs(F))^2 + 18*I*b^3*c^3*
f^3*g^3*n^3*log(abs(F))^3 - 9*I*pi^2*b^3*d^3*f^2*g^2*n^2*x^2*sgn(F) + 18*pi
*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 9*I*pi^2*b^3*d^3*f^2*g^2*n^2*
x^2 - 18*pi*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F)) - 18*I*b^3*d^3*f^2*g^2*n^2*
x^2*log(abs(F))^2 - 18*I*pi^2*b^3*c*d^2*f^2*g^2*n^2*x*sgn(F) + 36*pi*b^3*c*
d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 18*I*pi^2*b^3*c*d^2*f^2*g^2*n^2*x -
36*pi*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F)) - 36*I*b^3*c*d^2*f^2*g^2*n^2*x*lo
g(abs(F))^2 - 9*I*pi^2*b^3*c^2*d*f^2*g^2*n^2*sgn(F) + 18*pi*b^3*c^2*d*f^2*g
^2*n^2*log(abs(F))*sgn(F) + 9*I*pi^2*b^3*c^2*d*f^2*g^2*n^2 - 18*pi*b^3*c^2*
d*f^2*g^2*n^2*log(abs(F)) - 18*I*b^3*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 6*pi
*b^3*d^3*f*g*n*x*sgn(F) + 6*pi*b^3*d^3*f*g*n*x + 12*I*b^3*d^3*f*g*n*x*log(a
bs(F)) - 6*pi*b^3*c*d^2*f*g*n*sgn(F) + 6*pi*b^3*c*d^2*f*g*n + 12*I*b^3*c*d^
2*f*g*n*log(abs(F)) - 4*I*b^3*d^3)*e^(3/2*I*pi*f*g*n*x*sgn(F) - 3/2*I*pi*f*
g*n*x + 3/2*I*pi*e*g*n*sgn(F) - 3/2*I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) +
4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*
sgn(F) - 4*I*pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 - 4*I*p
i^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 + 4*I*pi*f^4

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*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^4*log(abs(F))^4) + (9*pi^3*b^3*d^3*f^3
*g^3*n^3*x^3*sgn(F) - 27*I*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F)
- 27*pi*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - 9*pi^3*b^3*d^3*f^3*g
^3*n^3*x^3 + 27*I*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 27*pi*b^3*d^3*
f^3*g^3*n^3*x^3*log(abs(F))^2 - 18*I*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3
+ 27*pi^3*b^3*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 81*I*pi^2*b^3*c*d^2*f^3*g^3*n^
3*x^2*log(abs(F))*sgn(F) - 81*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sg
n(F) - 27*pi^3*b^3*c*d^2*f^3*g^3*n^3*x^2 + 81*I*pi^2*b^3*c*d^2*f^3*g^3*n^3*
x^2*log(abs(F)) + 81*pi*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 - 54*I*b^3*
c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 27*pi^3*b^3*c^2*d*f^3*g^3*n^3*x*sgn(F
) - 81*I*pi^2*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 81*pi*b^3*c^2*d*
f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 27*pi^3*b^3*c^2*d*f^3*g^3*n^3*x + 81*I
*pi^2*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 81*pi*b^3*c^2*d*f^3*g^3*n^3*x*l
og(abs(F))^2 - 54*I*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 9*pi^3*b^3*c^3*
f^3*g^3*n^3*sgn(F) - 27*I*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 27*
pi*b^3*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 9*pi^3*b^3*c^3*f^3*g^3*n^3 +
27*I*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F)) + 27*pi*b^3*c^3*f^3*g^3*n^3*log(a
bs(F))^2 - 18*I*b^3*c^3*f^3*g^3*n^3*log(abs(F))^3 + 9*I*pi^2*b^3*d^3*f^2*g^
2*n^2*x^2*sgn(F) + 18*pi*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 9*I*p
i^2*b^3*d^3*f^2*g^2*n^2*x^2 - 18*pi*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 1
8*I*b^3*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 + 18*I*pi^2*b^3*c*d^2*f^2*g^2*n^2
*x*sgn(F) + 36*pi*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 18*I*pi^2*b^
3*c*d^2*f^2*g^2*n^2*x - 36*pi*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 36*I*b^
3*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 + 9*I*pi^2*b^3*c^2*d*f^2*g^2*n^2*sgn(F)
 + 18*pi*b^3*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 9*I*pi^2*b^3*c^2*d*f^2*
g^2*n^2 - 18*pi*b^3*c^2*d*f^2*g^2*n^2*log(abs(F)) + 18*I*b^3*c^2*d*f^2*g^2*
n^2*log(abs(F))^2 - 6*pi*b^3*d^3*f*g*n*x*sgn(F) + 6*pi*b^3*d^3*f*g*n*x - 12
*I*b^3*d^3*f*g*n*x*log(abs(F)) - 6*pi*b^3*c*d^2*f*g*n*sgn(F) + 6*pi*b^3*c*d
^2*f*g*n - 12*I*b^3*c*d^2*f*g*n*log(abs(F)) + 4*I*b^3*d^3)*e^(-3/2*I*pi*f*g
*n*x*sgn(F) + 3/2*I*pi*f*g*n*x - 3/2*I*pi*e*g*n*sgn(F) + 3/2*I*pi*e*g*n)/(p
i^4*f^4*g^4*n^4*sgn(F) - 4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - 6*pi^2*f
^4*g^4*n^4*log(abs(F))^2*sgn(F) + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) -
 pi^4*f^4*g^4*n^4 + 4*I*pi^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*f^4*g^4*n^4*l
og(abs(F))^2 - 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^4*log(abs(F))
^4))*e^(3*f*g*n*x*log(abs(F)) + 3*e*g*n*log(abs(F))) - 3/4*(((6*pi^2*a*b^2*
d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi^2*a*b^2*d^3*f^3*g^3*n^3*x^3*l
og(abs(F)) + 4*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 18*pi^2*a*b^2*c*d^
2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi^2*a*b^2*c*d^2*f^3*g^3*n^3*x^2*
log(abs(F)) + 12*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 18*pi^2*a*b^2*
c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi^2*a*b^2*c^2*d*f^3*g^3*n^3*x*
log(abs(F)) + 12*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 6*pi^2*a*b^2*c^3
*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi^2*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))
+ 4*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*a*b^2*d^3*f^2*g^2*n^2*x^2*
sgn(F) + 3*pi^2*a*b^2*d^3*f^2*g^2*n^2*x^2 - 6*a*b^2*d^3*f^2*g^2*n^2*x^2*log
(abs(F))^2 - 6*pi^2*a*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*a*b^2*c*d^2*f
```

^2*g^2*n^2*x - 12*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*a*b^2*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*a*b^2*c^2*d*f^2*g^2*n^2 - 6*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 6*a*b^2*d^3*f*g*n*x*log(abs(F)) + 6*a*b^2*c*d^2*f*g*n*log(abs(F)) - 3*a*b^2*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) - 4*(2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) - 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - 2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3 + 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 6*pi^3*a*b^2*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 18*pi*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c*d^2*f^3*g^3*n^3*x^2 + 18*pi*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x*sgn(F) - 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x + 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 2*pi^3*a*b^2*c^3*f^3*g^3*n^3*sgn(F) - 6*pi*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 2*pi^3*a*b^2*c^3*f^3*g^3*n^3 + 6*pi*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F)) - 3*pi*a*b^2*d^3*f*g*n*x*sgn(F) + 3*pi*a*b^2*d^3*f*g*n*x - 3*pi*a*b^2*c*d^2*f*g*n*sgn(F) + 3*pi*a*b^2*c*d^2*f*g*n)*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*cos(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n) - ((2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) - 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - 2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3 + 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 6*pi^3*a*b^2*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 18*pi*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c*d^2*f^3*g^3*n^3*x^2 + 18*pi*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x*sgn(F) - 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x + 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + 2*pi^3*a*b^2*c^3*f^3*g^3*n^3*sgn(F) - 6*pi*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 2*pi^3*a*b^2*c^3*f^3*g^3*n^3 + 6*pi*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F)) - 3*pi*a*b^2*d^3*f*g*n*x*sgn(F) + 3*pi*a*b^2*d

^3*f*g*n*x - 3*pi*a*b^2*c*d^2*f*g*n*sgn(F) + 3*pi*a*b^2*c*d^2*f*g*n)*(pi^4*
f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4
*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi
^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*
g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2
 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*s
gn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) + 4
*(6*pi^2*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi^2*a*b^2*d^3*f^
3*g^3*n^3*x^3*log(abs(F)) + 4*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 18*
pi^2*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi^2*a*b^2*c*d^2*f
^3*g^3*n^3*x^2*log(abs(F)) + 12*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 +
 18*pi^2*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi^2*a*b^2*c^2*d
*f^3*g^3*n^3*x*log(abs(F)) + 12*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 6
*pi^2*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi^2*a*b^2*c^3*f^3*g^3*n
^3*log(abs(F)) + 4*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*a*b^2*d^3*f
^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*a*b^2*d^3*f^2*g^2*n^2*x^2 - 6*a*b^2*d^3*f^2*
g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2*a*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^
2*a*b^2*c*d^2*f^2*g^2*n^2*x - 12*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 -
3*pi^2*a*b^2*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*a*b^2*c^2*d*f^2*g^2*n^2 - 6*
a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 6*a*b^2*d^3*f*g*n*x*log(abs(F)) + 6
*a*b^2*c*d^2*f*g*n*log(abs(F)) - 3*a*b^2*d^3)*(pi^3*f^4*g^4*n^4*log(abs(F))
*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)
) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^
4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(
F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*s
gn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F))
+ pi*f^4*g^4*n^4*log(abs(F))^3)^2))*sin(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - p
i*e*g*n*sgn(F) + pi*e*g*n))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F)))
 - 3/8*I*((2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) + 6*I*pi^2*a*b^2*d^3*f^3
*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F)
)^2*sgn(F) - 2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3 - 6*I*pi^2*a*b^2*d^3*f^3*g^3*
n^3*x^3*log(abs(F)) + 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 4*I*a*
b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 6*pi^3*a*b^2*c*d^2*f^3*g^3*n^3*x^2*
sgn(F) + 18*I*pi^2*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi*a
*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c*d^2*f^3*g^
3*n^3*x^2 - 18*I*pi^2*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 18*pi*a*b^2
*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 12*I*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log
(abs(F))^3 + 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x*sgn(F) + 18*I*pi^2*a*b^2*c^2*
d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(ab
s(F))^2*sgn(F) - 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x - 18*I*pi^2*a*b^2*c^2*d*f
^3*g^3*n^3*x*log(abs(F)) + 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 +
12*I*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 2*pi^3*a*b^2*c^3*f^3*g^3*n^3
*sgn(F) + 6*I*pi^2*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi*a*b^2*c^
3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 2*pi^3*a*b^2*c^3*f^3*g^3*n^3 - 6*I*pi^
2*a*b^2*c^3*f^3*g^3*n^3*log(abs(F)) + 6*pi*a*b^2*c^3*f^3*g^3*n^3*log(abs(F)

)^2 + 4*I*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*I*pi^2*a*b^2*d^3*f^2*g^2*
n^2*x^2*sgn(F) + 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 3*I*pi
^2*a*b^2*d^3*f^2*g^2*n^2*x^2 - 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F)) -
6*I*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*I*pi^2*a*b^2*c*d^2*f^2*g^2
*n^2*x*sgn(F) + 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 6*I*pi
^2*a*b^2*c*d^2*f^2*g^2*n^2*x - 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))
- 12*I*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*I*pi^2*a*b^2*c^2*d*f^2*g
^2*n^2*sgn(F) + 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) + 3*I*pi^2*
a*b^2*c^2*d*f^2*g^2*n^2 - 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*I*a*
b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 3*pi*a*b^2*d^3*f*g*n*x*sgn(F) + 3*pi*
a*b^2*d^3*f*g*n*x + 6*I*a*b^2*d^3*f*g*n*x*log(abs(F)) - 3*pi*a*b^2*c*d^2*f*
g*n*sgn(F) + 3*pi*a*b^2*c*d^2*f*g*n + 6*I*a*b^2*c*d^2*f*g*n*log(abs(F)) - 3
*I*a*b^2*d^3)*e^(I*pi*f*g*n*x*sgn(F) - I*pi*f*g*n*x + I*pi*e*g*n*sgn(F) - I
*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) + 4*I*pi^3*f^4*g^4*n^4*log(abs(F))*sgn(
F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - 4*I*pi*f^4*g^4*n^4*log(abs(F
))^3*sgn(F) - pi^4*f^4*g^4*n^4 - 4*I*pi^3*f^4*g^4*n^4*log(abs(F)) + 6*pi^2*
f^4*g^4*n^4*log(abs(F))^2 + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*f^4*g^4*n^
4*log(abs(F))^4) + (2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3*sgn(F) - 6*I*pi^2*a*b^
2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*l
og(abs(F))^2sgn(F) - 2*pi^3*a*b^2*d^3*f^3*g^3*n^3*x^3 + 6*I*pi^2*a*b^2*d^3
*f^3*g^3*n^3*x^3*log(abs(F)) + 6*pi*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2
- 4*I*a*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 6*pi^3*a*b^2*c*d^2*f^3*g^3
*n^3*x^2*sgn(F) - 18*I*pi^2*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F)
- 18*pi*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c*d
^2*f^3*g^3*n^3*x^2 + 18*I*pi^2*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 18
*pi*a*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 - 12*I*a*b^2*c*d^2*f^3*g^3*n^
3*x^2*log(abs(F))^3 + 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x*sgn(F) - 18*I*pi^2*a
*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi*a*b^2*c^2*d*f^3*g^3*n^3
*x*log(abs(F))^2*sgn(F) - 6*pi^3*a*b^2*c^2*d*f^3*g^3*n^3*x + 18*I*pi^2*a*b^
2*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 18*pi*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs
(F))^2 - 12*I*a*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 2*pi^3*a*b^2*c^3*f^
3*g^3*n^3*sgn(F) - 6*I*pi^2*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi
*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 2*pi^3*a*b^2*c^3*f^3*g^3*n^3
+ 6*I*pi^2*a*b^2*c^3*f^3*g^3*n^3*log(abs(F)) + 6*pi*a*b^2*c^3*f^3*g^3*n^3*l
og(abs(F))^2 - 4*I*a*b^2*c^3*f^3*g^3*n^3*log(abs(F))^3 + 3*I*pi^2*a*b^2*d^3
*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F)
- 3*I*pi^2*a*b^2*d^3*f^2*g^2*n^2*x^2 - 6*pi*a*b^2*d^3*f^2*g^2*n^2*x^2*log(
abs(F)) + 6*I*a*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 + 6*I*pi^2*a*b^2*c*d^
2*f^2*g^2*n^2*x*sgn(F) + 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F)
- 6*I*pi^2*a*b^2*c*d^2*f^2*g^2*n^2*x - 12*pi*a*b^2*c*d^2*f^2*g^2*n^2*x*log
(abs(F)) + 12*I*a*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 + 3*I*pi^2*a*b^2*c^
2*d*f^2*g^2*n^2*sgn(F) + 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) -
3*I*pi^2*a*b^2*c^2*d*f^2*g^2*n^2 - 6*pi*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))
+ 6*I*a*b^2*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 3*pi*a*b^2*d^3*f*g*n*x*sgn(F
) + 3*pi*a*b^2*d^3*f*g*n*x - 6*I*a*b^2*d^3*f*g*n*x*log(abs(F)) - 3*pi*a*b^2

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*c*d^2*f*g*n*sgn(F) + 3*pi*a*b^2*c*d^2*f*g*n - 6*I*a*b^2*c*d^2*f*g*n*log(ab
s(F)) + 3*I*a*b^2*d^3)*e^(-I*pi*f*g*n*x*sgn(F) + I*pi*f*g*n*x - I*pi*e*g*n*
sgn(F) + I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) - 4*I*pi^3*f^4*g^4*n^4*log(ab
s(F))*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) + 4*I*pi*f^4*g^4*n^4
*log(abs(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 + 4*I*pi^3*f^4*g^4*n^4*log(abs(F))
 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 4*I*pi*f^4*g^4*n^4*log(abs(F))^3 - 2*
f^4*g^4*n^4*log(abs(F))^4))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F)))
 - 3*(((3*pi^2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*a^2*b*
d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3
 + 9*pi^2*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi^2*a^2*b*c*d
^2*f^3*g^3*n^3*x^2*log(abs(F)) + 6*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^
3 + 9*pi^2*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 9*pi^2*a^2*b*c^2*
d*f^3*g^3*n^3*x*log(abs(F)) + 6*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 3
*pi^2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*a^2*b*c^3*f^3*g^3*n
^3*log(abs(F)) + 2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*a^2*b*d^3*f
^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*a^2*b*d^3*f^2*g^2*n^2*x^2 - 6*a^2*b*d^3*f^2*
g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2*a^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^
2*a^2*b*c*d^2*f^2*g^2*n^2*x - 12*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 -
3*pi^2*a^2*b*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*a^2*b*c^2*d*f^2*g^2*n^2 - 6*
a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 12*a^2*b*d^3*f*g*n*x*log(abs(F)) +
12*a^2*b*c*d^2*f*g*n*log(abs(F)) - 12*a^2*b*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) -
 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^
4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F
) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4
*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*
n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4
*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2) - 4*(pi^3*a^2*b*d^3*f^3
*g^3*n^3*x^3*sgn(F) - 3*pi*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) -
 pi^3*a^2*b*d^3*f^3*g^3*n^3*x^3 + 3*pi*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)
)^2 + 3*pi^3*a^2*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*pi*a^2*b*c*d^2*f^3*g^3*
n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*a^2*b*c*d^2*f^3*g^3*n^3*x^2 + 9*pi*a^
2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x*
sgn(F) - 9*pi*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*a^2*b
*c^2*d*f^3*g^3*n^3*x + 9*pi*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + pi^3*
a^2*b*c^3*f^3*g^3*n^3*sgn(F) - 3*pi*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn
(F) - pi^3*a^2*b*c^3*f^3*g^3*n^3 + 3*pi*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^2
 + 6*pi*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 6*pi*a^2*b*d^3*f^2*g
^2*n^2*x^2*log(abs(F)) + 12*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F)
 - 12*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F)) + 6*pi*a^2*b*c^2*d*f^2*g^2*n
^2*log(abs(F))*sgn(F) - 6*pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*pi*a^2
*b*d^3*f*g*n*x*sgn(F) + 6*pi*a^2*b*d^3*f*g*n*x - 6*pi*a^2*b*c*d^2*f*g*n*sgn
(F) + 6*pi*a^2*b*c*d^2*f*g*n)*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4
*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n
^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F)
)^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^
```

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4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g
^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4
*log(abs(F))^3)^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*
g*n*sgn(F) + 1/2*pi*e*g*n) - ((pi^3*a^2*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*pi
*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - pi^3*a^2*b*d^3*f^3*g^3*n^
3*x^3 + 3*pi*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^2 + 3*pi^3*a^2*b*c*d^2*f
^3*g^3*n^3*x^2*sgn(F) - 9*pi*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(
F) - 3*pi^3*a^2*b*c*d^2*f^3*g^3*n^3*x^2 + 9*pi*a^2*b*c*d^2*f^3*g^3*n^3*x^2*
log(abs(F))^2 + 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x*sgn(F) - 9*pi*a^2*b*c^2*d*
f^3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x + 9*p
i*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^2 + pi^3*a^2*b*c^3*f^3*g^3*n^3*sgn(
F) - 3*pi*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*a^2*b*c^3*f^3*g
^3*n^3 + 3*pi*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^2 + 6*pi*a^2*b*d^3*f^2*g^2*
n^2*x^2*log(abs(F))*sgn(F) - 6*pi*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F)) + 1
2*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 12*pi*a^2*b*c*d^2*f^2*g
^2*n^2*x*log(abs(F)) + 6*pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) - 6*
pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*pi*a^2*b*d^3*f*g*n*x*sgn(F) + 6*
pi*a^2*b*d^3*f*g*n*x - 6*pi*a^2*b*c*d^2*f*g*n*sgn(F) + 6*pi*a^2*b*c*d^2*f*g
*n)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi
^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F
))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) -
 pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log(ab
s(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(a
bs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))
^3)^2) + 4*(3*pi^2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*a^
2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F
))^3 + 9*pi^2*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 9*pi^2*a^2*b
*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 6*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(
F))^3 + 9*pi^2*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 9*pi^2*a^2*b*
c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 6*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3
 + 3*pi^2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*a^2*b*c^3*f^3*g
^3*n^3*log(abs(F)) + 2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*a^2*b*d
^3*f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*a^2*b*d^3*f^2*g^2*n^2*x^2 - 6*a^2*b*d^3*
f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2*a^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6
*pi^2*a^2*b*c*d^2*f^2*g^2*n^2*x - 12*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^
2 - 3*pi^2*a^2*b*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*a^2*b*c^2*d*f^2*g^2*n^2
- 6*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 + 12*a^2*b*d^3*f*g*n*x*log(abs(F)
) + 12*a^2*b*c*d^2*f*g*n*log(abs(F)) - 12*a^2*b*d^3)*(pi^3*f^4*g^4*n^4*log(
abs(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log
(abs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2
*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*l
og(abs(F))^2 - 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(ab
s(F))*sgn(F) - pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(a
bs(F)) + pi*f^4*g^4*n^4*log(abs(F))^3)^2))*sin(-1/2*pi*f*g*n*x*sgn(F) + 1/2
*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*e^(f*g*n*x*log(abs(F)) +
```

e*g*n*log(abs(F))) - 3/2*I*((pi^3*a^2*b*d^3*f^3*g^3*n^3*x^3*sgn(F) + 3*I*p
i^2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi*a^2*b*d^3*f^3*g^3*n
^3*x^3*log(abs(F))^2*sgn(F) - pi^3*a^2*b*d^3*f^3*g^3*n^3*x^3 - 3*I*pi^2*a^2
*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 3*pi*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs
(F))^2 + 2*I*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 3*pi^3*a^2*b*c*d^2*f
^3*g^3*n^3*x^2*sgn(F) + 9*I*pi^2*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sg
n(F) - 9*pi*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*pi^3*a^2*b
*c*d^2*f^3*g^3*n^3*x^2 - 9*I*pi^2*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) +
9*pi*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 + 6*I*a^2*b*c*d^2*f^3*g^3*n
^3*x^2*log(abs(F))^3 + 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x*sgn(F) + 9*I*pi^2*a
^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 9*pi*a^2*b*c^2*d*f^3*g^3*n^3*
x*log(abs(F))^2*sgn(F) - 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x - 9*I*pi^2*a^2*b*
c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 9*pi*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)
)^2 + 6*I*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + pi^3*a^2*b*c^3*f^3*g^3*
n^3*sgn(F) + 3*I*pi^2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi*a^2*b
*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*a^2*b*c^3*f^3*g^3*n^3 - 3*I*pi
^2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F)) + 3*pi*a^2*b*c^3*f^3*g^3*n^3*log(abs(F
))^2 + 2*I*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*I*pi^2*a^2*b*d^3*f^2*g^2
*n^2*x^2*sgn(F) + 6*pi*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 3*I*p
i^2*a^2*b*d^3*f^2*g^2*n^2*x^2 - 6*pi*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))
- 6*I*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 - 6*I*pi^2*a^2*b*c*d^2*f^2*g^
2*n^2*x*sgn(F) + 12*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 6*I*p
i^2*a^2*b*c*d^2*f^2*g^2*n^2*x - 12*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))
- 12*I*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*I*pi^2*a^2*b*c^2*d*f^2*
g^2*n^2*sgn(F) + 6*pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F) + 3*I*pi^2
*a^2*b*c^2*d*f^2*g^2*n^2 - 6*pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F)) - 6*I*a
^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 6*pi*a^2*b*d^3*f*g*n*x*sgn(F) + 6*pi
*a^2*b*d^3*f*g*n*x + 12*I*a^2*b*d^3*f*g*n*x*log(abs(F)) - 6*pi*a^2*b*c*d^2*
f*g*n*sgn(F) + 6*pi*a^2*b*c*d^2*f*g*n + 12*I*a^2*b*c*d^2*f*g*n*log(abs(F))
- 12*I*a^2*b*d^3)*e^(1/2*I*pi*f*g*n*x*sgn(F) - 1/2*I*pi*f*g*n*x + 1/2*I*pi*
e*g*n*sgn(F) - 1/2*I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) + 4*I*pi^3*f^4*g^4*
n^4*log(abs(F))*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F) - 4*I*pi*f
^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 - 4*I*pi^3*f^4*g^4*n^4*l
og(abs(F)) + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 + 4*I*pi*f^4*g^4*n^4*log(abs(
F))^3 - 2*f^4*g^4*n^4*log(abs(F))^4) + (pi^3*a^2*b*d^3*f^3*g^3*n^3*x^3*sgn(
F) - 3*I*pi^2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi*a^2*b*d^3
*f^3*g^3*n^3*x^3*log(abs(F))^2*sgn(F) - pi^3*a^2*b*d^3*f^3*g^3*n^3*x^3 + 3*
I*pi^2*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F)) + 3*pi*a^2*b*d^3*f^3*g^3*n^3*x
^3*log(abs(F))^2 - 2*I*a^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 3*pi^3*a^2
*b*c*d^2*f^3*g^3*n^3*x^2*sgn(F) - 9*I*pi^2*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(
abs(F))*sgn(F) - 9*pi*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2*sgn(F) - 3*
pi^3*a^2*b*c*d^2*f^3*g^3*n^3*x^2 + 9*I*pi^2*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log
(abs(F)) + 9*pi*a^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^2 - 6*I*a^2*b*c*d^2
*f^3*g^3*n^3*x^2*log(abs(F))^3 + 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x*sgn(F) -
9*I*pi^2*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 9*pi*a^2*b*c^2*d*f^

```
3*g^3*n^3*x*log(abs(F))^2*sgn(F) - 3*pi^3*a^2*b*c^2*d*f^3*g^3*n^3*x + 9*I*p
i^2*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 9*pi*a^2*b*c^2*d*f^3*g^3*n^3*x*
log(abs(F))^2 - 6*I*a^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + pi^3*a^2*b*c^
3*f^3*g^3*n^3*sgn(F) - 3*I*pi^2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) -
3*pi*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*a^2*b*c^3*f^3*g^3*n^
3 + 3*I*pi^2*a^2*b*c^3*f^3*g^3*n^3*log(abs(F)) + 3*pi*a^2*b*c^3*f^3*g^3*n^3
*log(abs(F))^2 - 2*I*a^2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 + 3*I*pi^2*a^2*b*d
^3*f^2*g^2*n^2*x^2*sgn(F) + 6*pi*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))*sgn(
F) - 3*I*pi^2*a^2*b*d^3*f^2*g^2*n^2*x^2 - 6*pi*a^2*b*d^3*f^2*g^2*n^2*x^2*lo
g(abs(F)) + 6*I*a^2*b*d^3*f^2*g^2*n^2*x^2*log(abs(F))^2 + 6*I*pi^2*a^2*b*c*
d^2*f^2*g^2*n^2*x*sgn(F) + 12*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))*sgn(
F) - 6*I*pi^2*a^2*b*c*d^2*f^2*g^2*n^2*x - 12*pi*a^2*b*c*d^2*f^2*g^2*n^2*x*l
og(abs(F)) + 12*I*a^2*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 + 3*I*pi^2*a^2*b*
c^2*d*f^2*g^2*n^2*sgn(F) + 6*pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))*sgn(F)
- 3*I*pi^2*a^2*b*c^2*d*f^2*g^2*n^2 - 6*pi*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F
)) + 6*I*a^2*b*c^2*d*f^2*g^2*n^2*log(abs(F))^2 - 6*pi*a^2*b*d^3*f*g*n*x*sgn
(F) + 6*pi*a^2*b*d^3*f*g*n*x - 12*I*a^2*b*d^3*f*g*n*x*log(abs(F)) - 6*pi*a^
2*b*c*d^2*f*g*n*sgn(F) + 6*pi*a^2*b*c*d^2*f*g*n - 12*I*a^2*b*c*d^2*f*g*n*lo
g(abs(F)) + 12*I*a^2*b*d^3)*e^(-1/2*I*pi*f*g*n*x*sgn(F) + 1/2*I*pi*f*g*n*x
- 1/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(pi^4*f^4*g^4*n^4*sgn(F) - 4*I*pi
^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F)
 + 4*I*pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^4*f^4*g^4*n^4 + 4*I*pi^3*f^
4*g^4*n^4*log(abs(F)) + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 4*I*pi*f^4*g^4*n
^4*log(abs(F))^3 - 2*f^4*g^4*n^4*log(abs(F))^4))*e^(f*g*n*x*log(abs(F)) + e
*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.31

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^3\,dx = a^3\,c^3\,x - \left(F^{f\,g\,x}\,F^{e\,g}\right)^n\left(\frac{3\,a^2\,b\,\left(-c^3\,f^3\,g^3\,n^3\,\ln\left(F\right)^3+3\,c^2\,d\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-6\,c\,d^2\,f\,g\,n\,\ln\left(F\right)+6\,d^3\right)}{f^4\,g^4\,n^4\,\ln\left(F\right)^4} - \frac{3\,a^2\,b\,d^3\,x^3}{f\,g\,n\,\ln\left(F\right)} - \frac{9\,a^2\,b\,d\,x\,\left(c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{f^3\,g^3\,n^3\,\ln\left(F\right)^3} + \frac{9\,a^2\,b\,d^2\,x^2\,\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{3\,a\,b^2\,\left(-4\,c^3\,f^3\,g^3\,n^3\,\ln\left(F\right)^3+6\,c^2\,d\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-6\,c\,d^2\,f\,g\,n\,\ln\left(F\right)+3\,d^3\right)}{8\,f^4\,g^4\,n^4\,\ln\left(F\right)^4} - \frac{3\,a\,b^2\,d^3\,x^3}{2\,f\,g\,n\,\ln\left(F\right)} - \frac{9\,a\,b^2\,d\,x\,\left(2\,c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+d^2\right)}{4\,f^3\,g^3\,n^3\,\ln\left(F\right)^3} + \frac{9\,a\,b^2\,d^2\,x^2\,\left(d-2\,c\,f\,g\,n\,\ln\left(F\right)\right)}{4\,f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{3\,n}\left(\frac{b^3\,\left(-9\,c^3\,f^3\,g^3\,n^3\,\ln\left(F\right)^3+9\,c^2\,d\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-6\,c\,d^2\,f\,g\,n\,\ln\left(F\right)+2\,d^3\right)}{27\,f^4\,g^4\,n^4\,\ln\left(F\right)^4} - \frac{b^3\,d^3\,x^3}{3\,f\,g\,n\,\ln\left(F\right)} - \frac{b^3\,d\,x\,\left(9\,c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-6\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{9\,f^3\,g^3\,n^3\,\ln\left(F\right)^3} + \frac{b^3\,d^2\,x^2\,\left(d-3\,c\,f\,g\,n\,\ln\left(F\right)\right)}{3\,f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right) + \frac{a^3\,d^3\,x^4}{4} + \frac{3\,a^3\,c^2\,d\,x^2}{2} + a^3\,c\,d^2\,x^3$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^3,x)
```

```
[Out] a^3*c^3*x - (F^(f*g*x)*F^(e*g))^n*((3*a^2*b*(6*d^3 - c^3*f^3*g^3*n^3*log(F)
^3 - 6*c*d^2*f*g*n*log(F) + 3*c^2*d*f^2*g^2*n^2*log(F)^2))/(f^4*g^4*n^4*log
(F)^4) - (3*a^2*b*d^3*x^3)/(f*g*n*log(F)) - (9*a^2*b*d*x*(2*d^2 + c^2*f^2*g
^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/(f^3*g^3*n^3*log(F)^3) + (9*a^2*b*d^
2*x^2*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) - (F^(f*g*x)*F^(e*g))^(
2*n)*((3*a*b^2*(3*d^3 - 4*c^3*f^3*g^3*n^3*log(F)^3 - 6*c*d^2*f*g*n*log(F) +
 6*c^2*d*f^2*g^2*n^2*log(F)^2))/(8*f^4*g^4*n^4*log(F)^4) - (3*a*b^2*d^3*x^3
)/(2*f*g*n*log(F)) - (9*a*b^2*d*x*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d
*f*g*n*log(F)))/(4*f^3*g^3*n^3*log(F)^3) + (9*a*b^2*d^2*x^2*(d - 2*c*f*g*n*
log(F)))/(4*f^2*g^2*n^2*log(F)^2)) - (F^(f*g*x)*F^(e*g))^(3*n)*((b^3*(2*d^3
 - 9*c^3*f^3*g^3*n^3*log(F)^3 - 6*c*d^2*f*g*n*log(F) + 9*c^2*d*f^2*g^2*n^2*
log(F)^2))/(27*f^4*g^4*n^4*log(F)^4) - (b^3*d^3*x^3)/(3*f*g*n*log(F)) - (b^
3*d*x*(2*d^2 + 9*c^2*f^2*g^2*n^2*log(F)^2 - 6*c*d*f*g*n*log(F)))/(9*f^3*g^3
```

```
*n^3*log(F)^3) + (b^3*d^2*x^2*(d - 3*c*f*g*n*log(F)))/(3*f^2*g^2*n^2*log(F)
^2)) + (a^3*d^3*x^4)/4 + (3*a^3*c^2*d*x^2)/2 + a^3*c*d^2*x^3
```

3.40 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 366

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx &= \frac{a^3(c+dx)^3}{3d}+\frac{6a^2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}+\frac{3ab^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}\\
&+\frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}}{27f^3g^3n^3\log^3(F)}-\frac{6a^2bd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}\\
&-\frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2f^2g^2n^2\log^2(F)}\\
&-\frac{2b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)}{9f^2g^2n^2\log^2(F)}\\
&+\frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}\\
&+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)}\\
&+\frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3fgn\log(F)}\end{aligned}$$

```
[Out] 1/3*a^3*(d*x+c)^3/d+6*a^2*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/ln(F)^3+3/4*a
*b^2*d^2*(F^(f*g*x+e*g))^(2*n)/f^3/g^3/n^3/ln(F)^3+2/27*b^3*d^2*(F^(f*g*x+e
*g))^(3*n)/f^3/g^3/n^3/ln(F)^3-6*a^2*b*d*(F^(f*g*x+e*g))^n*(d*x+c)/f^2/g^2/
n^2/ln(F)^2-3/2*a*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^2/g^2/n^2/ln(F)^2-2
/9*b^3*d*(F^(f*g*x+e*g))^(3*n)*(d*x+c)/f^2/g^2/n^2/ln(F)^2+3*a^2*b*(F^(f*g*
x+e*g))^n*(d*x+c)^2/f/g/n/ln(F)+3/2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^2/f
/g/n/ln(F)+1/3*b^3*(F^(f*g*x+e*g))^(3*n)*(d*x+c)^2/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2\,dx = \frac{a^3(c+dx)^3}{3d}-\frac{6a^2bd(c+dx)\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{3a^2b(c+dx)^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{6a^2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-\frac{3ab^2d(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2f^2g^2n^2\log^2(F)}+\frac{3ab^2(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}+\frac{3ab^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}-\frac{2b^3d(c+dx)\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)}+\frac{b^3(c+dx)^2\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)}+\frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}}{27f^3g^3n^3\log^3(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2,x]
```

```
[Out] (a^3*(c + d*x)^3)/(3*d) + (6*a^2*b*d^2*(F^(e*g + f*g*x))^n)/(f^3*g^3*n^3*Lo
g[F]^3) + (3*a*b^2*d^2*(F^(e*g + f*g*x))^(2*n))/(4*f^3*g^3*n^3*Log[F]^3) +
(2*b^3*d^2*(F^(e*g + f*g*x))^(3*n))/(27*f^3*g^3*n^3*Log[F]^3) - (6*a^2*b*d*
(F^(e*g + f*g*x))^n*(c + d*x))/(f^2*g^2*n^2*Log[F]^2) - (3*a*b^2*d*(F^(e*g
+ f*g*x))^(2*n)*(c + d*x))/(2*f^2*g^2*n^2*Log[F]^2) - (2*b^3*d*(F^(e*g + f*
g*x))^(3*n)*(c + d*x))/(9*f^2*g^2*n^2*Log[F]^2) + (3*a^2*b*(F^(e*g + f*g*x)
)^n*(c + d*x)^2)/(f*g*n*Log[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*(c + d*x
)^2)/(2*f*g*n*Log[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x)^2)/(3*f*g*n*
Log[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \int \left(a^3(c+dx)^2 + 3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^2 + 3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^2 + b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^2\right) dx$$

$$= \frac{a^3(c+dx)^3}{3d} + \left(3a^2b\right)\int \left(F^{eg+fgx}\right)^n (c+dx)^2\,dx + \left(3ab^2\right)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)^2\,dx + b^3\int \left(F^{eg+fgx}\right)^{3n} (c+dx)^2\,dx$$

$$= \frac{a^3(c+dx)^3}{3d} + \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^2}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^2}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^2}{3fgn\log(F)} - \frac{\left(6a^2bd\right)\int \left(F^{eg+fgx}\right)^n (c+dx)\,dx}{fgn\log(F)} - \frac{\left(3ab^2d\right)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)\,dx}{fgn\log(F)} - \frac{\left(2b^3d\right)\int \left(F^{eg+fgx}\right)^{3n} (c+dx)\,dx}{3fgn\log(F)}$$

$$= \frac{a^3(c+dx)^3}{3d} - \frac{6a^2bd\left(F^{eg+fgx}\right)^n (c+dx)}{f^2g^2n^2\log^2(F)} - \frac{3ab^2d\left(F^{eg+fgx}\right)^{2n} (c+dx)}{2f^2g^2n^2\log^2(F)} - \frac{2b^3d\left(F^{eg+fgx}\right)^{3n} (c+dx)}{9f^2g^2n^2\log^2(F)} + \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^2}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^2}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^2}{3fgn\log(F)} + \frac{\left(6a^2bd^2\right)\int \left(F^{eg+fgx}\right)^n\,dx}{f^2g^2n^2\log^2(F)} + \frac{\left(3ab^2d^2\right)\int \left(F^{eg+fgx}\right)^{2n}\,dx}{2f^2g^2n^2\log^2(F)} + \frac{\left(2b^3d^2\right)\int \left(F^{eg+fgx}\right)^{3n}\,dx}{9f^2g^2n^2\log^2(F)}$$

$$= \frac{a^3(c+dx)^3}{3d} + \frac{6a^2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} + \frac{3ab^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)} + \frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}}{27f^3g^3n^3\log^3(F)} - \frac{6a^2bd\left(F^{eg+fgx}\right)^n (c+dx)}{f^2g^2n^2\log^2(F)} - \frac{3ab^2d\left(F^{eg+fgx}\right)^{2n} (c+dx)}{2f^2g^2n^2\log^2(F)} - \frac{2b^3d\left(F^{eg+fgx}\right)^{3n} (c+dx)}{9f^2g^2n^2\log^2(F)} + \frac{3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^2}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^2}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^2}{3fgn\log(F)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.68

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx \\
&= a^3c^2x + a^3cdx^2 + \frac{1}{3}a^3d^2x^3 \\
&\quad + \frac{3a^2b\left(F^{g(e+fx)}\right)^n \left(2d^2 - 2dfgn(c+dx)\log(F) + f^2g^2n^2(c+dx)^2\log^2(F)\right)}{f^3g^3n^3\log^3(F)} \\
&\quad + \frac{3ab^2\left(F^{g(e+fx)}\right)^{2n} \left(d^2 - 2dfgn(c+dx)\log(F) + 2f^2g^2n^2(c+dx)^2\log^2(F)\right)}{4f^3g^3n^3\log^3(F)} \\
&\quad + \frac{b^3\left(F^{g(e+fx)}\right)^{3n} \left(2d^2 - 6dfgn(c+dx)\log(F) + 9f^2g^2n^2(c+dx)^2\log^2(F)\right)}{27f^3g^3n^3\log^3(F)}
\end{aligned}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2,x]
```

```
[Out] a^3*c^2*x + a^3*c*d*x^2 + (a^3*d^2*x^3)/3 + (3*a^2*b*(F^(g*(e + f*x)))^n*(2
*d^2 - 2*d*f*g*n*(c + d*x)*Log[F] + f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(f^3
*g^3*n^3*Log[F]^3) + (3*a*b^2*(F^(g*(e + f*x)))^(2*n)*(d^2 - 2*d*f*g*n*(c +
 d*x)*Log[F] + 2*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(4*f^3*g^3*n^3*Log[F]^3
) + (b^3*(F^(g*(e + f*x)))^(3*n)*(2*d^2 - 6*d*f*g*n*(c + d*x)*Log[F] + 9*f^
2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(27*f^3*g^3*n^3*Log[F]^3)
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.69

method	result
parallelrisch	$\frac{36a^3d^2x^3n^3g^3f^3\ln(F)^3+108a^3dcx^2n^3g^3f^3\ln(F)^3+108a^3c^2xn^3g^3f^3\ln(F)^3+36d^2b^3\left(F^{g(fx+e)}\right)^{3n}x^2n^2g^2f^2\ln(F)^2+162ab^2\cdots}{}$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(36*a^3*d^2*x^3*n^3*g^3*f^3*ln(F)^3+108*a^3*d*c*x^2*n^3*g^3*f^3*ln(F)
^3+108*a^3*c^2*x*n^3*g^3*f^3*ln(F)^3+36*d^2*b^3*((F^(g*(f*x+e)))^n)^3*x^2*n
^2*g^2*f^2*ln(F)^2+162*a*b^2*d^2*((F^(g*(f*x+e)))^n)^2*x^2*n^2*g^2*f^2*ln(F
)^2+72*ln(F)^2*x*((F^(g*(f*x+e)))^n)^3*b^3*c*d*f^2*g^2*n^2+324*a^2*b*d^2*(F
^(g*(f*x+e)))^n*x^2*n^2*g^2*f^2*ln(F)^2+324*ln(F)^2*x*((F^(g*(f*x+e)))^n)^2
*a*b^2*c*d*f^2*g^2*n^2+36*ln(F)^2*((F^(g*(f*x+e)))^n)^3*b^3*c^2*f^2*g^2*n^2
+648*ln(F)^2*x*(F^(g*(f*x+e)))^n*a^2*b*c*d*f^2*g^2*n^2+162*ln(F)^2*((F^(g*(
f*x+e)))^n)^2*a*b^2*c^2*f^2*g^2*n^2+324*ln(F)^2*(F^(g*(f*x+e)))^n*a^2*b*c^2
*f^2*g^2*n^2-24*ln(F)*x*((F^(g*(f*x+e)))^n)^3*b^3*d^2*f*g*n-162*ln(F)*x*((F
^(g*(f*x+e)))^n)^2*a*b^2*d^2*f*g*n-24*ln(F)*((F^(g*(f*x+e)))^n)^3*b^3*c*d*f
```

```
*g*n-648*ln(F)*x*(F^(g*(f*x+e)))^n*a^2*b*d^2*f*g*n-162*ln(F)*((F^(g*(f*x+e)
))^n)^2*a*b^2*c*d*f*g*n-648*ln(F)*(F^(g*(f*x+e)))^n*a^2*b*c*d*f*g*n+8*((F^(
g*(f*x+e)))^n)^3*b^3*d^2+81*((F^(g*(f*x+e)))^n)^2*a*b^2*d^2+648*(F^(g*(f*x+
e)))^n*a^2*b*d^2)/n^3/g^3/f^3/ln(F)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.13

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$$
$$= \frac{36\,(a^3d^2f^3g^3n^3x^3+3\,a^3cdf^3g^3n^3x^2+3\,a^3c^2f^3g^3n^3x)\log(F)^3+4\,\left(2\,b^3d^2+9\,(b^3d^2f^2g^2n^2x^2+2\,b^3cdf^2g^2n\right.}{}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/108*(36*(a^3*d^2*f^3*g^3*n^3*x^3 + 3*a^3*c*d*f^3*g^3*n^3*x^2 + 3*a^3*c^2*
f^3*g^3*n^3*x)*log(F)^3 + 4*(2*b^3*d^2 + 9*(b^3*d^2*f^2*g^2*n^2*x^2 + 2*b^3
*c*d*f^2*g^2*n^2*x + b^3*c^2*f^2*g^2*n^2)*log(F)^2 - 6*(b^3*d^2*f*g*n*x + b
^3*c*d*f*g*n)*log(F))*F^(3*f*g*n*x + 3*e*g*n) + 81*(a*b^2*d^2 + 2*(a*b^2*d^
2*f^2*g^2*n^2*x^2 + 2*a*b^2*c*d*f^2*g^2*n^2*x + a*b^2*c^2*f^2*g^2*n^2)*log(
F)^2 - 2*(a*b^2*d^2*f*g*n*x + a*b^2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g
*n) + 324*(2*a^2*b*d^2 + (a^2*b*d^2*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d*f^2*g^2*n
^2*x + a^2*b*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(a^2*b*d^2*f*g*n*x + a^2*b*c*d*f
*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^3*g^3*n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. 2(362) = 724.

Time = 5.04 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.12

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$$
$$= \begin{cases} (a+b)^3\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ (a+b(F^{eg})^n)^3\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ (a+b)^3\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ a^3c^2x+a^3cdx^2+\frac{a^3d^2x^3}{3}+\frac{3a^2bc^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{6a^2bcdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{6a^2bcd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{3a^2bd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{6a^2b\cdots}{f^2\cdots} \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c)**2,x)
```

```
[Out] Piecewise(((a + b)**3*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) & Eq(f, 0
) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**3*(c**2*x + c*d*x**2 + d*
*2*x**3/3), Eq(f, 0)), ((a + b)**3*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F,
 1) | Eq(g, 0) | Eq(n, 0)), (a**3*c**2*x + a**3*c*d*x**2 + a**3*d**2*x**3/3
 + 3*a**2*b*c**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 6*a**2*b*c*d*x*(F**
(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*a**2*b*c*d*(F**(e*g + f*g*x))**n/(f**2
*g**2*n**2*log(F)**2) + 3*a**2*b*d**2*x**2*(F**(e*g + f*g*x))**n/(f*g*n*log
(F)) - 6*a**2*b*d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 6
*a**2*b*d**2*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) + 3*a*b**2*c*
*2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + 3*a*b**2*c*d*x*(F**(e*g + f
*g*x))**(2*n)/(f*g*n*log(F)) - 3*a*b**2*c*d*(F**(e*g + f*g*x))**(2*n)/(2*f*
*2*g**2*n**2*log(F)**2) + 3*a*b**2*d**2*x**2*(F**(e*g + f*g*x))**(2*n)/(2*f
*g*n*log(F)) - 3*a*b**2*d**2*x*(F**(e*g + f*g*x))**(2*n)/(2*f**2*g**2*n**2*
log(F)**2) + 3*a*b**2*d**2*(F**(e*g + f*g*x))**(2*n)/(4*f**3*g**3*n**3*log(
F)**3) + b**3*c**2*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)) + 2*b**3*c*d*
x*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)) - 2*b**3*c*d*(F**(e*g + f*g*x)
)**(3*n)/(9*f**2*g**2*n**2*log(F)**2) + b**3*d**2*x**2*(F**(e*g + f*g*x))**
(3*n)/(3*f*g*n*log(F)) - 2*b**3*d**2*x*(F**(e*g + f*g*x))**(3*n)/(9*f**2*g*
*2*n**2*log(F)**2) + 2*b**3*d**2*(F**(e*g + f*g*x))**(3*n)/(27*f**3*g**3*n*
*3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.43

$$
\begin{aligned}
\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx
&= \frac{1}{3}\,a^3 d^2 x^3 + a^3 c d x^2 + a^3 c^2 x + \frac{3\,F^{fgnx+egn} a^2 b c^2}{fgn\log\left(F\right)} + \frac{3\,F^{2\,fgnx+2\,egn} a b^2 c^2}{2\,fgn\log\left(F\right)} \\
&\quad + \frac{F^{3\,fgnx+3\,egn} b^3 c^2}{3\,fgn\log\left(F\right)} + \frac{6\,(F^{egn} fgnx\log\left(F\right) - F^{egn})F^{fgnx} a^2 bcd}{f^2 g^2 n^2 \log\left(F\right)^2} \\
&\quad + \frac{3\,(2\,F^{2\,egn} fgnx\log\left(F\right) - F^{2\,egn})F^{2\,fgnx} a b^2 cd}{2\,f^2 g^2 n^2 \log\left(F\right)^2} \\
&\quad + \frac{2\,(3\,F^{3\,egn} fgnx\log\left(F\right) - F^{3\,egn})F^{3\,fgnx} b^3 cd}{9\,f^2 g^2 n^2 \log\left(F\right)^2} \\
&\quad + \frac{3\,\left(F^{egn} f^2 g^2 n^2 x^2 \log\left(F\right)^2 - 2\,F^{egn} fgnx\log\left(F\right) + 2\,F^{egn}\right)F^{fgnx} a^2 b d^2}{f^3 g^3 n^3 \log\left(F\right)^3} \\
&\quad + \frac{3\,\left(2\,F^{2\,egn} f^2 g^2 n^2 x^2 \log\left(F\right)^2 - 2\,F^{2\,egn} fgnx\log\left(F\right) + F^{2\,egn}\right)F^{2\,fgnx} a b^2 d^2}{4\,f^3 g^3 n^3 \log\left(F\right)^3} \\
&\quad + \frac{\left(9\,F^{3\,egn} f^2 g^2 n^2 x^2 \log\left(F\right)^2 - 6\,F^{3\,egn} fgnx\log\left(F\right) + 2\,F^{3\,egn}\right)F^{3\,fgnx} b^3 d^2}{27\,f^3 g^3 n^3 \log\left(F\right)^3}
\end{aligned}
$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + a^3*c^2*x + 3*F^(f*g*n*x + e*g*n)*a^2*b*c^2
/(f*g*n*log(F)) + 3/2*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*c^2/(f*g*n*log(F)) + 1/
3*F^(3*f*g*n*x + 3*e*g*n)*b^3*c^2/(f*g*n*log(F)) + 6*(F^(e*g*n)*f*g*n*x*log
(F) - F^(e*g*n))*F^(f*g*n*x)*a^2*b*c*d/(f^2*g^2*n^2*log(F)^2) + 3/2*(2*F^(2
*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*c*d/(f^2*g^2*n^2*
log(F)^2) + 2/9*(3*F^(3*e*g*n)*f*g*n*x*log(F) - F^(3*e*g*n))*F^(3*f*g*n*x)*
b^3*c*d/(f^2*g^2*n^2*log(F)^2) + 3*(F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*
F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*a^2*b*d^2/(f^3*g^3*n^3*
log(F)^3) + 3/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(2*e*g*n)*f*g
*n*x*log(F) + F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*d^2/(f^3*g^3*n^3*log(F)^3) +
 1/27*(9*F^(3*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 6*F^(3*e*g*n)*f*g*n*x*log(F
) + 2*F^(3*e*g*n))*F^(3*f*g*n*x)*b^3*d^2/(f^3*g^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 8820, normalized size of antiderivative = 24.10

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + a^3*c^2*x - 1/27*((6*(3*pi*b^3*d^2*f^2*g^2*
n^2*x^2*log(abs(F))*sgn(F) - 3*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 6*p
i*b^3*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 6*pi*b^3*c*d*f^2*g^2*n^2*x*log
(abs(F)) + 3*pi*b^3*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 3*pi*b^3*c^2*f^2*g
^2*n^2*log(abs(F)) - pi*b^3*d^2*f*g*n*x*sgn(F) + pi*b^3*d^2*f*g*n*x - pi*b^
3*c*d*f*g*n*sgn(F) + pi*b^3*c*d*f*g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*
g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(
F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) -
pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*
log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs
(F))^3)^2) - (9*pi^2*b^3*d^2*f^2*g^2*n^2*x^2*sgn(F) - 9*pi^2*b^3*d^2*f^2*g^
2*n^2*x^2 + 18*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 18*pi^2*b^3*c*d*f^2*
g^2*n^2*x*sgn(F) - 18*pi^2*b^3*c*d*f^2*g^2*n^2*x + 36*b^3*c*d*f^2*g^2*n^2*x
*log(abs(F))^2 + 9*pi^2*b^3*c^2*f^2*g^2*n^2*sgn(F) - 9*pi^2*b^3*c^2*f^2*g^2
*n^2 + 18*b^3*c^2*f^2*g^2*n^2*log(abs(F))^2 - 12*b^3*d^2*f*g*n*x*log(abs(F)
) - 12*b^3*c*d*f*g*n*log(abs(F)) + 4*b^3*d^2)*(3*pi^2*f^3*g^3*n^3*log(abs(F
))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/(
(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3
*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(
F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^
```

2))*cos(-3/2*pi*f*g*n*x*sgn(F) + 3/2*pi*f*g*n*x - 3/2*pi*e*g*n*sgn(F) + 3/2
*pi*e*g*n) - ((9*pi^2*b^3*d^2*f^2*g^2*n^2*x^2*sgn(F) - 9*pi^2*b^3*d^2*f^2*g
^2*n^2*x^2 + 18*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 18*pi^2*b^3*c*d*f^2
*g^2*n^2*x*sgn(F) - 18*pi^2*b^3*c*d*f^2*g^2*n^2*x + 36*b^3*c*d*f^2*g^2*n^2*
x*log(abs(F))^2 + 9*pi^2*b^3*c^2*f^2*g^2*n^2*sgn(F) - 9*pi^2*b^3*c^2*f^2*g^
2*n^2 + 18*b^3*c^2*f^2*g^2*n^2*log(abs(F))^2 - 12*b^3*d^2*f*g*n*x*log(abs(F
)) - 12*b^3*c*d*f*g*n*log(abs(F)) + 4*b^3*d^2)*(pi^3*f^3*g^3*n^3*sgn(F) - 3
*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*
log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*s
gn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*
g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3
log(abs(F))^3)^2) + 6(3*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 3
*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 6*pi*b^3*c*d*f^2*g^2*n^2*x*log(ab
s(F))*sgn(F) - 6*pi*b^3*c*d*f^2*g^2*n^2*x*log(abs(F)) + 3*pi*b^3*c^2*f^2*g^
2*n^2*log(abs(F))*sgn(F) - 3*pi*b^3*c^2*f^2*g^2*n^2*log(abs(F)) - pi*b^3*d^
2*f*g*n*x*sgn(F) + pi*b^3*d^2*f*g*n*x - pi*b^3*c*d*f*g*n*sgn(F) + pi*b^3*c*
d*f*g*n)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(ab
s(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g
^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F
))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(a
bs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))*sin(-3/2*pi*f*g*n*x*sgn(F) + 3/2*
pi*f*g*n*x - 3/2*pi*e*g*n*sgn(F) + 3/2*pi*e*g*n))*e^(3*f*g*n*x*log(abs(F))
+ 3*e*g*n*log(abs(F))) - 2*I*((-9*I*pi^2*b^3*d^2*f^2*g^2*n^2*x^2*sgn(F) + 1
8*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 9*I*pi^2*b^3*d^2*f^2*g^2*
n^2*x^2 - 18*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F)) - 18*I*b^3*d^2*f^2*g^2*
n^2*x^2*log(abs(F))^2 - 18*I*pi^2*b^3*c*d*f^2*g^2*n^2*x*sgn(F) + 36*pi*b^3*
c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 18*I*pi^2*b^3*c*d*f^2*g^2*n^2*x - 36
*pi*b^3*c*d*f^2*g^2*n^2*x*log(abs(F)) - 36*I*b^3*c*d*f^2*g^2*n^2*x*log(abs(
F))^2 - 9*I*pi^2*b^3*c^2*f^2*g^2*n^2*sgn(F) + 18*pi*b^3*c^2*f^2*g^2*n^2*log
(abs(F))*sgn(F) + 9*I*pi^2*b^3*c^2*f^2*g^2*n^2 - 18*pi*b^3*c^2*f^2*g^2*n^2*
log(abs(F)) - 18*I*b^3*c^2*f^2*g^2*n^2*log(abs(F))^2 - 6*pi*b^3*d^2*f*g*n*x
*sgn(F) + 6*pi*b^3*d^2*f*g*n*x + 12*I*b^3*d^2*f*g*n*x*log(abs(F)) - 6*pi*b^
3*c*d*f*g*n*sgn(F) + 6*pi*b^3*c*d*f*g*n + 12*I*b^3*c*d*f*g*n*log(abs(F)) -
4*I*b^3*d^2)*e^(3/2*I*pi*f*g*n*x*sgn(F) - 3/2*I*pi*f*g*n*x + 3/2*I*pi*e*g*n
*sgn(F) - 3/2*I*pi*e*g*n)/(-108*I*pi^3*f^3*g^3*n^3*sgn(F) + 324*pi^2*f^3*g^
3*n^3*log(abs(F))*sgn(F) + 324*I*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) + 108*
I*pi^3*f^3*g^3*n^3 - 324*pi^2*f^3*g^3*n^3*log(abs(F)) - 324*I*pi*f^3*g^3*n^
3*log(abs(F))^2 + 216*f^3*g^3*n^3*log(abs(F))^3) - (-9*I*pi^2*b^3*d^2*f^2*g
^2*n^2*x^2*sgn(F) - 18*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + 9*I*
pi^2*b^3*d^2*f^2*g^2*n^2*x^2 + 18*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F)) -
18*I*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 - 18*I*pi^2*b^3*c*d*f^2*g^2*n^2*
x*sgn(F) - 36*pi*b^3*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 18*I*pi^2*b^3*c
*d*f^2*g^2*n^2*x + 36*pi*b^3*c*d*f^2*g^2*n^2*x*log(abs(F)) - 36*I*b^3*c*d*f
^2*g^2*n^2*x*log(abs(F))^2 - 9*I*pi^2*b^3*c^2*f^2*g^2*n^2*sgn(F) - 18*pi*b^
3*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + 9*I*pi^2*b^3*c^2*f^2*g^2*n^2 + 18*pi

*b^3*c^2*f^2*g^2*n^2*log(abs(F)) - 18*I*b^3*c^2*f^2*g^2*n^2*log(abs(F))^2 + 6*pi*b^3*d^2*f*g*n*x*sgn(F) - 6*pi*b^3*d^2*f*g*n*x + 12*I*b^3*d^2*f*g*n*x*log(abs(F)) + 6*pi*b^3*c*d*f*g*n*sgn(F) - 6*pi*b^3*c*d*f*g*n + 12*I*b^3*c*d*f*g*n*log(abs(F)) - 4*I*b^3*d^2)*e^(-3/2*I*pi*f*g*n*x*sgn(F) + 3/2*I*pi*f*g*n*x - 3/2*I*pi*e*g*n*sgn(F) + 3/2*I*pi*e*g*n)/(108*I*pi^3*f^3*g^3*n^3*sgn(F) + 324*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 324*I*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - 108*I*pi^3*f^3*g^3*n^3 - 324*pi^2*f^3*g^3*n^3*log(abs(F)) + 324*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 216*f^3*g^3*n^3*log(abs(F))^3))*e^(3*f*g*n*x*log(abs(F)) + 3*e*g*n*log(abs(F))) - 3/2*(((2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)) + 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F)) - pi*a*b^2*d^2*f*g*n*x*sgn(F) + pi*a*b^2*d^2*f*g*n*x - pi*a*b^2*c*d*f*g*n*sgn(F) + pi*a*b^2*c*d*f*g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) - (pi^2*a*b^2*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*a*b^2*d^2*f^2*g^2*n^2*x^2 + 2*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*a*b^2*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*a*b^2*c*d*f^2*g^2*n^2*x + 4*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*a*b^2*c^2*f^2*g^2*n^2*sgn(F) - pi^2*a*b^2*c^2*f^2*g^2*n^2 + 2*a*b^2*c^2*f^2*g^2*n^2*log(abs(F))^2 - 2*a*b^2*d^2*f*g*n*x*log(abs(F)) - 2*a*b^2*c*d*f*g*n*log(abs(F)) + a*b^2*d^2)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))*cos(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n) - ((pi^2*a*b^2*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*a*b^2*d^2*f^2*g^2*n^2*x^2 + 2*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*a*b^2*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*a*b^2*c*d*f^2*g^2*n^2*x + 4*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*a*b^2*c^2*f^2*g^2*n^2*sgn(F) - pi^2*a*b^2*c^2*f^2*g^2*n^2 + 2*a*b^2*c^2*f^2*g^2*n^2*log(abs(F))^2 - 2*a*b^2*d^2*f*g*n*x*log(abs(F)) - 2*a*b^2*c*d*f*g*n*log(abs(F)) + a*b^2*d^2)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) + (2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) - 2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)) + 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F)) - pi*a*b^2*d^2*f*g*n*x*sgn(F) + pi*a*b^2*d^2*f*g*n*x - pi*a*b^2*c*d*f*g*n*sgn(F) + pi*a*b^2*c*d*f*g*n)*(3*pi^2*f^3*g^3*n^3*log(abs(F))

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*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((p
i^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g
^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F)
)*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2)
)*sin(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n))*e^(2*f
*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F))) - 3*I*((-I*pi^2*a*b^2*d^2*f^2*g^2
*n^2*x^2*sgn(F) + 2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + I*pi^
2*a*b^2*d^2*f^2*g^2*n^2*x^2 - 2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) -
2*I*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 - 2*I*pi^2*a*b^2*c*d*f^2*g^2*n^
2*x*sgn(F) + 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 2*I*pi^2*a*b
^2*c*d*f^2*g^2*n^2*x - 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)) - 4*I*a*b^2
*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*a*b^2*c^2*f^2*g^2*n^2*sgn(F) + 2*
pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*a*b^2*c^2*f^2*g^2*n^2
- 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*a*b^2*c^2*f^2*g^2*n^2*log(ab
s(F))^2 - pi*a*b^2*d^2*f*g*n*x*sgn(F) + pi*a*b^2*d^2*f*g*n*x + 2*I*a*b^2*d^
2*f*g*n*x*log(abs(F)) - pi*a*b^2*c*d*f*g*n*sgn(F) + pi*a*b^2*c*d*f*g*n + 2*
I*a*b^2*c*d*f*g*n*log(abs(F)) - I*a*b^2*d^2)*e^(I*pi*f*g*n*x*sgn(F) - I*pi*
f*g*n*x + I*pi*e*g*n*sgn(F) - I*pi*e*g*n)/(-4*I*pi^3*f^3*g^3*n^3*sgn(F) + 1
2*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) + 12*I*pi*f^3*g^3*n^3*log(abs(F))^2*s
gn(F) + 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)) - 12*I*pi*f^
3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3) - (-I*pi^2*a*b^2*d^2
*f^2*g^2*n^2*x^2*sgn(F) - 2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F)
 + I*pi^2*a*b^2*d^2*f^2*g^2*n^2*x^2 + 2*pi*a*b^2*d^2*f^2*g^2*n^2*x^2*log(ab
s(F)) - 2*I*a*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 - 2*I*pi^2*a*b^2*c*d*f^
2*g^2*n^2*x*sgn(F) - 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 2*I*
pi^2*a*b^2*c*d*f^2*g^2*n^2*x + 4*pi*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F)) - 4
*I*a*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*a*b^2*c^2*f^2*g^2*n^2*sgn
(F) - 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*a*b^2*c^2*f^2*
g^2*n^2 + 2*pi*a*b^2*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*a*b^2*c^2*f^2*g^2*n^
2*log(abs(F))^2 + pi*a*b^2*d^2*f*g*n*x*sgn(F) - pi*a*b^2*d^2*f*g*n*x + 2*I*
a*b^2*d^2*f*g*n*x*log(abs(F)) + pi*a*b^2*c*d*f*g*n*sgn(F) - pi*a*b^2*c*d*f*
g*n + 2*I*a*b^2*c*d*f*g*n*log(abs(F)) - I*a*b^2*d^2)*e^(-I*pi*f*g*n*x*sgn(F
) + I*pi*f*g*n*x - I*pi*e*g*n*sgn(F) + I*pi*e*g*n)/(4*I*pi^3*f^3*g^3*n^3*sg
n(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 12*I*pi*f^3*g^3*n^3*log(abs
(F))^2*sgn(F) - 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)) + 12
*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3))*e^(2*f*g*n*
x*log(abs(F)) + 2*e*g*n*log(abs(F))) - 3*((2*(pi*a^2*b*d^2*f^2*g^2*n^2*x^2*
log(abs(F))*sgn(F) - pi*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*a^2*b*
c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 2*pi*a^2*b*c*d*f^2*g^2*n^2*x*log(abs
(F)) + pi*a^2*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*a^2*b*c^2*f^2*g^2*n
^2*log(abs(F)) - pi*a^2*b*d^2*f*g*n*x*sgn(F) + pi*a^2*b*d^2*f*g*n*x - pi*a^
2*b*c*d*f*g*n*sgn(F) + pi*a^2*b*c*d*f*g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*
f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(
abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F
) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*
```

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n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log
(abs(F))^3)^2) - (pi^2*a^2*b*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*a^2*b*d^2*f^
2*g^2*n^2*x^2 + 2*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*a^2*b*c*
d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*a^2*b*c*d*f^2*g^2*n^2*x + 4*a^2*b*c*d*f^2*g
^2*n^2*x*log(abs(F))^2 + pi^2*a^2*b*c^2*f^2*g^2*n^2*sgn(F) - pi^2*a^2*b*c^2
*f^2*g^2*n^2 + 2*a^2*b*c^2*f^2*g^2*n^2*log(abs(F))^2 - 4*a^2*b*d^2*f*g*n*x*
log(abs(F)) - 4*a^2*b*c*d*f*g*n*log(abs(F)) + 4*a^2*b*d^2)*(3*pi^2*f^3*g^3*
n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log
(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(
F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3
*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*lo
g(abs(F))^3)^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n
*sgn(F) + 1/2*pi*e*g*n) - ((pi^2*a^2*b*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*a^
2*b*d^2*f^2*g^2*n^2*x^2 + 2*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^
2*a^2*b*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*a^2*b*c*d*f^2*g^2*n^2*x + 4*a^2*b
*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*a^2*b*c^2*f^2*g^2*n^2*sgn(F) - pi^2
*a^2*b*c^2*f^2*g^2*n^2 + 2*a^2*b*c^2*f^2*g^2*n^2*log(abs(F))^2 - 4*a^2*b*d^
2*f*g*n*x*log(abs(F)) - 4*a^2*b*c*d*f*g*n*log(abs(F)) + 4*a^2*b*d^2)*(pi^3*
f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n
^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g
^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F
))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(a
bs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) + 2*(pi*a^2*b*d^2*f^2*g^2*n^2*x^2*
log(abs(F))*sgn(F) - pi*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*a^2*b*
c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 2*pi*a^2*b*c*d*f^2*g^2*n^2*x*log(abs
(F)) + pi*a^2*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*a^2*b*c^2*f^2*g^2*n
^2*log(abs(F)) - pi*a^2*b*d^2*f*g*n*x*sgn(F) + pi*a^2*b*d^2*f*g*n*x - pi*a^
2*b*c*d*f*g*n*sgn(F) + pi*a^2*b*c*d*f*g*n)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*
sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi
^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^
3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))
*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2))
*sin(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F) + 1/2*pi
*e*g*n))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F))) - 6*I*((-I*pi^2*a^2*b*
d^2*f^2*g^2*n^2*x^2*sgn(F) + 2*pi*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn
(F) + I*pi^2*a^2*b*d^2*f^2*g^2*n^2*x^2 - 2*pi*a^2*b*d^2*f^2*g^2*n^2*x^2*log
(abs(F)) - 2*I*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 - 2*I*pi^2*a^2*b*c*d
*f^2*g^2*n^2*x*sgn(F) + 4*pi*a^2*b*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) + 2
*I*pi^2*a^2*b*c*d*f^2*g^2*n^2*x - 4*pi*a^2*b*c*d*f^2*g^2*n^2*x*log(abs(F))
- 4*I*a^2*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 - I*pi^2*a^2*b*c^2*f^2*g^2*n^2*
sgn(F) + 2*pi*a^2*b*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) + I*pi^2*a^2*b*c^2*f
^2*g^2*n^2 - 2*pi*a^2*b*c^2*f^2*g^2*n^2*log(abs(F)) - 2*I*a^2*b*c^2*f^2*g^2
*n^2*log(abs(F))^2 - 2*pi*a^2*b*d^2*f*g*n*x*sgn(F) + 2*pi*a^2*b*d^2*f*g*n*x
 + 4*I*a^2*b*d^2*f*g*n*x*log(abs(F)) - 2*pi*a^2*b*c*d*f*g*n*sgn(F) + 2*pi*a
^2*b*c*d*f*g*n + 4*I*a^2*b*c*d*f*g*n*log(abs(F)) - 4*I*a^2*b*d^2)*e^(1/2*I*
```

```
pi*f*g*n*x*sgn(F) - 1/2*I*pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2*I*pi*e*g
*n)/(-4*I*pi^3*f^3*g^3*n^3*sgn(F) + 12*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F)
+ 12*I*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) + 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2
*f^3*g^3*n^3*log(abs(F)) - 12*I*pi*f^3*g^3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^
3*log(abs(F))^3) - (-I*pi^2*a^2*b*d^2*f^2*g^2*n^2*x^2*sgn(F) - 2*pi*a^2*b*d
^2*f^2*g^2*n^2*x^2*log(abs(F))*sgn(F) + I*pi^2*a^2*b*d^2*f^2*g^2*n^2*x^2 +
2*pi*a^2*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) - 2*I*a^2*b*d^2*f^2*g^2*n^2*x^2*
log(abs(F))^2 - 2*I*pi^2*a^2*b*c*d*f^2*g^2*n^2*x*sgn(F) - 4*pi*a^2*b*c*d*f^
2*g^2*n^2*x*log(abs(F))*sgn(F) + 2*I*pi^2*a^2*b*c*d*f^2*g^2*n^2*x + 4*pi*a^
2*b*c*d*f^2*g^2*n^2*x*log(abs(F)) - 4*I*a^2*b*c*d*f^2*g^2*n^2*x*log(abs(F))
^2 - I*pi^2*a^2*b*c^2*f^2*g^2*n^2*sgn(F) - 2*pi*a^2*b*c^2*f^2*g^2*n^2*log(a
bs(F))*sgn(F) + I*pi^2*a^2*b*c^2*f^2*g^2*n^2 + 2*pi*a^2*b*c^2*f^2*g^2*n^2*l
og(abs(F)) - 2*I*a^2*b*c^2*f^2*g^2*n^2*log(abs(F))^2 + 2*pi*a^2*b*d^2*f*g*n
*x*sgn(F) - 2*pi*a^2*b*d^2*f*g*n*x + 4*I*a^2*b*d^2*f*g*n*x*log(abs(F)) + 2*
pi*a^2*b*c*d*f*g*n*sgn(F) - 2*pi*a^2*b*c*d*f*g*n + 4*I*a^2*b*c*d*f*g*n*log(
abs(F)) - 4*I*a^2*b*d^2)*e^(-1/2*I*pi*f*g*n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1
/2*I*pi*e*g*n*sgn(F) + 1/2*I*pi*e*g*n)/(4*I*pi^3*f^3*g^3*n^3*sgn(F) + 12*pi
^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 12*I*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F
) - 4*I*pi^3*f^3*g^3*n^3 - 12*pi^2*f^3*g^3*n^3*log(abs(F)) + 12*I*pi*f^3*g^
3*n^3*log(abs(F))^2 + 8*f^3*g^3*n^3*log(abs(F))^3))*e^(f*g*n*x*log(abs(F))
+ e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.09

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$$
$$= \left(F^{f\,g\,x}\,F^{e\,g}\right)^{3\,n}\left(\frac{b^3\,\left(9\,c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-6\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{27\,f^3\,g^3\,n^3\,\ln\left(F\right)^3}+\frac{b^3\,d^2\,x^2}{3\,f\,g\,n\,\ln\left(F\right)}-\frac{2\,b^3\,d\,x\,\left(d-3\,c\,f\,g\,n\,\ln\left(F\right)\right)}{9\,f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right)+\left(F^{f\,g\,x}\,F^{e\,g}\right)^{n}\left(\frac{3\,a^2\,b\,\left(c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+2\,d^2\right)}{f^3\,g^3\,n^3\,\ln\left(F\right)^3}+\frac{3\,a^2\,b\,d^2\,x^2}{f\,g\,n\,\ln\left(F\right)}-\frac{6\,a^2\,b\,d\,x\,\left(d-c\,f\,g\,n\,\ln\left(F\right)\right)}{f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right)+\left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{3\,a\,b^2\,\left(2\,c^2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2-2\,c\,d\,f\,g\,n\,\ln\left(F\right)+d^2\right)}{4\,f^3\,g^3\,n^3\,\ln\left(F\right)^3}+\frac{3\,a\,b^2\,d^2\,x^2}{2\,f\,g\,n\,\ln\left(F\right)}-\frac{3\,a\,b^2\,d\,x\,\left(d-2\,c\,f\,g\,n\,\ln\left(F\right)\right)}{2\,f^2\,g^2\,n^2\,\ln\left(F\right)^2}\right)+a^3\,c^2\,x+\frac{a^3\,d^2\,x^3}{3}+a^3\,c\,d\,x^2$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2,x)
```

```
[Out] (F^(f*g*x)*F^(e*g))^(3*n)*((b^3*(2*d^2 + 9*c^2*f^2*g^2*n^2*log(F)^2 - 6*c*d
*f*g*n*log(F)))/(27*f^3*g^3*n^3*log(F)^3) + (b^3*d^2*x^2)/(3*f*g*n*log(F))
- (2*b^3*d*x*(d - 3*c*f*g*n*log(F)))/(9*f^2*g^2*n^2*log(F)^2)) + (F^(f*g*x)
*F^(e*g))^n*((3*a^2*b*(2*d^2 + c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F
)))/(f^3*g^3*n^3*log(F)^3) + (3*a^2*b*d^2*x^2)/(f*g*n*log(F)) - (6*a^2*b*d*
x*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + (F^(f*g*x)*F^(e*g))^(2*n)
*((3*a*b^2*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/(4*f^3*
g^3*n^3*log(F)^3) + (3*a*b^2*d^2*x^2)/(2*f*g*n*log(F)) - (3*a*b^2*d*x*(d -
2*c*f*g*n*log(F)))/(2*f^2*g^2*n^2*log(F)^2)) + a^3*c^2*x + (a^3*d^2*x^3)/3
+ a^3*c*d*x^2
```

3.41 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 236

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx = \frac{a^3(c+dx)^2}{2d} - \frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} - \frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} - \frac{b^3d\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)} + \frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)}{3fgn\log(F)}$$

```
[Out] 1/2*a^3*(d*x+c)^2/d-3*a^2*b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/ln(F)^2-3/4*a*b
^2*d*(F^(f*g*x+e*g))^(2*n)/f^2/g^2/n^2/ln(F)^2-1/9*b^3*d*(F^(f*g*x+e*g))^(3
*n)/f^2/g^2/n^2/ln(F)^2+3*a^2*b*(F^(f*g*x+e*g))^n*(d*x+c)/f/g/n/ln(F)+3/2*a
*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f/g/n/ln(F)+1/3*b^3*(F^(f*g*x+e*g))^(3*n
)*(d*x+c)/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2214, 2207, 2225}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx=\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{3ab^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}+\frac{b^3(c+dx)\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)}-\frac{b^3d\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x),x]
```

```
[Out] (a^3*(c + d*x)^2)/(2*d) - (3*a^2*b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*Log[
F]^2) - (3*a*b^2*d*(F^(e*g + f*g*x))^(2*n))/(4*f^2*g^2*n^2*Log[F]^2) - (b^3
*d*(F^(e*g + f*g*x))^(3*n))/(9*f^2*g^2*n^2*Log[F]^2) + (3*a^2*b*(F^(e*g + f
*g*x))^n*(c + d*x))/(f*g*n*Log[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*(c +
d*x))/(2*f*g*n*Log[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x))/(3*f*g*n*L
og[F])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \int \left(a^3(c+dx)+3a^2b\left(F^{eg+fgx}\right)^n(c+dx)+3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)+b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)\right)dx$$

$$=\frac{a^3(c+dx)^2}{2d}+\left(3a^2b\right)\int\left(F^{eg+fgx}\right)^n(c+dx)\,dx+\left(3ab^2\right)\int\left(F^{eg+fgx}\right)^{2n}(c+dx)\,dx+b^3\int\left(F^{eg+fgx}\right)^{3n}(c+dx)\,dx$$

$$=\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)}{3fgn\log(F)}-\frac{\left(3a^2bd\right)\int\left(F^{eg+fgx}\right)^n\,dx}{fgn\log(F)}-\frac{\left(3ab^2d\right)\int\left(F^{eg+fgx}\right)^{2n}\,dx}{2fgn\log(F)}-\frac{\left(b^3d\right)\int\left(F^{eg+fgx}\right)^{3n}\,dx}{3fgn\log(F)}$$

$$=\frac{a^3(c+dx)^2}{2d}-\frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}-\frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}-\frac{b^3d\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)}+\frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)}{3fgn\log(F)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

$$\int\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)\,dx$$

$$=\frac{-bd\left(F^{g(e+fx)}\right)^n\left(108a^2+27ab\left(F^{g(e+fx)}\right)^n+4b^2\left(F^{g(e+fx)}\right)^{2n}\right)+6bf\left(F^{g(e+fx)}\right)^n\left(18a^2+9ab\left(F^{g(e+fx)}\right)^n+\right.}{36f^2g^2n^2\log^2(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x),x]
```

```
[Out] (-(b*d*(F^(g*(e + f*x)))^n*(108*a^2 + 27*a*b*(F^(g*(e + f*x)))^n + 4*b^2*(F
^(g*(e + f*x)))^(2*n))) + 6*b*f*(F^(g*(e + f*x)))^n*(18*a^2 + 9*a*b*(F^(g*(
e + f*x)))^n + 2*b^2*(F^(g*(e + f*x)))^(2*n))*g*n*(c + d*x)*Log[F] + 18*a^3
*f^2*g^2*n^2*x*(2*c + d*x)*Log[F]^2)/(36*f^2*g^2*n^2*Log[F]^2)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{18a^3 d\, x^2 n^2 g^2 f^2 \ln(F)^2+36a^3 cx\, n^2 g^2 f^2 \ln(F)^2+12x\left(F^{g(fx+e)}\right)^{3n} b^3 dngf \ln(F)+54x\left(F^{g(fx+e)}\right)^{2n} a\, b^2 dngf \ln(F)+12 \ln(F)}{}$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(18*a^3*d*x^2*n^2*g^2*f^2*ln(F)^2+36*a^3*c*x*n^2*g^2*f^2*ln(F)^2+12*x*
((F^(g*(f*x+e)))^n)^3*b^3*d*n*g*f*ln(F)+54*x*((F^(g*(f*x+e)))^n)^2*a*b^2*d*
n*g*f*ln(F)+12*ln(F)*((F^(g*(f*x+e)))^n)^3*b^3*c*f*g*n+108*x*(F^(g*(f*x+e))
)^n*a^2*b*d*n*g*f*ln(F)+54*ln(F)*((F^(g*(f*x+e)))^n)^2*a*b^2*c*f*g*n+108*ln
(F)*(F^(g*(f*x+e)))^n*a^2*b*c*f*g*n-4*((F^(g*(f*x+e)))^n)^3*b^3*d-27*((F^(g
*(f*x+e)))^n)^2*a*b^2*d-108*(F^(g*(f*x+e)))^n*a^2*b*d)/n^2/g^2/f^2/ln(F)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$$
$$=\frac{18\left(a^3df^2g^2n^2x^2+2\,a^3cf^2g^2n^2x\right)\log\left(F\right)^2-4\left(b^3d-3\left(b^3dfgnx+b^3cfgn\right)\log\left(F\right)\right)F^{3\,fgnx+3\,egn}-27\left(ab^2\right.}{36\,f^2g^2n^2}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/36*(18*(a^3*d*f^2*g^2*n^2*x^2 + 2*a^3*c*f^2*g^2*n^2*x)*log(F)^2 - 4*(b^3*
d - 3*(b^3*d*f*g*n*x + b^3*c*f*g*n)*log(F))*F^(3*f*g*n*x + 3*e*g*n) - 27*(a
*b^2*d - 2*(a*b^2*d*f*g*n*x + a*b^2*c*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n
) - 108*(a^2*b*d - (a^2*b*d*f*g*n*x + a^2*b*c*f*g*n)*log(F))*F^(f*g*n*x + e
*g*n))/(f^2*g^2*n^2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.56

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$$
$$= \begin{cases} (a+b)^3\left(cx+\frac{dx^2}{2}\right) \\ (a+b(F^{eg})^n)^3\left(cx+\frac{dx^2}{2}\right) \\ (a+b)^3\left(cx+\frac{dx^2}{2}\right) \\ a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bc\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{3a^2bdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{3ab^2c\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}+\frac{3ab^2dx\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{3ab}{4f} \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c),x)
```

```
[Out] Piecewise(((a + b)**3*(c*x + d*x**2/2), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq
(n, 0)), ((a + b*(F**(e*g))**n)**3*(c*x + d*x**2/2), Eq(f, 0)), ((a + b)**3
*(c*x + d*x**2/2), Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), (a**3*c*x + a**3*d*x**2
/2 + 3*a**2*b*c*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 3*a**2*b*d*x*(F**(e*
g + f*g*x))**n/(f*g*n*log(F)) - 3*a**2*b*d*(F**(e*g + f*g*x))**n/(f**2*g**2
*n**2*log(F)**2) + 3*a*b**2*c*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) +
3*a*b**2*d*x*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 3*a*b**2*d*(F**(e
*g + f*g*x))**(2*n)/(4*f**2*g**2*n**2*log(F)**2) + b**3*c*(F**(e*g + f*g*x)
)**(3*n)/(3*f*g*n*log(F)) + b**3*d*x*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log
(F)) - b**3*d*(F**(e*g + f*g*x))**(3*n)/(9*f**2*g**2*n**2*log(F)**2), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx &= \frac{1}{2}\,a^3dx^2+a^3cx+\frac{3\,F^{fgnx+egn}a^2bc}{fgn\log(F)} \\ &\quad+\frac{3\,F^{2\,fgnx+2\,egn}ab^2c}{2\,fgn\log(F)}+\frac{F^{3\,fgnx+3\,egn}b^3c}{3\,fgn\log(F)} \\ &\quad+\frac{3\,(F^{egn}fgnx\log(F)-F^{egn})F^{fgnx}a^2bd}{f^2g^2n^2\log(F)^2} \\ &\quad+\frac{3\,(2\,F^{2\,egn}fgnx\log(F)-F^{2\,egn})F^{2\,fgnx}ab^2d}{4\,f^2g^2n^2\log(F)^2} \\ &\quad+\frac{(3\,F^{3\,egn}fgnx\log(F)-F^{3\,egn})F^{3\,fgnx}b^3d}{9\,f^2g^2n^2\log(F)^2}\end{aligned}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*x^2 + a^3*c*x + 3*F^(f*g*n*x + e*g*n)*a^2*b*c/(f*g*n*log(F)) + 3/
2*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*c/(f*g*n*log(F)) + 1/3*F^(3*f*g*n*x + 3*e*g
*n)*b^3*c/(f*g*n*log(F)) + 3*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*
n*x)*a^2*b*d/(f^2*g^2*n^2*log(F)^2) + 3/4*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F
^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*d/(f^2*g^2*n^2*log(F)^2) + 1/9*(3*F^(3*e*g*
n)*f*g*n*x*log(F) - F^(3*e*g*n))*F^(3*f*g*n*x)*b^3*d/(f^2*g^2*n^2*log(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 3528, normalized size of antiderivative = 14.95

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*a^3*d*x^2 + a^3*c*x + 1/9*(2*((3*b^3*d*f*g*n*x*log(abs(F)) + 3*b^3*c*f*
g*n*log(abs(F)) - b^3*d)*(pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^
2*g^2*n^2*log(abs(F))^2)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f
^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2
*g^2*n^2*log(abs(F)))^2) + 3*(pi*b^3*d*f*g*n*x*sgn(F) - pi*b^3*d*f*g*n*x +
pi*b^3*c*f*g*n*sgn(F) - pi*b^3*c*f*g*n)*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F)
- pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2
+ 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - p
i*f^2*g^2*n^2*log(abs(F)))^2))*cos(-3/2*pi*f*g*n*x*sgn(F) + 3/2*pi*f*g*n*x
- 3/2*pi*e*g*n*sgn(F) + 3/2*pi*e*g*n) + (3*(pi*b^3*d*f*g*n*x*sgn(F) - pi*b^
3*d*f*g*n*x + pi*b^3*c*f*g*n*sgn(F) - pi*b^3*c*f*g*n)*(pi^2*f^2*g^2*n^2*sgn
(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)/((pi^2*f^2*g^2*n^2*sg
n(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^
2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) - 4*(3*b^3*d*f*g*n*x*
log(abs(F)) + 3*b^3*c*f*g*n*log(abs(F)) - b^3*d)*(pi*f^2*g^2*n^2*log(abs(F)
)*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2
*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*s
gn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*sin(-3/2*pi*f*g*n*x*sgn(F) + 3/2*pi
*f*g*n*x - 3/2*pi*e*g*n*sgn(F) + 3/2*pi*e*g*n))*e^(3*f*g*n*x*log(abs(F)) +
3*e*g*n*log(abs(F))) - 1/18*I*((3*pi*b^3*d*f*g*n*x*sgn(F) - 3*pi*b^3*d*f*g*
n*x - 6*I*b^3*d*f*g*n*x*log(abs(F)) + 3*pi*b^3*c*f*g*n*sgn(F) - 3*pi*b^3*c*
f*g*n - 6*I*b^3*c*f*g*n*log(abs(F)) + 2*I*b^3*d)*e^(3/2*I*pi*f*g*n*x*sgn(F)
 - 3/2*I*pi*f*g*n*x + 3/2*I*pi*e*g*n*sgn(F) - 3/2*I*pi*e*g*n)/(pi^2*f^2*g^2
*n^2*sgn(F) + 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^2*f^2*g^2*n^2 - 2*
I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^2) + (3*pi*b^3*d*f
*g*n*x*sgn(F) - 3*pi*b^3*d*f*g*n*x + 6*I*b^3*d*f*g*n*x*log(abs(F)) + 3*pi*b
```

```
^3*c*f*g*n*sgn(F) - 3*pi*b^3*c*f*g*n + 6*I*b^3*c*f*g*n*log(abs(F)) - 2*I*b^
3*d)*e^(-3/2*I*pi*f*g*n*x*sgn(F) + 3/2*I*pi*f*g*n*x - 3/2*I*pi*e*g*n*sgn(F)
 + 3/2*I*pi*e*g*n)/(pi^2*f^2*g^2*n^2*sgn(F) - 2*I*pi*f^2*g^2*n^2*log(abs(F)
)*sgn(F) - pi^2*f^2*g^2*n^2 + 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^
2*log(abs(F))^2))*e^(3*f*g*n*x*log(abs(F)) + 3*e*g*n*log(abs(F))) + 3/2*(((
2*a*b^2*d*f*g*n*x*log(abs(F)) + 2*a*b^2*c*f*g*n*log(abs(F)) - a*b^2*d)*(pi^
2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)/((pi
^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 +
 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) + 2*
(pi*a*b^2*d*f*g*n*x*sgn(F) - pi*a*b^2*d*f*g*n*x + pi*a*b^2*c*f*g*n*sgn(F) -
 pi*a*b^2*c*f*g*n)*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(
abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(a
bs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs
(F)))^2))*cos(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e*g*n)
 + ((pi*a*b^2*d*f*g*n*x*sgn(F) - pi*a*b^2*d*f*g*n*x + pi*a*b^2*c*f*g*n*sgn(
F) - pi*a*b^2*c*f*g*n)*(pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*
g^2*n^2*log(abs(F))^2)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2
*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g
^2*n^2*log(abs(F)))^2) - 2*(2*a*b^2*d*f*g*n*x*log(abs(F)) + 2*a*b^2*c*f*g*n
*log(abs(F)) - a*b^2*d)*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2
*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*
log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*lo
g(abs(F)))^2))*sin(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x - pi*e*g*n*sgn(F) + pi*e
*g*n))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F))) - 3/4*I*((pi*a*b^2*d
*f*g*n*x*sgn(F) - pi*a*b^2*d*f*g*n*x - 2*I*a*b^2*d*f*g*n*x*log(abs(F)) + pi
*a*b^2*c*f*g*n*sgn(F) - pi*a*b^2*c*f*g*n - 2*I*a*b^2*c*f*g*n*log(abs(F)) +
I*a*b^2*d)*e^(I*pi*f*g*n*x*sgn(F) - I*pi*f*g*n*x + I*pi*e*g*n*sgn(F) - I*pi
*e*g*n)/(pi^2*f^2*g^2*n^2*sgn(F) + 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) -
pi^2*f^2*g^2*n^2 - 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F
))^2) + (pi*a*b^2*d*f*g*n*x*sgn(F) - pi*a*b^2*d*f*g*n*x + 2*I*a*b^2*d*f*g*n
*x*log(abs(F)) + pi*a*b^2*c*f*g*n*sgn(F) - pi*a*b^2*c*f*g*n + 2*I*a*b^2*c*f
*g*n*log(abs(F)) - I*a*b^2*d)*e^(-I*pi*f*g*n*x*sgn(F) + I*pi*f*g*n*x - I*pi
*e*g*n*sgn(F) + I*pi*e*g*n)/(pi^2*f^2*g^2*n^2*sgn(F) - 2*I*pi*f^2*g^2*n^2*l
og(abs(F))*sgn(F) - pi^2*f^2*g^2*n^2 + 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f
^2*g^2*n^2*log(abs(F))^2))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F)))
+ 3*(2*((a^2*b*d*f*g*n*x*log(abs(F)) + a^2*b*c*f*g*n*log(abs(F)) - a^2*b*d)
*(pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)
/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2
)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2)
 + (pi*a^2*b*d*f*g*n*x*sgn(F) - pi*a^2*b*d*f*g*n*x + pi*a^2*b*c*f*g*n*sgn(F
) - pi*a^2*b*c*f*g*n)*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*l
og(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*lo
g(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(
abs(F)))^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn
(F) + 1/2*pi*e*g*n) + ((pi*a^2*b*d*f*g*n*x*sgn(F) - pi*a^2*b*d*f*g*n*x + pi
```

```
*a^2*b*c*f*g*n*sgn(F) - pi*a^2*b*c*f*g*n)*(pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f
^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*
f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F)
)*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) - 4*(a^2*b*d*f*g*n*x*log(abs(F))
+ a^2*b*c*f*g*n*log(abs(F)) - a^2*b*d)*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) -
 pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 +
 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi
*f^2*g^2*n^2*log(abs(F)))^2))*sin(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x -
 1/2*pi*e*g*n*sgn(F) + 1/2*pi*e*g*n))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(ab
s(F))) - 3/2*I*((pi*a^2*b*d*f*g*n*x*sgn(F) - pi*a^2*b*d*f*g*n*x - 2*I*a^2*b
*d*f*g*n*x*log(abs(F)) + pi*a^2*b*c*f*g*n*sgn(F) - pi*a^2*b*c*f*g*n - 2*I*a
^2*b*c*f*g*n*log(abs(F)) + 2*I*a^2*b*d)*e^(1/2*I*pi*f*g*n*x*sgn(F) - 1/2*I*
pi*f*g*n*x + 1/2*I*pi*e*g*n*sgn(F) - 1/2*I*pi*e*g*n)/(pi^2*f^2*g^2*n^2*sgn(
F) + 2*I*pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi^2*f^2*g^2*n^2 - 2*I*pi*f^2*
g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*log(abs(F))^2) + (pi*a^2*b*d*f*g*n*x*sg
n(F) - pi*a^2*b*d*f*g*n*x + 2*I*a^2*b*d*f*g*n*x*log(abs(F)) + pi*a^2*b*c*f*
g*n*sgn(F) - pi*a^2*b*c*f*g*n + 2*I*a^2*b*c*f*g*n*log(abs(F)) - 2*I*a^2*b*d
)*e^(-1/2*I*pi*f*g*n*x*sgn(F) + 1/2*I*pi*f*g*n*x - 1/2*I*pi*e*g*n*sgn(F) +
1/2*I*pi*e*g*n)/(pi^2*f^2*g^2*n^2*sgn(F) - 2*I*pi*f^2*g^2*n^2*log(abs(F))*s
gn(F) - pi^2*f^2*g^2*n^2 + 2*I*pi*f^2*g^2*n^2*log(abs(F)) + 2*f^2*g^2*n^2*l
og(abs(F))^2))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F)))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx = \frac{a^3\,d\,x^2}{2} - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{3\,a\,b^2\,(d-2\,c\,f\,g\,n\,\ln(F))}{4\,f^2\,g^2\,n^2\,\ln(F)^2} - \frac{3\,a\,b^2\,d\,x}{2\,f\,g\,n\,\ln(F)}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{3\,n}\left(\frac{b^3\,(d-3\,c\,f\,g\,n\,\ln(F))}{9\,f^2\,g^2\,n^2\,\ln(F)^2} - \frac{b^3\,d\,x}{3\,f\,g\,n\,\ln(F)}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{n}\left(\frac{3\,a^2\,b\,(d-c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\,\ln(F)^2} - \frac{3\,a^2\,b\,d\,x}{f\,g\,n\,\ln(F)}\right) + a^3\,c\,x$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x),x)
```

```
[Out] (a^3*d*x^2)/2 - (F^(f*g*x)*F^(e*g))^(2*n)*((3*a*b^2*(d - 2*c*f*g*n*log(F)))
/(4*f^2*g^2*n^2*log(F)^2) - (3*a*b^2*d*x)/(2*f*g*n*log(F))) - (F^(f*g*x)*F^
```

```
(e*g))^(3*n)*((b^3*(d - 3*c*f*g*n*log(F)))/(9*f^2*g^2*n^2*log(F)^2) - (b^3*
d*x)/(3*f*g*n*log(F))) - (F^(f*g*x)*F^(e*g))^n*((3*a^2*b*(d - c*f*g*n*log(F
)))/(f^2*g^2*n^2*log(F)^2) - (3*a^2*b*d*x)/(f*g*n*log(F))) + a^3*c*x
```

3.42 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3\,dx=a^3x+\frac{3a^2b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}+\frac{b^3\left(F^{g(e+fx)}\right)^{3n}}{3fgn\log(F)}$$

```
[Out] a^3*x+3*a^2*b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)+3/2*a*b^2*(F^(g*(f*x+e)))^(2*n)
/f/g/n/ln(F)+1/3*b^3*(F^(g*(f*x+e)))^(3*n)/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2320, 272, 45}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3\,dx=a^3x+\frac{3a^2b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}+\frac{b^3\left(F^{g(e+fx)}\right)^{3n}}{3fgn\log(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*(F^(g*(e + f*x)))^n)/(f*g*n*Log[F]) + (3*a*b^2*(F^(g*(e +
f*x)))^(2*n))/(2*f*g*n*Log[F]) + (b^3*(F^(g*(e + f*x)))^(3*n))/(3*f*g*n*Log
[F])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^3}{x}\,dx, x, F^{g(e+fx)}\right)}{fg\log(F)}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)}\\
&= \frac{\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)}\\
&= a^3x+\frac{3a^2b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}+\frac{b^3\left(F^{g(e+fx)}\right)^{3n}}{3fgn\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3\,dx\\
&=\frac{b\left(F^{g(e+fx)}\right)^n\left(18a^2+9ab\left(F^{g(e+fx)}\right)^n+2b^2\left(F^{g(e+fx)}\right)^{2n}\right)+6a^3\log\left(\left(F^{g(e+fx)}\right)^n\right)}{6fgn\log(F)}
\end{aligned}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3,x]
[Out] (b*(F^(g*(e + f*x)))^n*(18*a^2 + 9*a*b*(F^(g*(e + f*x)))^n + 2*b^2*(F^(g*(e
 + f*x)))^(2*n)) + 6*a^3*Log[(F^(g*(e + f*x)))^n])/(6*f*g*n*Log[F])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{6a^{3}x\ln\left(F\right)fgn+2b^{3}\left(F^{g(fx+e)}\right)^{3n}+9a\,b^{2}\left(F^{g(fx+e)}\right)^{2n}+18a^{2}b\left(F^{g(fx+e)}\right)^{n}}{6\ln\left(F\right)fgn}$	82
derivativedivides	$\frac{\frac{b^{3}\left(F^{g(fx+e)}\right)^{3n}}{3}+\frac{3a\,b^{2}\left(F^{g(fx+e)}\right)^{2n}}{2}+3a^{2}b\left(F^{g(fx+e)}\right)^{n}+a^{3}\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{gf\ln\left(F\right)n}$	86
default	$\frac{\frac{b^{3}\left(F^{g(fx+e)}\right)^{3n}}{3}+\frac{3a\,b^{2}\left(F^{g(fx+e)}\right)^{2n}}{2}+3a^{2}b\left(F^{g(fx+e)}\right)^{n}+a^{3}\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{gf\ln\left(F\right)n}$	86
norman	$a^{3}x+\frac{b^{3}{\mathrm e}^{3n\ln\left({\mathrm e}^{g(fx+e)\ln(F)}\right)}}{3ngf\ln\left(F\right)}+\frac{3a\,b^{2}{\mathrm e}^{2n\ln\left({\mathrm e}^{g(fx+e)\ln(F)}\right)}}{2ngf\ln\left(F\right)}+\frac{3a^{2}b\,{\mathrm e}^{n\ln\left({\mathrm e}^{g(fx+e)\ln(F)}\right)}}{ngf\ln\left(F\right)}$	109

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(6*a^3*x*ln(F)*f*g*n+2*b^3*((F^(g*(f*x+e)))^n)^3+9*a*b^2*((F^(g*(f*x+e)
))^n)^2+18*a^2*b*(F^(g*(f*x+e)))^n)/ln(F)/f/g/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3\,dx = \frac{6\,a^{3}fgnx\log\left(F\right)+18\,F^{fgnx+egn}a^{2}b+9\,F^{2\,fgnx+2\,egn}ab^{2}+2\,F^{3\,fgnx+3\,egn}b^{3}}{6\,fgn\log\left(F\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*f*g*n*x*log(F) + 18*F^(f*g*n*x + e*g*n)*a^2*b + 9*F^(2*f*g*n*x +
 2*e*g*n)*a*b^2 + 2*F^(3*f*g*n*x + 3*e*g*n)*b^3)/(f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3\,dx = \begin{cases} x(a+b)^3 & \text{for}\ F=1\wedge f=0\wedge g=0\wedge n=0 \\ x\left(a+b\left(F^{eg}\right)^n\right)^3 & \text{for}\ f=0 \\ x(a+b)^3 & \text{for}\ F=1\vee g=0\vee n=0 \\ a^3x+\frac{3a^2b\left(F^{eg+fgx}\right)^n}{fgn\log\left(F\right)}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log\left(F\right)}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{3fgn\log\left(F\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3,x)
```

```
[Out] Piecewise((x*(a + b)**3, Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), (x*(a
+ b*(F**(e*g))**n)**3, Eq(f, 0)), (x*(a + b)**3, Eq(F, 1) | Eq(g, 0) | Eq(n
, 0)), (a**3*x + 3*a**2*b*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 3*a*b**2*(
F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + b**3*(F**(e*g + f*g*x))**(3*n)/
(3*f*g*n*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 \, dx = a^3x + \frac{3\,F^{(fx+e)gn}a^2b}{fgn\log\left(F\right)} + \frac{3\,F^{2\,(fx+e)gn}ab^2}{2\,fgn\log\left(F\right)} + \frac{F^{3\,(fx+e)gn}b^3}{3\,fgn\log\left(F\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3*F^((f*x + e)*g*n)*a^2*b/(f*g*n*log(F)) + 3/2*F^(2*(f*x + e)*g*n)*
a*b^2/(f*g*n*log(F)) + 1/3*F^(3*(f*x + e)*g*n)*b^3/(f*g*n*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 \, dx = \frac{18\,F^{fgnx}F^{egn}a^2b + 9\,F^{2\,fgnx}F^{2\,egn}ab^2 + 2\,F^{3\,fgnx}F^{3\,egn}b^3 + 6\,a^3\log\left(|F|^{fgnx}|F|^{egn}\right)}{6\,fgn\log\left(F\right)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*F^(f*g*n*x)*F^(e*g*n)*a^2*b + 9*F^(2*f*g*n*x)*F^(2*e*g*n)*a*b^2 + 2
*F^(3*f*g*n*x)*F^(3*e*g*n)*b^3 + 6*a^3*log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))
)/(f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx = \frac{a^3 \ln\left(F^{f\,g\,x}\right)}{f\,g\,\ln(F)} + \frac{b^3\left(F^{f\,g\,x}\,F^{e\,g}\right)^{3\,n}}{3\,f\,g\,n\,\ln(F)} + \frac{3\,a^2\,b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n}{f\,g\,n\,\ln(F)} + \frac{3\,a\,b^2\left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}}{2\,f\,g\,n\,\ln(F)}$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3,x)
```

```
[Out] (a^3*log(F^(f*g*x)))/(f*g*log(F)) + (b^3*(F^(f*g*x)*F^(e*g))^(3*n))/(3*f*g*
n*log(F)) + (3*a^2*b*(F^(f*g*x)*F^(e*g))^n)/(f*g*n*log(F)) + (3*a*b^2*(F^(f
*g*x)*F^(e*g))^(2*n))/(2*f*g*n*log(F))
```

3.43 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \frac{3a^2bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{3ab^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} \operatorname{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{b^3F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} \operatorname{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{a^3\log(c+dx)}{d}$$

```
[Out] 3*a^2*b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*Ei(f*g*n*(d*x+c)*ln
(F)/d)/d+3*a*b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*Ei
(2*f*g*n*(d*x+c)*ln(F)/d)/d+b^3*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^(f*g*x
+e*g))^(3*n)*Ei(3*f*g*n*(d*x+c)*ln(F)/d)/d+a^3*ln(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used

$= \{2214, 2213, 2209\}$

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$$
$$=\frac{a^3\log(c+dx)}{d}+\frac{3a^2b\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}$$
$$+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$
$$+\frac{b^3\left(F^{eg+fgx}\right)^{3n} F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x),x]
```

```
[Out] (3*a^2*b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*ExpInteg
ralEi[(f*g*n*(c + d*x)*Log[F])/d])/d + (3*a*b^2*F^(2*(e - (c*f)/d)*g*n - 2*
g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log
[F])/d])/d + (b^3*F^(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x
))^(3*n)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d])/d + (a^3*Log[c + d*x]
)/d
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\text{integral}=\int\left(\frac{a^3}{c+dx}+\frac{3a^2b\left(F^{eg+fgx}\right)^n}{c+dx}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{c+dx}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{c+dx}\right)dx$$

$$= \frac{a^3 \log(c+dx)}{d} + (3a^2 b) \int \frac{(F^{eg+fgx})^n}{c+dx} \, dx + (3ab^2) \int \frac{(F^{eg+fgx})^{2n}}{c+dx} \, dx + b^3 \int \frac{(F^{eg+fgx})^{3n}}{c+dx} \, dx$$

$$= \frac{a^3 \log(c+dx)}{d} + \left(3a^2 b F^{-n(eg+fgx)} (F^{eg+fgx})^n\right) \int \frac{F^{n(eg+fgx)}}{c+dx} \, dx + \left(3ab^2 F^{-2n(eg+fgx)} (F^{eg+fgx})^{2n}\right) \int \frac{F^{2n(eg+fgx)}}{c+dx} \, dx + \left(b^3 F^{-3n(eg+fgx)} (F^{eg+fgx})^{3n}\right) \int \frac{F^{3n(eg+fgx)}}{c+dx} \, dx$$

$$= \frac{3a^2 b F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)} (F^{eg+fgx})^n \operatorname{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{3ab^2 F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)} (F^{eg+fgx})^{2n} \operatorname{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{b^3 F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)} (F^{eg+fgx})^{3n} \operatorname{Ei}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{a^3 \log(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx} \, dx = \frac{3a^2 b F^{-\frac{fgn(c+dx)}{d}} \left(F^{g(e+fx)}\right)^n \text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + 3ab^2 F^{-\frac{2fgn(c+dx)}{d}} \left(F^{g(e+fx)}\right)^{2n} \text{ExpIntegralEi}\left(\cdots\right.}{d}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x),x]
```

```
[Out] ((3*a^2*b*(F^(g*(e + f*x)))^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/F^
((f*g*n*(c + d*x))/d) + (3*a*b^2*(F^(g*(e + f*x)))^(2*n)*ExpIntegralEi[(2*f
*g*n*(c + d*x)*Log[F])/d])/F^((2*f*g*n*(c + d*x))/d) + (b^3*(F^(g*(e + f*x)
))^(3*n)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d])/F^((3*f*g*n*(c + d*x)
)/d) + a^3*Log[c + d*x])/d
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3}{dx+c}\,dx$$

[In] int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)

[Out] int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \frac{F^{\frac{3(de-cf)gn}{d}}b^3\text{Ei}\left(\frac{3(dfgnx+cfgn)\log(F)}{d}\right)+3F^{\frac{2(de-cf)gn}{d}}ab^2\text{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right)+3F^{\frac{(de-cf)gn}{d}}a^2b\text{Ei}\left(\frac{(dfgnx+cf}{d}\right.}{d}$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="fricas")

[Out] (F^(3*(d*e - c*f)*g*n/d)*b^3*Ei(3*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 3*F^(2*(d*e - c*f)*g*n/d)*a*b^2*Ei(2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 3*F^((d*e - c*f)*g*n/d)*a^2*b*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d) + a^3*log(d*x + c))/d

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}{c+dx}\,dx$$

[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c),x)

[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**3/(c + d*x), x)

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{dx+c}\,dx$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="maxima")

[Out] F^(3*e*g*n)*b^3*integrate(F^(3*f*g*n*x)/(d*x + c), x) + 3*F^(2*e*g*n)*a*b^2*integrate(F^(2*f*g*n*x)/(d*x + c), x) + 3*F^(e*g*n)*a^2*b*integrate(F^(f*g*n*x)/(d*x + c), x) + a^3*log(d*x + c)/d

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{dx+c}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^3/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx=\int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}{c+d\,x}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x),x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x), x)
```

3.44 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 305

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = -\frac{a^3}{d(c+dx)}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}-\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}+\frac{3a^2bfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\,\mathrm{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}+\frac{6ab^2fF^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} gn\,\mathrm{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}+\frac{3b^3fF^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} gn\,\mathrm{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

```
[Out] -a^3/d/(d*x+c)-3*a^2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)-3*a*b^2*(F^(f*g*x+e*g))^
(2*n)/d/(d*x+c)-b^3*(F^(f*g*x+e*g))^(3*n)/d/(d*x+c)+3*a^2*b*f*F^((e-c*f/d)*
g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g*n*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2+
6*a*b^2*f*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g*n*Ei(2*
f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2+3*b^3*f*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*
(F^(f*g*x+e*g))^(3*n)*g*n*Ei(3*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2214, 2208, 2213, 2209}

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx$$
$$= -\frac{a^3}{d(c+dx)} + \frac{3a^2bfgn\log(F)\left(F^{eg+fgx}\right)^nF^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)} + \frac{6ab^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n}F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)} + \frac{3b^3fgn\log(F)\left(F^{eg+fgx}\right)^{3n}F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)}\text{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2,x]
```

```
[Out] -(a^3/(d*(c + d*x))) - (3*a^2*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) - (3*a*b
^2*(F^(e*g + f*g*x))^(2*n))/(d*(c + d*x)) - (b^3*(F^(e*g + f*g*x))^(3*n))/(
d*(c + d*x)) + (3*a^2*b*f*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f
*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (6*a*b
^2*f*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*g*n*
ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (3*b^3*f*F^(3*(e
- (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*g*n*ExpIntegralEi
[(3*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{a^3}{(c+dx)^2}+\frac{3a^2b\left(F^{eg+fgx}\right)^n}{(c+dx)^2}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^2}\right)dx\\
&= -\frac{a^3}{d(c+dx)}+\left(3a^2b\right)\int \frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx+\left(3ab^2\right)\int \frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}\,dx+b^3\int \frac{\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^2}\,dx\\
&= -\frac{a^3}{d(c+dx)}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}\\
&\quad -\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}+\frac{\left(3a^2bfgn\log(F)\right)\int \frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx}{d}\\
&\quad +\frac{\left(6ab^2fgn\log(F)\right)\int \frac{\left(F^{eg+fgx}\right)^{2n}}{c+dx}\,dx}{d}+\frac{\left(3b^3fgn\log(F)\right)\int \frac{\left(F^{eg+fgx}\right)^{3n}}{c+dx}\,dx}{d}\\
&= -\frac{a^3}{d(c+dx)}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}-\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}\\
&\quad +\frac{\left(3a^2bfF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n gn\log(F)\right)\int \frac{F^{n(eg+fgx)}}{c+dx}\,dx}{d}\\
&\quad +\frac{\left(6ab^2fF^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n} gn\log(F)\right)\int \frac{F^{2n(eg+fgx)}}{c+dx}\,dx}{d}\\
&\quad +\frac{\left(3b^3fF^{-3n(eg+fgx)}\left(F^{eg+fgx}\right)^{3n} gn\log(F)\right)\int \frac{F^{3n(eg+fgx)}}{c+dx}\,dx}{d}
\end{aligned}
$$

$$= -\frac{a^3}{d(c+dx)} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}$$
$$+ \frac{3a^2bfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$
$$+ \frac{6ab^2fF^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} gn\text{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$
$$+ \frac{3b^3fF^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} gn\text{Ei}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx =$$
$$-\frac{a^3d+3a^2bd\left(F^{g(e+fx)}\right)^n+3ab^2d\left(F^{g(e+fx)}\right)^{2n}+b^3d\left(F^{g(e+fx)}\right)^{3n}-3a^2bfF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n gn(c+dx)}{}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2,x]
```

```
[Out] -((a^3*d + 3*a^2*b*d*(F^(g*(e + f*x)))^n + 3*a*b^2*d*(F^(g*(e + f*x)))^(2*n
) + b^3*d*(F^(g*(e + f*x)))^(3*n) - (3*a^2*b*f*(F^(g*(e + f*x)))^n*g*n*(c +
 d*x)*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((f*g*n*(c + d*x)
)/d) - (6*a*b^2*f*(F^(g*(e + f*x)))^(2*n)*g*n*(c + d*x)*ExpIntegralEi[(2*f*
g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((2*f*g*n*(c + d*x))/d) - (3*b^3*f*(F^(g
*(e + f*x)))^(3*n)*g*n*(c + d*x)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d
]*Log[F])/F^((3*f*g*n*(c + d*x))/d))/(d^2*(c + d*x)))
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3}{\left(dx+c\right)^2}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

```
[Out] int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = -\frac{3\,F^{fgnx+egn}a^2bd+3\,F^{2\,fgnx+2\,egn}ab^2d+F^{3\,fgnx+3\,egn}b^3d+a^3d-3\,(b^3dfgnx+b^3cfgn)F^{\frac{3\,(de-cf)gn}{d}}\mathrm{Ei}\Big(\frac{3\,(}{}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(3*F^(f*g*n*x + e*g*n)*a^2*b*d + 3*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*d + F^(3*
f*g*n*x + 3*e*g*n)*b^3*d + a^3*d - 3*(b^3*d*f*g*n*x + b^3*c*f*g*n)*F^(3*(d*
e - c*f)*g*n/d)*Ei(3*(d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F) - 6*(a*b^2*d*f*
g*n*x + a*b^2*c*f*g*n)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n)*l
og(F)/d)*log(F) - 3*(a^2*b*d*f*g*n*x + a^2*b*c*f*g*n)*F^((d*e - c*f)*g*n/d)
*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**3/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] F^(3*e*g*n)*b^3*integrate(F^(3*f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) + 3*F
^(2*e*g*n)*a*b^2*integrate(F^(2*f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) + 3*
F^(e*g*n)*a^2*b*integrate(F^(f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) - a^3/(
d^2*x + c*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^3/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}{(c+d\,x)^2}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2, x)
```

3.45 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 447

$$\begin{aligned}
\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx
&= -\frac{a^3}{2d(c+dx)^2}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2}\\
&\quad-\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2}-\frac{3a^2bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)}\\
&\quad-\frac{3ab^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)}-\frac{3b^3f\left(F^{eg+fgx}\right)^{3n} gn\log(F)}{2d^2(c+dx)}\\
&\quad+\frac{3a^2bf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}\\
&\quad+\frac{6ab^2f^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3}\\
&\quad+\frac{9b^3f^2F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}
\end{aligned}$$

```
[Out] -1/2*a^3/d/(d*x+c)^2-3/2*a^2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-3/2*a*b^2*(F^(
f*g*x+e*g))^(2*n)/d/(d*x+c)^2-1/2*b^3*(F^(f*g*x+e*g))^(3*n)/d/(d*x+c)^2-3/2
*a^2*b*f*(F^(f*g*x+e*g))^n*g*n*ln(F)/d^2/(d*x+c)-3*a*b^2*f*(F^(f*g*x+e*g))^
(2*n)*g*n*ln(F)/d^2/(d*x+c)-3/2*b^3*f*(F^(f*g*x+e*g))^(3*n)*g*n*ln(F)/d^2/(
d*x+c)+3/2*a^2*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g^2*n^
2*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3+6*a*b^2*f^2*F^(2*(e-c*f/d)*g*n-2*g*
n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g^2*n^2*Ei(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)^
```

```
2/d^3+9/2*b^3*f^2*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^(f*g*x+e*g))^(3*n)*g
^2*n^2*Ei(3*f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2214, 2208, 2213, 2209}

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx$$
$$= -\frac{a^3}{2d(c+dx)^2} + \frac{3a^2bf^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^nF^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{2d^3} - \frac{3a^2bfgn\log(F)\left(F^{eg+fgx}\right)^n}{2d^2(c+dx)} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} + \frac{6ab^2f^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^{2n}F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^3} - \frac{3ab^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n}}{d^2(c+dx)} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} + \frac{9b^3f^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^{3n}F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)}\text{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{2d^3} - \frac{3b^3fgn\log(F)\left(F^{eg+fgx}\right)^{3n}}{2d^2(c+dx)} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3,x]
```

```
[Out] -1/2*a^3/(d*(c + d*x)^2) - (3*a^2*b*(F^(e*g + f*g*x))^n)/(2*d*(c + d*x)^2)
- (3*a*b^2*(F^(e*g + f*g*x))^(2*n))/(2*d*(c + d*x)^2) - (b^3*(F^(e*g + f*g*
x))^(3*n))/(2*d*(c + d*x)^2) - (3*a^2*b*f*(F^(e*g + f*g*x))^n*g*n*Log[F])/(
2*d^2*(c + d*x)) - (3*a*b^2*f*(F^(e*g + f*g*x))^(2*n)*g*n*Log[F])/(d^2*(c +
 d*x)) - (3*b^3*f*(F^(e*g + f*g*x))^(3*n)*g*n*Log[F])/(2*d^2*(c + d*x)) + (
3*a^2*b*f^2*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g^2*n
^2*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/(2*d^3) + (6*a*b^2*f
^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*g^2*n^
2*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/d^3 + (9*b^3*f^2*F^
(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*g^2*n^2*Exp
IntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/(2*d^3)
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^3}{(c+dx)^3}+\frac{3a^2b\left(F^{eg+fgx}\right)^n}{(c+dx)^3}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^3}\right)\,dx\\
&=-\frac{a^3}{2d(c+dx)^2}+\left(3a^2b\right)\int\frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^3}\,dx+\left(3ab^2\right)\int\frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3}\,dx+b^3\int\frac{\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^3}\,dx\\
&=-\frac{a^3}{2d(c+dx)^2}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2}\\
&\quad-\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2}+\frac{\left(3a^2bfgn\log(F)\right)\int\frac{\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx}{2d}\\
&\quad+\frac{\left(3ab^2fgn\log(F)\right)\int\frac{\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}\,dx}{d}+\frac{\left(3b^3fgn\log(F)\right)\int\frac{\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^2}\,dx}{2d}
\end{aligned}$$

$$= -\frac{a^3}{2d(c+dx)^2} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2} - \frac{3a^2bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} - \frac{3ab^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} - \frac{3b^3f\left(F^{eg+fgx}\right)^{3n} gn\log(F)}{2d^2(c+dx)} + \frac{\left(3a^2bf^2g^2n^2\log^2(F)\right)\int\frac{\left(F^{eg+fgx}\right)^n}{c+dx}\,dx}{2d^2} + \frac{\left(6ab^2f^2g^2n^2\log^2(F)\right)\int\frac{\left(F^{eg+fgx}\right)^{2n}}{c+dx}\,dx}{d^2} + \frac{\left(9b^3f^2g^2n^2\log^2(F)\right)\int\frac{\left(F^{eg+fgx}\right)^{3n}}{c+dx}\,dx}{2d^2}$$

$$= -\frac{a^3}{2d(c+dx)^2} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2} - \frac{3a^2bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} - \frac{3ab^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} - \frac{3b^3f\left(F^{eg+fgx}\right)^{3n} gn\log(F)}{2d^2(c+dx)} + \frac{\left(3a^2bf^2F^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n g^2n^2\log^2(F)\right)\int\frac{F^{n(eg+fgx)}}{c+dx}\,dx}{2d^2} + \frac{\left(6ab^2f^2F^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2\log^2(F)\right)\int\frac{F^{2n(eg+fgx)}}{c+dx}\,dx}{d^2} + \frac{\left(9b^3f^2F^{-3n(eg+fgx)}\left(F^{eg+fgx}\right)^{3n} g^2n^2\log^2(F)\right)\int\frac{F^{3n(eg+fgx)}}{c+dx}\,dx}{2d^2}$$

$$= -\frac{a^3}{2d(c+dx)^2} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2} - \frac{3a^2bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} - \frac{3ab^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} - \frac{3b^3f\left(F^{eg+fgx}\right)^{3n} gn\log(F)}{2d^2(c+dx)} + \frac{3a^2bf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\text{Ei}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3} + \frac{6ab^2f^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2\text{Ei}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3} + \frac{9b^3f^2F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} g^2n^2\text{Ei}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = -\frac{a^3d^2-3a^2bf^2F^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n g^2n^2(c+dx)^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)-12ab^2f^2F}{}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3,x]
```

```
[Out] -1/2*(a^3*d^2 - (3*a^2*b*f^2*(F^(g*(e + f*x)))^n*g^2*n^2*(c + d*x)^2*ExpInt
egralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((f*g*n*(c + d*x))/d) - (12
*a*b^2*f^2*(F^(g*(e + f*x)))^(2*n)*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(2*f*g
*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((2*f*g*n*(c + d*x))/d) - (9*b^3*f^2*(F
^(g*(e + f*x)))^(3*n)*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(3*f*g*n*(c + d*x)*
Log[F])/d]*Log[F]^2)/F^((3*f*g*n*(c + d*x))/d) + 3*a^2*b*d*(F^(g*(e + f*x))
)^n*(d + f*g*n*(c + d*x)*Log[F]) + 3*a*b^2*d*(F^(g*(e + f*x)))^(2*n)*(d + 2
*f*g*n*(c + d*x)*Log[F]) + b^3*d*(F^(g*(e + f*x)))^(3*n)*(d + 3*f*g*n*(c +
d*x)*Log[F]))/(d^3*(c + d*x)^2)
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3}{\left(dx+c\right)^3}dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x)
```

```
[Out] int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = -\frac{a^3d^2-9\left(b^3d^2f^2g^2n^2x^2+2\,b^3cdf^2g^2n^2x+b^3c^2f^2g^2n^2\right)F^{\frac{3\,(de-cf)gn}{d}}\text{Ei}\left(\frac{3\,(dfgnx+cfgn)\log(F)}{d}\right)\log\left(F\right)^2-12\,(}{}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^3*d^2 - 9*(b^3*d^2*f^2*g^2*n^2*x^2 + 2*b^3*c*d*f^2*g^2*n^2*x + b^3*
c^2*f^2*g^2*n^2)*F^(3*(d*e - c*f)*g*n/d)*Ei(3*(d*f*g*n*x + c*f*g*n)*log(F)/
```

```
d)*log(F)^2 - 12*(a*b^2*d^2*f^2*g^2*n^2*x^2 + 2*a*b^2*c*d*f^2*g^2*n^2*x + a
*b^2*c^2*f^2*g^2*n^2)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n)*lo
g(F)/d)*log(F)^2 - 3*(a^2*b*d^2*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d*f^2*g^2*n^2*x
 + a^2*b*c^2*f^2*g^2*n^2)*F^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*lo
g(F)/d)*log(F)^2 + (b^3*d^2 + 3*(b^3*d^2*f*g*n*x + b^3*c*d*f*g*n)*log(F))*F
^(3*f*g*n*x + 3*e*g*n) + 3*(a*b^2*d^2 + 2*(a*b^2*d^2*f*g*n*x + a*b^2*c*d*f*
g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 3*(a^2*b*d^2 + (a^2*b*d^2*f*g*n*x +
a^2*b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(d^5*x^2 + 2*c*d^4*x + c^2*d^
3)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx=\int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}{\left(c+dx\right)^3}\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c)**3,x)
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**3/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x, algorithm="maxima")
[Out] F^(3*e*g*n)*b^3*integrate(F^(3*f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3), x) + 3*F^(2*e*g*n)*a*b^2*integrate(F^(2*f*g*n*x)/(d^3*x^3 + 3*c*d^2
*x^2 + 3*c^2*d*x + c^3), x) + 3*F^(e*g*n)*a^2*b*integrate(F^(f*g*n*x)/(d^3*
x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x) - 1/2*a^3/(d^3*x^2 + 2*c*d^2*x + c
^2*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx=\int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{\left(dx+c\right)^3}\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^3/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx=\int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}{\left(c+d\,x\right)^3}\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3, x)
```

3.46 $\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{6d^3\operatorname{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^4g^4n^4\log^4(F)}$$

```
[Out] 1/4*(d*x+c)^4/a/d-(d*x+c)^3*ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/ln(F)-3*d*(
d*x+c)^2*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2+6*d^2*(d*x
+c)*polylog(3,-b*(F^(g*(f*x+e)))^n/a)/a/f^3/g^3/n^3/ln(F)^3-6*d^3*polylog(4
,-b*(F^(g*(f*x+e)))^n/a)/a/f^4/g^4/n^4/ln(F)^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2215, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{afgn\log(F)} - \frac{6d^3\operatorname{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^4g^4n^4\log^4(F)} + \frac{(c+dx)^4}{4ad}$$

```
[In] Int[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n),x]
```

```
[Out] (c + d*x)^4/(4*a*d) - ((c + d*x)^3*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a*f
*g*n*Log[F]) - (3*d*(c + d*x)^2*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(
a*f^2*g^2*n^2*Log[F]^2) + (6*d^2*(c + d*x)*PolyLog[3, -((b*(F^(g*(e + f*x))
)^n)/a)])/(a*f^3*g^3*n^3*Log[F]^3) - (6*d^3*PolyLog[4, -((b*(F^(g*(e + f*x)
))^n)/a)])/(a*f^4*g^4*n^4*Log[F]^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^4}{4ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a}$$

$$= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} + \frac{(3d)\int (c+dx)^2 \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{afgn\log(F)}$$

$$= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{(6d^2)\int (c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{af^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{(6d^3)\int \text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{af^3g^3n^3\log^3(F)}$$

$$= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn\log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{(6d^3)\,\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx^n}{a}\right)}{x}\,dx, x, F^{g(e+fx)}\right)}{af^4g^4n^3\log^4(F)}$$

$$= \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1+\frac{b(F^{g(e+fx)})^n}{a}\right)}{afgn\log(F)} - \frac{3d(c+dx)^2 \text{Li}_2\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{6d^3\text{Li}_4\left(-\frac{b(F^{g(e+fx)})^n}{a}\right)}{af^4g^4n^4\log^4(F)}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$= \frac{-(c+dx)^3 \log\left(1+\frac{a(F^{g(e+fx)})^{-n}}{b}\right) + \frac{3d\left(f^2g^2n^2(c+dx)^2\log^2(F)\,\text{PolyLog}\left(2,-\frac{a(F^{g(e+fx)})^{-n}}{b}\right)+2d\left(fgn(c+dx)\log(F)\,\text{PolyLo}\right.\right.}{f^3g^3n^3\log^3(F)}}{afgn\log(F)}$$

```
[In] Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n),x]
```

```
[Out] (-((c + d*x)^3*Log[1 + a/(b*(F^(g*(e + f*x)))^n)]) + (3*d*(f^2*g^2*n^2*(c +
 d*x)^2*Log[F]^2*PolyLog[2, -(a/(b*(F^(g*(e + f*x)))^n))] + 2*d*(f*g*n*(c +
 d*x)*Log[F]*PolyLog[3, -(a/(b*(F^(g*(e + f*x)))^n))] + d*PolyLog[4, -(a/(b
*(F^(g*(e + f*x)))^n))])))/(f^3*g^3*n^3*Log[F]^3))/(a*f*g*n*Log[F])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3074 vs. 2(190) = 380.

Time = 0.41 (sec) , antiderivative size = 3075, normalized size of antiderivative = 16.02

method	result	size
risch	Expression too large to display	3075

```
[In] int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -3/n/g^3/f^4/ln(F)^3*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^
n/a)*e*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2+3/n/g/f^3/ln(F)*c*d^2*e^2/a*ln
(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+3/n/g/f^2/ln(F)*c^2*d*e/a*ln((
```

```
F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-3/n/g/f^2/ln(F)*c^2*d*e/a*ln
(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-3/n/g/f^3/ln(F)*c*d^2*e^2/a*ln
((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+3/n/g^3/f^4/ln(F)^3*d^3*e*
(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F
^(n*g*f*x)*b+a)-3/n/g^3/f^4/ln(F)^3*d^3*e*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F
))^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-1/n/g/f/ln(F)*c^3/a*l
n((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/n/g/f/ln(F)*c^3/a*ln(F^
(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/g^3/f^3/ln(F)^3*d^3/a*x*(ln(F^(
g*(f*x+e)))-g*(f*x+e)*ln(F))^3+3*c^2*d/a*x^2+3*c*d^2/a*x^3+6/n/g^2/f^3/ln(F
)^2*c*d^2/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*e*(ln(F^(g
*(f*x+e)))-g*(f*x+e)*ln(F))-6/n/g^2/f^3/ln(F)^2*c*d^2*e*(ln(F^(g*(f*x+e)))-
g*(f*x+e)*ln(F))/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+6/n/g
^2/f^3/ln(F)^2*c*d^2*e*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))/a*ln(F^(n*g*f*x)
*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-3/n/g^3/f^3/ln(F)^3*c*d^2*(ln(F^(g*(f*x+e)
))-g*(f*x+e)*ln(F))^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+
3/n/g^3/f^3/ln(F)^3*c*d^2*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2/a*ln(F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+3/n/g^2/f^4/ln(F)^2*d^3*e^2*(ln(F^(g*
(f*x+e)))-g*(f*x+e)*ln(F))/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*
b+a)-3/n/g^2/f^4/ln(F)^2*d^3*e^2*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))/a*ln(F
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+3/n/g^2/f^2/ln(F)^2*c^2*d*(ln(F^
(g*(f*x+e)))-g*(f*x+e)*ln(F))/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*
x)*b+a)-3/n/g^2/f^2/ln(F)^2*c^2*d*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))/a*ln(
F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-3/n/g/f/ln(F)*c^2*d/a*ln(1+b*F^
(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-3/n/g/f^2/ln(F)*c^2*d/a*ln(1+
b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*e-3/n/g^2/f^2/ln(F)^2*c^2*d
/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*(ln(F^(g*(f*x+e)))-
g*(f*x+e)*ln(F))+d^3/a*x^4+1/f^3*d^3/a*x*e^3-3/g/f/ln(F)*d^3/a*ln(F^(g*(f*x
+e)))*x^3+9/2/g^2/f^2/ln(F)^2*d^3/a*ln(F^(g*(f*x+e)))^2*x^2-3/g^3/f^3/ln(F)
^3*d^3/a*ln(F^(g*(f*x+e)))^3*x-3/g/f/ln(F)*c^2*d/a*ln(F^(g*(f*x+e)))*x-6/g/
f/ln(F)*c*d^2/a*ln(F^(g*(f*x+e)))*x^2+6/g^2/f^2/ln(F)^2*c*d^2/a*ln(F^(g*(f*
x+e)))^2*x+3/2/g^2/f^2/ln(F)^2*c^2*d/a*ln(F^(g*(f*x+e)))^2-2/g^3/f^3/ln(F)^
3*c*d^2/a*ln(F^(g*(f*x+e)))^3+3/g/f/ln(F)*c^2*d/a*x*(ln(F^(g*(f*x+e)))-g*(f
*x+e)*ln(F))+3/g/f^3/ln(F)*d^3/a*x*e^2*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))+
3/g^2/f^3/ln(F)^2*d^3/a*x*e*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2+1/n/g/f^4
/ln(F)*d^3*e^3/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-1/n/g/f
^4/ln(F)*d^3*e^3/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/n/g^4/f
^4/ln(F)^4*d^3*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^3/a*ln((F^(g*(f*x+e)))^n
*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-1/n/g^4/f^4/ln(F)^4*d^3*(ln(F^(g*(f*x+e)))-g
*(f*x+e)*ln(F))^3/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-3/n^2/g^
2/f^2/ln(F)^2*c^2*d/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))
^n/a)+6/n^3/g^3/f^3/ln(F)^3*c*d^2/a*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(
F^(g*(f*x+e)))^n/a)-1/n/g/f/ln(F)*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^
(g*(f*x+e)))^n/a)*x^3-3/n^2/g^2/f^2/ln(F)^2*d^3/a*polylog(2,-b*F^(n*g*f*x)*
F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x^2+6/n^3/g^3/f^3/ln(F)^3*d^3/a*polylog(3
,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-1/n/g^4/f^4/ln(F)^4*d^3
```

```
/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*(ln(F^(g*(f*x+e)))-
g*(f*x+e)*ln(F))^3-1/n/g/f^4/ln(F)*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F
^(g*(f*x+e)))^n/a)*e^3-3/g^2/f^2/ln(F)^2*c*d^2/a*x*(ln(F^(g*(f*x+e)))-g*(f*
x+e)*ln(F))^2+3/4/g^4/f^4/ln(F)^4*d^3/a*ln(F^(g*(f*x+e)))^4-6/n^4/g^4/f^4/l
n(F)^4*d^3/a*polylog(4,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+3/f
*c^2*d/a*x*e-3/f^2*c*d^2/a*x*e^2-6/g/f^2/ln(F)*c*d^2/a*x*e*(ln(F^(g*(f*x+e)
))-g*(f*x+e)*ln(F))-3/n/g/f/ln(F)*c*d^2/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(
F^(g*(f*x+e)))^n/a)*x^2-3/n/g^2/f^4/ln(F)^2*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-n*
g*f*x)*(F^(g*(f*x+e)))^n/a)*e^2*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))+3/n/g/f
^3/ln(F)*c*d^2/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*e^2+3
/n/g^3/f^3/ln(F)^3*c*d^2/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^
n/a)*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2-6/n^2/g^2/f^2/ln(F)^2*c*d^2/a*po
lylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(189) = 378$.

Time = 0.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{4\left(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3\right)g^3n^3\log\left(F^{fgnx+egn}b+a\right)\log\left(F\right)^3+\left(d^3f^4g^4n^4x^4+4\,cd^2f^4g^4n^4x^3+\right.}{}$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*g^3*n^3*log(F^(f
*g*n*x + e*g*n)*b + a)*log(F)^3 + (d^3*f^4*g^4*n^4*x^4 + 4*c*d^2*f^4*g^4*n^
4*x^3 + 6*c^2*d*f^4*g^4*n^4*x^2 + 4*c^3*f^4*g^4*n^4*x)*log(F)^4 - 4*(d^3*f^
3*g^3*n^3*x^3 + 3*c*d^2*f^3*g^3*n^3*x^2 + 3*c^2*d*f^3*g^3*n^3*x + (d^3*e^3
- 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*g^3*n^3)*log(F)^3*log((F^(f*g*n*x + e*g*n)
*b + a)/a) - 12*(d^3*f^2*g^2*n^2*x^2 + 2*c*d^2*f^2*g^2*n^2*x + c^2*d*f^2*g^
2*n^2)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1)*log(F)^2 - 24*d^3*polylog(
4, -F^(f*g*n*x + e*g*n)*b/a) + 24*(d^3*f*g*n*x + c*d^2*f*g*n)*log(F)*polylo
g(3, -F^(f*g*n*x + e*g*n)*b/a))/(a*f^4*g^4*n^4*log(F)^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+dx)^3}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

```
[In] integrate((d*x+c)**3/(a+b*(F**(g*(f*x+e)))**n),x)
[Out] Integral((c + d*x)**3/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(189) = 378.

Time = 0.28 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.50

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = c^3\left(\frac{fgnx+egn}{afgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{afgn\log(F)}\right) - \frac{3\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)c^2d}{af^2g^2n^2\log(F)^2} - \frac{3\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)cd^2}{af^3g^3n^3\log(F)^3} - \frac{\left(f^3g^3n^3x^3\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^3+3\,f^2g^2n^2x^2\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)^2-6\,fgnx\log(F)\,\mathrm{Li}_3(-\cdots}{af^4g^4n^4\log(F)^4} + \frac{d^3f^4g^4n^4x^4\log(F)^4+4\,cd^2f^4g^4n^4x^3\log(F)^4+6\,c^2df^4g^4n^4x^2\log(F)^4}{4\,af^4g^4n^4\log(F)^4}$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
[Out] c^3*((f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n*
log(F))) - 3*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^
(f*g*n*x)*F^(e*g*n)*b/a))*c^2*d/(a*f^2*g^2*n^2*log(F)^2) - 3*(f^2*g^2*n^2*x
^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*
x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*c*d^2/
(a*f^3*g^3*n^3*log(F)^3) - (f^3*g^3*n^3*x^3*log(F^(f*g*n*x)*F^(e*g*n)*b/a +
 1)*log(F)^3 + 3*f^2*g^2*n^2*x^2*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F)^2
 - 6*f*g*n*x*log(F)*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a) + 6*polylog(4, -
F^(f*g*n*x)*F^(e*g*n)*b/a))*d^3/(a*f^4*g^4*n^4*log(F)^4) + 1/4*(d^3*f^4*g^4
*n^4*x^4*log(F)^4 + 4*c*d^2*f^4*g^4*n^4*x^3*log(F)^4 + 6*c^2*d*f^4*g^4*n^4*
x^2*log(F)^4)/(a*f^4*g^4*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^3}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
[Out] integrate((d*x + c)^3/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+d\,x)^3}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

```
[In] int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n),x)
[Out] int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n), x)
```

3.47 $\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n} dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n} dx = \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2 \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)}$$

```
[Out] 1/3*(d*x+c)^3/a/d-(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/ln(F)-2*d*(
d*x+c)*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2+2*d^2*polylo
g(3,-b*(F^(g*(f*x+e)))^n/a)/a/f^3/g^3/n^3/ln(F)^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2215, 2221, 2611, 2320, 6724}

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = -\frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{afgn\log(F)} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)} + \frac{(c+dx)^3}{3ad}$$

```
[In] Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n),x]
```

```
[Out] (c + d*x)^3/(3*a*d) - ((c + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a*f
*g*n*Log[F]) - (2*d*(c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a*
f^2*g^2*n^2*Log[F]^2) + (2*d^2*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(a
*f^3*g^3*n^3*Log[F]^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^3}{3ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a}$$

$$= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} + \frac{(2d)\int (c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{afgn\log(F)}$$

$$= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{(2d^2)\int \mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{af^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2\left(-\frac{bx^n}{a}\right)}{x}\,dx, x, F^{g(e+fx)}\right)}{af^3g^3n^2\log^3(F)}$$

$$= \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{2d^2\mathrm{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{-f^2g^2n^2(c+dx)^2\log^2(F)\log\left(1+\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)+2dfgn(c+dx)\log(F)\,\text{PolyLog}\left(2,-\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)+}{af^3g^3n^3\log^3(F)}$$

```
[In] Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n),x]
```

```
[Out] (-(f^2*g^2*n^2*(c + d*x)^2*Log[F]^2*Log[1 + a/(b*(F^(g*(e + f*x)))^n)]) + 2
*d*f*g*n*(c + d*x)*Log[F]*PolyLog[2, -(a/(b*(F^(g*(e + f*x)))^n))] + 2*d^2*
PolyLog[3, -(a/(b*(F^(g*(e + f*x)))^n))])/(a*f^3*g^3*n^3*Log[F]^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. 2(143) = 286.

Time = 0.30 (sec) , antiderivative size = 1124, normalized size of antiderivative = 7.75

method	result	size
risch	Expression too large to display	1124

```
[In] int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/ln(F)/f/g/n*c^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/l
n(F)/f/g/n*c^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-2/ln(F)^2/f
^2/g^2/n*c*d/a*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x
+e)))^n/a)-2/ln(F)^2/f^2/g^2/n*d^2/a*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F
^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x+2/ln(F)^2/f^2/g^2/n*c*d/a*ln((F^(g*(f*x+
e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+2/ln(F)/f/g/n*c*d/a*
ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-2/ln(F)^2/f^2/g^2/n*c*d/a*
ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))+1/ln(F)^2/
f^2/g^2*d^2/a*ln(F^(g*(f*x+e)))^2*x-2/ln(F)^2/f^2/g^2/n^2*d^2/a*polylog(2,-
b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-1/ln(F)^3/f^3/g^3/n*d^2/a
*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+1/l
n(F)/f/g/n*d^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2+1/ln(F)
^3/f^3/g^3/n*d^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(
f*x+e)))^2-2/3/ln(F)^3/f^3/g^3*d^2/a*ln(F^(g*(f*x+e)))^3+2/ln(F)^3/f^3/g^3/
n^3*d^2/a*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F
)/f/g/n*d^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2+1/ln(F
)^2/f^2/g^2*c*d/a*ln(F^(g*(f*x+e)))^2-2/ln(F)^2/f^2/g^2/n^2*c*d/a*polylog(2
,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/ln(F)^3/f^3/g^3/n*d^2/a
```

```
*ln(F^(g*(f*x+e)))^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+2
/ln(F)^2/f^2/g^2/n*d^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)
*ln(F^(g*(f*x+e)))*x-2/ln(F)^2/f^2/g^2/n*d^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*
(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x-2/ln(F)/f/g/n*c*d/a*ln((F^(g*(f*x+e)
))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = -\frac{3\,(d^2e^2-2\,cdef+c^2f^2)g^2n^2\log\left(F^{fgnx+egn}b+a\right)\log\left(F\right)^2-(d^2f^3g^3n^3x^3+3\,cdf^3g^3n^3x^2+3\,c^2f^3g^3n^3x}{}$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*g^2*n^2*log(F^(f*g*n*x + e*g*n)*b +
 a)*log(F)^2 - (d^2*f^3*g^3*n^3*x^3 + 3*c*d*f^3*g^3*n^3*x^2 + 3*c^2*f^3*g^3
*n^3*x)*log(F)^3 + 3*(d^2*f^2*g^2*n^2*x^2 + 2*c*d*f^2*g^2*n^2*x - (d^2*e^2
- 2*c*d*e*f)*g^2*n^2)*log(F)^2*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 6*(d^2*
f*g*n*x + c*d*f*g*n)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1)*log(F) - 6*d
^2*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(a*f^3*g^3*n^3*log(F)^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+dx)^2}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

```
[In] integrate((d*x+c)**2/(a+b*(F**(g*(f*x+e)))**n),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(142) = 284.

Time = 0.25 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = c^2\left(\frac{fgnx+egn}{afgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{afgn\log(F)}\right) - \frac{2\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)cd}{af^2g^2n^2\log(F)^2} - \frac{\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3(-\frac{F^{fgnx}F^{egn}b}{a})\right)d^2}{af^3g^3n^3\log(F)^3} + \frac{d^2f^3g^3n^3x^3\log(F)^3+3\,cdf^3g^3n^3x^2\log(F)^3}{3\,af^3g^3n^3\log(F)^3}$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

```
[Out] c^2*((f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n*
log(F))) - 2*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^
(f*g*n*x)*F^(e*g*n)*b/a))*c*d/(a*f^2*g^2*n^2*log(F)^2) - (f^2*g^2*n^2*x^2*l
og(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*x)*F
^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^2/(a*f^3
*g^3*n^3*log(F)^3) + 1/3*(d^2*f^3*g^3*n^3*x^3*log(F)^3 + 3*c*d*f^3*g^3*n^3*
x^2*log(F)^3)/(a*f^3*g^3*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^2}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{\left(c+d\,x\right)^2}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

```
[In] int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n),x)
[Out] int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n), x)
```

3.48 $\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n} dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n} dx = \frac{(c+dx)^2}{2ad} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{d\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)}$$

```
[Out] 1/2*(d*x+c)^2/a/d-(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/ln(F)-d*polyl
og(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2215, 2221, 2317, 2438}

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n} dx = -\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{afgn\log(F)} - \frac{d\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{(c+dx)^2}{2ad}$$

```
[In] Int[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n),x]
```

```
[Out] (c + d*x)^2/(2*a*d) - ((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a*f*g
*n*Log[F]) - (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a*f^2*g^2*n^2*Lo
g[F]^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \frac{(c+dx)^2}{2ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a}$$

$$= \frac{(c+dx)^2}{2ad} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} + \frac{d\int \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{afgn\log(F)}$$

$$= \frac{(c+dx)^2}{2ad} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} + \frac{d\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{af^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^2}{2ad} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{d\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{-fgn(c+dx)\log(F)\log\left(1+\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)+d\,\text{PolyLog}\left(2,-\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)}{af^2g^2n^2\log^2(F)}$$

```
[In] Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n),x]
[Out] (-(f*g*n*(c + d*x)*Log[F]*Log[1 + a/(b*(F^(g*(e + f*x)))^n)]) + d*PolyLog[2
, -(a/(b*(F^(g*(e + f*x)))^n))])/(a*f^2*g^2*n^2*Log[F]^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(96) = 192.

Time = 0.06 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.65

method	result
risch	$-\frac{c\ln\left(\left(F^{g(fx+e)}\right)^n F^{-ngfx}F^{ngfx}b+a\right)}{\ln(F)fgna}+\frac{c\ln\left(F^{ngfx}F^{-ngfx}\left(F^{g(fx+e)}\right)^n\right)}{\ln(F)fgna}+\frac{d\ln\left(F^{g(fx+e)}\right)^2}{2\ln(F)^2f^2g^2a}-\frac{d\ln\left(F^{g(fx+e)}\right)\ln\left(1+\frac{bF^{n}\cdots}{\cdots}\right)}{\ln(F)^2\cdots}$

```
[In] int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
[Out] -1/ln(F)/f/g/n*c/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/ln(
F)/f/g/n*c/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/2/ln(F)^2/f^2
/g^2*d/a*ln(F^(g*(f*x+e)))^2-1/ln(F)^2/f^2/g^2/n*d/a*ln(F^(g*(f*x+e)))*ln(1
+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F)^2/f^2/g^2/n^2*d/a*
polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F)/f/g/n*d/
a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+1/ln(F)^2/f^2/g^2/n*
d/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+1/
ln(F)/f/g/n*d/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-1/ln(F)^2/
f^2/g^2/n*d/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$= \frac{2\,(de-cf)gn\log\left(F^{fgnx+egn}b+a\right)\log\left(F\right)+\left(df^2g^2n^2x^2+2\,cf^2g^2n^2x\right)\log\left(F\right)^2-2\,(dfgnx+degn)\log\left(F\right.}{2\,af^2g^2n^2\log\left(F\right)^2}$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d*e - c*f)*g*n*log(F^(f*g*n*x + e*g*n)*b + a)*log(F) + (d*f^2*g^2*n
^2*x^2 + 2*c*f^2*g^2*n^2*x)*log(F)^2 - 2*(d*f*g*n*x + d*e*g*n)*log(F)*log((
F^(f*g*n*x + e*g*n)*b + a)/a) - 2*d*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a +
1))/(a*f^2*g^2*n^2*log(F)^2)
```

Sympy [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{c+dx}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n),x)
```

```
[Out] Integral((c + d*x)/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{dx+c}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

```
[Out] c*((f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n*lo
g(F))) + d*integrate(x/(F^(f*g*n*x)*F^(e*g*n)*b + a), x)
```

Giac [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{dx+c}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
[Out] integrate((d*x + c)/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{c+d\,x}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

```
[In] int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n),x)
[Out] int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n), x)
```

3.49 $\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n} dx = \frac{x}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{afgn\log(F)}$$

```
[Out] x/a-ln(a+b*(F^(g*(f*x+e)))^n)/a/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2320, 272, 36, 29, 31}

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n} dx = \frac{x}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{afgn\log(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^(-1),x]
```

```
[Out] x/a - Log[a + b*(F^(g*(e + f*x)))^n]/(a*f*g*n*Log[F])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^n)}\,dx, x, F^{g(e+fx)}\right)}{fg\log(F)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{afgn\log(F)} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{afgn\log(F)} \\
&= \frac{x}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{afgn\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{\log\left(\left(F^{g(e+fx)}\right)^n\right) - \log\left(af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)\right)}{afgn\log(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-1),x]
[Out] (Log[(F^(g*(e + f*x)))^n] - Log[a*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F]]
)/(a*f*g*n*Log[F])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{x}{a}-\frac{\ln\left(a+b\,{\mathrm e}^{n\ln\left({\mathrm e}^{g(fx+e)\ln(F)}\right)}\right)}{\ln(F)afgn}$	44
parallelrisch	$\frac{ngf\ln(F)x-\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}{\ln(F)fgan}$	44
derivativedivides	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}{a}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{a}}{gf\ln(F)n}$	53
default	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}{a}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{a}}{gf\ln(F)n}$	53
risch	$\frac{\ln\left(F^{g(fx+e)}\right)}{\ln(F)afg}-\frac{\ln\left(\left(F^{g(fx+e)}\right)^{n}+\frac{a}{b}\right)}{\ln(F)afgn}$	62

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
[Out] x/a-1/ln(F)/a/f/g/n*ln(a+b*exp(n*ln(exp(g*(f*x+e)*ln(F)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^{n}}\,dx=\frac{fgnx\log\left(F\right)-\log\left(F^{fgnx+egn}b+a\right)}{afgn\log\left(F\right)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
[Out] (f*g*n*x*log(F) - log(F^(f*g*n*x + e*g*n)*b + a))/(a*f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^{n}}\,dx=\frac{2\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+\left(F^{g(e+fx)}\right)^{n}\right)}{\sqrt{-\frac{a^{2}}{b^{2}}}}\right)}{bfgn\sqrt{-\frac{a^{2}}{b^{2}}}\log\left(F\right)}$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n),x)
[Out] 2*atan(2*(a/(2*b) + (F**(g*(e + f*x)))**n)/sqrt(-a**2/b**2))/(b*f*g*n*sqrt(
-a**2/b**2)*log(F))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{fgnx+egn}{afgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{afgn\log\left(F\right)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

```
[Out] (f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n*log(F
))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{\log\left(|F|^{fgnx}|F|^{egn}\right)}{afgn\log\left(F\right)} - \frac{\log\left(\left|F^{fgnx}F^{egn}b+a\right|\right)}{afgn\log\left(F\right)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

```
[Out] log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))/(a*f*g*n*log(F)) - log(abs(F^(f*g*n*x)
*F^(e*g*n)*b + a))/(a*f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = -\frac{\ln\left(a+b\left(F^{e\,g+f\,g\,x}\right)^n\right)-f\,g\,n\,x\,\ln\left(F\right)}{a\,f\,g\,n\,\ln\left(F\right)}$$

```
[In] int(1/(a + b*(F^(g*(e + f*x)))^n),x)
```

```
[Out] -(log(a + b*(F^(e*g + f*g*x))^n) - f*g*n*x*log(F))/(a*f*g*n*log(F))
```

3.50 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\text{Int}\left(\frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*(F^(f*g*x+e*g))^n)/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx$$

```
[In] Int[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)),x]
```

```
[Out] Defer[Int][1/((a + b*(F^(e*g + f*g*x))^n)*(c + d*x)), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx$$

```
[In] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)),x]
[Out] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)), x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)(dx+c)}dx$$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
[Out] int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="fricas")
[Out] integral(1/(a*d*x + (b*d*x + b*c)*(F^(f*g*x + e*g))^n + a*c), x)
```

Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)/(d*x+c),x)
[Out] Integral(1/((a + b*(F**(e*g + f*g*x))**n)*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="maxima")
[Out] integrate(1/((F^((f*x + e)*g*n)*b + a)*(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="giac")
[Out] integrate(1/(((F^((f*x + e)*g))^n*b + a)*(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)\,(c+d\,x)}\,dx$$

```
[In] int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)),x)
[Out] int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)), x)
```

3.51 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*(F^(f*g*x+e*g))^n)/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$$

```
[In] Int[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2),x]
[Out] Defer[Int][1/((a + b*(F^(e*g + f*g*x))^n)*(c + d*x)^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$$

```
[In] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2),x]
[Out] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2), x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)(dx+c)^2}dx$$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
[Out] int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 3.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^2}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)/(d*x+c)**2,x)
[Out] Integral(1/((a + b*(F**(e*g + f*g*x))**n)*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(1/((F^((f*x + e)*g*n)*b + a)*(d*x + c)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(1/(((F^((f*x + e)*g))^n*b + a)*(d*x + c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)\,\left(c+d\,x\right)^2}\,dx$$

```
[In] int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2),x)
[Out] int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2), x)
```

3.52 $\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 388

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{6d^3\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)} + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{6d^3\,\mathrm{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)}$$

```
[Out] 1/4*(d*x+c)^4/a^2/d-(d*x+c)^3/a^2/f/g/n/ln(F)+(d*x+c)^3/a/f/(a+b*(F^(g*(f*x
+e)))^n)/g/n/ln(F)+3*d*(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^
2/ln(F)^2-(d*x+c)^3*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/ln(F)+6*d^2*(d*x+
```

```
c)*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3-3*d*(d*x+c)^2*
polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/ln(F)^2-6*d^3*polylog(3,-
b*(F^(g*(f*x+e)))^n/a)/a^2/f^4/g^4/n^4/ln(F)^4+6*d^2*(d*x+c)*polylog(3,-b*(
F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3-6*d^3*polylog(4,-b*(F^(g*(f*x+e
)))^n/a)/a^2/f^4/g^4/n^4/ln(F)^4
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222}

$$\begin{aligned}\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = {} & \frac{6d^2(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} \\ & +\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} \\ & -\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} \\ & +\frac{3d(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^2f^2g^2n^2\log^2(F)} \\ & -\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^2fgn\log(F)} \\ & -\frac{6d^3\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)} \\ & -\frac{6d^3\operatorname{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)}-\frac{(c+dx)^3}{a^2fgn\log(F)} \\ & +\frac{(c+dx)^4}{4a^2d}+\frac{(c+dx)^3}{afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\end{aligned}$$

```
[In] Int[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

```
[Out] (c + d*x)^4/(4*a^2*d) - (c + d*x)^3/(a^2*f*g*n*Log[F]) + (c + d*x)^3/(a*f*(
a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F]) + (3*d*(c + d*x)^2*Log[1 + (b*(F^(g*
(e + f*x)))^n)/a])/(a^2*f^2*g^2*n^2*Log[F]^2) - ((c + d*x)^3*Log[1 + (b*(F^
(g*(e + f*x)))^n)/a])/(a^2*f*g*n*Log[F]) + (6*d^2*(c + d*x)*PolyLog[2, -((b
*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^3*g^3*n^3*Log[F]^3) - (3*d*(c + d*x)^2*Po
lyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^2*g^2*n^2*Log[F]^2) - (6*d^3
*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^4*g^4*n^4*Log[F]^4) + (6*
d^2*(c + d*x)*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^3*g^3*n^3*Lo
```

```
g[F]^3) - (6*d^3*PolyLog[4, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^4*g^4*n^4
*Log[F]^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a}\\
&= \frac{(c+dx)^4}{4a^2d} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)}\\
&\quad - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2} - \frac{(3d)\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{afgn\log(F)}\\
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)}\\
&\quad - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)}\\
&\quad + \frac{(3d)\int (c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2fgn\log(F)} + \frac{(3bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2fgn\log(F)}
\end{aligned}$$

$$= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(6d^2)\int(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2f^2g^2n^2\log^2(F)} + \frac{(6d^2)\int(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2f^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{6d^2(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{(6d^3)\int\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2f^3g^3n^3\log^3(F)} - \frac{(6d^3)\int\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2f^3g^3n^3\log^3(F)}$$

$$= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{6d^2(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{(6d^3)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx^n}{a}\right)}{x}dx, x, F^{g(e+fx)}\right)}{a^2f^4g^4n^3\log^4(F)} - \frac{(6d^3)\,\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{bx^n}{a}\right)}{x}dx, x, F^{g(e+fx)}\right)}{a^2f^4g^4n^3\log^4(F)}$$

$$
\begin{aligned}
&= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} + \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\
&+ \frac{3d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} \\
&+ \frac{6d^2(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} \\
&- \frac{6d^3\mathrm{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)} + \frac{6d^2(c+dx)\mathrm{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{6d^3\mathrm{Li}_4\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx
$$

```
[In] Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2,x]
[Out] Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3318 vs. 2(386) = 772.

Time = 0.46 (sec) , antiderivative size = 3319, normalized size of antiderivative = 8.55

method	result	size
risch	Expression too large to display	3319

```
[In] int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
[Out] 1/n/g/f/ln(F)/a*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(a+b*(F^(g*(f*x+e)))^n)
+3/4/g^4/f^4/ln(F)^4/a^2*d^3*ln(F^(g*(f*x+e)))^4+3/2/g^2/f^2/ln(F)^2/a^2*c^
2*d*ln(F^(g*(f*x+e)))^2-1/n/g/f/ln(F)/a^2*c^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*
f*x)*F^(n*g*f*x)*b+a)+1/n/g/f/ln(F)/a^2*c^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^
(g*(f*x+e)))^n)+2/n/g^4/f^4/ln(F)^4/a^2*d^3*ln(F^(g*(f*x+e)))^3-6/n^4/g^4/f
^4/ln(F)^4/a^2*d^3*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/
a)-6/n^4/g^4/f^4/ln(F)^4/a^2*d^3*polylog(4,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(
g*(f*x+e)))^n/a)+3/2/g^2/f^2/ln(F)^2/a^2*d^3*ln(F^(g*(f*x+e)))^2*x^2-2/g^3/
f^3/ln(F)^3/a^2*d^3*ln(F^(g*(f*x+e)))^3*x-2/g^3/f^3/ln(F)^3/a^2*c*d^2*ln(F^
(g*(f*x+e)))^3+3/n/g^3/f^3/ln(F)^3/a^2*c*d^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x
)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))^2+3/n/g/f/ln(F)/a^2*c^2*d*ln(F^(n*
g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-3/n/g^2/f^2/ln(F)^2/a^2*c^2*d*ln(F
```

```
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-6/n^2/g^2/f^2/
ln(F)^2/a^2*c*d^2*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a
)*x-6/n^2/g^3/f^3/ln(F)^3/a^2*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*
f*x)*b+a)*ln(F^(g*(f*x+e)))*x+6/n^2/g^3/f^3/ln(F)^3/a^2*d^3*ln(F^(n*g*f*x)*
F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x+6/n^2/g^3/f^3/ln(F)^3/a
^2*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)
))*x-3/n/g/f/ln(F)/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*
b+a)*x^2-3/n/g^3/f^3/ln(F)^3/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^
(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+3/n/g/f/ln(F)/a^2*c*d^2*ln(F^(n*g*f*x)*F
^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2+3/n/g^3/f^3/ln(F)^3/a^2*c*d^2*ln(F^(n*g*
f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2-3/n/g^2/f^2/ln(F)^
2/a^2*c^2*d*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f
*x+e)))-3/n/g/f/ln(F)/a^2*c^2*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*
x)*b+a)*x+3/n/g^2/f^2/ln(F)^2/a^2*c^2*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F
^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))-3/n/g^2/f^2/ln(F)^2/a^2*d^3*ln(F^(n*g*f*x
)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x^2+3/n/g^3/f^3/ln(F)^3
/a^2*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2
*x-3/n/g^2/f^2/ln(F)^2/a^2*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e
)))^n/a)*ln(F^(g*(f*x+e)))*x^2+3/n/g^3/f^3/ln(F)^3/a^2*d^3*ln(1+b*F^(n*g*f*
x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))^2*x+3/n/g^2/f^2/ln(F
)^2/a^2*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x
+e)))*x^2+6/n^3/g^3/f^3/ln(F)^3/a^2*d^3*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*
x)*(F^(g*(f*x+e)))^n/a)*x+6/n^3/g^3/f^3/ln(F)^3/a^2*c*d^2*polylog(2,-b*F^(n
*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+6/n^3/g^3/f^3/ln(F)^3/a^2*d^3*pol
ylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-3/n^2/g^2/f^2/ln(
F)^2/a^2*c^2*d*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+3
/n^2/g^2/f^2/ln(F)^2/a^2*c^2*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x
)*b+a)-3/n^2/g^2/f^2/ln(F)^2/a^2*c^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f
*x+e)))^n)+6/n^3/g^3/f^3/ln(F)^3/a^2*c*d^2*polylog(3,-b*F^(n*g*f*x)*F^(-n*g
*f*x)*(F^(g*(f*x+e)))^n/a)-3/n^2/g^2/f^2/ln(F)^2/a^2*d^3*polylog(2,-b*F^(n*
g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x^2+1/n/g/f/ln(F)/a^2*d^3*ln(F^(n*
g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^3-1/n/g^4/f^4/ln(F)^4/a^2*d^3*ln(F
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^3-1/n/g^4/f^4/
ln(F)^4/a^2*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(
g*(f*x+e)))^3-1/n/g/f/ln(F)/a^2*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*
g*f*x)*b+a)*x^3+1/n/g^4/f^4/ln(F)^4/a^2*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*
x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^3+3/n^2/g^2/f^2/ln(F)^2/a^2*d^3*ln((F
^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2-3/n/g^3/f^3/ln(F)^3/a^2*d
^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2*x
+6/n^2/g^2/f^2/ln(F)^2/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f
*x)*b+a)*x-6/n^2/g^3/f^3/ln(F)^3/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x
)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))-6/n^2/g^2/f^2/ln(F)^2/a^2*c*d^2*ln(F^(
n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x+6/n^2/g^3/f^3/ln(F)^3/a^2*c*d^2*
ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))+6/n^2/g^3/
f^3/ln(F)^3/a^2*c*d^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*
```

```
ln(F^(g*(f*x+e)))-3/n/g^3/f^3/ln(F)^3/a^2*d^3*ln(F^(g*(f*x+e)))^2*x+3/n^2/g
^4/f^4/ln(F)^4/a^2*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*l
n(F^(g*(f*x+e)))^2-3/n^2/g^2/f^2/ln(F)^2/a^2*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x
)*(F^(g*(f*x+e)))^n)*x^2-3/n^2/g^4/f^4/ln(F)^4/a^2*d^3*ln(F^(n*g*f*x)*F^(-n
*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2+3/g^2/f^2/ln(F)^2/a^2*c*d^2*
ln(F^(g*(f*x+e)))^2*x-3/n/g^3/f^3/ln(F)^3/a^2*d^2*c*ln(F^(g*(f*x+e)))^2-3/n
^2/g^4/f^4/ln(F)^4/a^2*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^
n/a)*ln(F^(g*(f*x+e)))^2-6/n/g^2/f^2/ln(F)^2/a^2*c*d^2*ln(1+b*F^(n*g*f*x)*F
^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))*x+6/n/g^2/f^2/ln(F)^2/a^
2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e))
)*x-6/n/g^2/f^2/ln(F)^2/a^2*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)
))^n)*ln(F^(g*(f*x+e)))*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(384) = 768$.

Time = 0.31 (sec) , antiderivative size = 1390, normalized size of antiderivative = 3.58

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*g^3*n^3
*log(F)^3 - (a*d^3*f^4*g^4*n^4*x^4 + 4*a*c*d^2*f^4*g^4*n^4*x^3 + 6*a*c^2*d*
f^4*g^4*n^4*x^2 + 4*a*c^3*f^4*g^4*n^4*x - (a*d^3*e^4 - 4*a*c*d^2*e^3*f + 6*
a*c^2*d*e^2*f^2 - 4*a*c^3*e*f^3)*g^4*n^4)*log(F)^4 - ((b*d^3*f^4*g^4*n^4*x^
4 + 4*b*c*d^2*f^4*g^4*n^4*x^3 + 6*b*c^2*d*f^4*g^4*n^4*x^2 + 4*b*c^3*f^4*g^4
*n^4*x - (b*d^3*e^4 - 4*b*c*d^2*e^3*f + 6*b*c^2*d*e^2*f^2 - 4*b*c^3*e*f^3)*
g^4*n^4)*log(F)^4 - 4*(b*d^3*f^3*g^3*n^3*x^3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 +
3*b*c^2*d*f^3*g^3*n^3*x + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*g
^3*n^3)*log(F)^3)*F^(f*g*n*x + e*g*n) + 12*((a*d^3*f^2*g^2*n^2*x^2 + 2*a*c*
d^2*f^2*g^2*n^2*x + a*c^2*d*f^2*g^2*n^2)*log(F)^2 + ((b*d^3*f^2*g^2*n^2*x^2
 + 2*b*c*d^2*f^2*g^2*n^2*x + b*c^2*d*f^2*g^2*n^2)*log(F)^2 - 2*(b*d^3*f*g*n
*x + b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 2*(a*d^3*f*g*n*x + a*c*d^
2*f*g*n)*log(F))*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 4*((a*d^3*e^3
- 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*g^3*n^3*log(F)^3 + 3*(a*d^
3*e^2 - 2*a*c*d^2*e*f + a*c^2*d*f^2)*g^2*n^2*log(F)^2 + ((b*d^3*e^3 - 3*b*c
*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*g^3*n^3*log(F)^3 + 3*(b*d^3*e^2 -
 2*b*c*d^2*e*f + b*c^2*d*f^2)*g^2*n^2*log(F)^2)*F^(f*g*n*x + e*g*n))*log(F^
(f*g*n*x + e*g*n)*b + a) + 4*((a*d^3*f^3*g^3*n^3*x^3 + 3*a*c*d^2*f^3*g^3*n^
3*x^2 + 3*a*c^2*d*f^3*g^3*n^3*x + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*
e*f^2)*g^3*n^3)*log(F)^3 - 3*(a*d^3*f^2*g^2*n^2*x^2 + 2*a*c*d^2*f^2*g^2*n^2
*x - (a*d^3*e^2 - 2*a*c*d^2*e*f)*g^2*n^2)*log(F)^2 + ((b*d^3*f^3*g^3*n^3*x^
```

```
3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + (b*d^3*e^3 - 3*b*
c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*g^3*n^3)*log(F)^3 - 3*(b*d^3*f^2*g^2*n^2*x^2
 + 2*b*c*d^2*f^2*g^2*n^2*x - (b*d^3*e^2 - 2*b*c*d^2*e*f)*g^2*n^2)*log(F)^2)
*F^(f*g*n*x + e*g*n))*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 24*(F^(f*g*n*x +
 e*g*n)*b*d^3 + a*d^3)*polylog(4, -F^(f*g*n*x + e*g*n)*b/a) + 24*(a*d^3 + (
b*d^3 - (b*d^3*f*g*n*x + b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n) - (a*d^
3*f*g*n*x + a*c*d^2*f*g*n)*log(F))*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(F
^(f*g*n*x + e*g*n)*a^2*b*f^4*g^4*n^4*log(F)^4 + a^3*f^4*g^4*n^4*log(F)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)**3/(a+b*(F**(g*(f*x+e)))**n)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$= c^3\left(\frac{fgnx+egn}{a^2fgn} + \frac{1}{(F^{fgnx+egn}ab+a^2)fgn\log(F)} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{a^2fgn\log(F)}\right)$$
$$+ \frac{d^3x^3+3\,cd^2x^2+3\,c^2dx}{F^{fgnx}F^{egn}abfgn\log(F)+a^2fgn\log(F)} - \frac{3\,c^2dx}{a^2fgn\log(F)} + \frac{3\,c^2d\log\left(F^{fgnx}F^{egn}b+a\right)}{a^2f^2g^2n^2\log(F)^2}$$
$$- \frac{3\,(c^2dfgn\log(F)-2\,cd^2)\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)}{a^2f^3g^3n^3\log(F)^3}$$
$$- \frac{\left(f^3g^3n^3x^3\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^3+3\,f^2g^2n^2x^2\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)^2-6\,fgnx\log(F)\,\mathrm{Li}_3(-\right.}{a^2f^4g^4n^4\log(F)^4}$$
$$- \frac{3\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3(-\frac{F^{fgnx}F^{egn}b}{a})\right)(cd^{}}{a^2f^4g^4n^4\log(F)^4}$$
$$+ \frac{d^3f^4g^4n^4x^4\log(F)^4+4\,(cd^2fgn\log(F)-d^3)f^3g^3n^3x^3\log(F)^3+6\left(c^2df^2g^2n^2\log(F)^2-2\,cd^2fgn\log\right.}{4\,a^2f^4g^4n^4\log(F)^4}$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

```
[Out] c^3*((f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a*b + a^2)*f*g
*n*log(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))) + (d^3*x^3
+ 3*c*d^2*x^2 + 3*c^2*d*x)/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*
g*n*log(F)) - 3*c^2*d*x/(a^2*f*g*n*log(F)) + 3*c^2*d*log(F^(f*g*n*x)*F^(e*g
*n)*b + a)/(a^2*f^2*g^2*n^2*log(F)^2) - 3*(c^2*d*f*g*n*log(F) - 2*c*d^2)*(f
*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^(f*g*n*x)*F^(e*
g*n)*b/a))/(a^2*f^3*g^3*n^3*log(F)^3) - (f^3*g^3*n^3*x^3*log(F^(f*g*n*x)*F^
(e*g*n)*b/a + 1)*log(F)^3 + 3*f^2*g^2*n^2*x^2*dilog(-F^(f*g*n*x)*F^(e*g*n)*
b/a)*log(F)^2 - 6*f*g*n*x*log(F)*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a) + 6
*polylog(4, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^3/(a^2*f^4*g^4*n^4*log(F)^4) - 3
*(f^2*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*d
ilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*
n)*b/a))*(c*d^2*f*g*n*log(F) - d^3)/(a^2*f^4*g^4*n^4*log(F)^4) + 1/4*(d^3*f
^4*g^4*n^4*x^4*log(F)^4 + 4*(c*d^2*f*g*n*log(F) - d^3)*f^3*g^3*n^3*x^3*log(
F)^3 + 6*(c^2*d*f^2*g^2*n^2*log(F)^2 - 2*c*d^2*f*g*n*log(F))*f^2*g^2*n^2*x^
2*log(F)^2)/(a^2*f^4*g^4*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(dx+c)^3}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(c+d\,x)^3}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

```
[In] int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2,x)
[Out] int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

3.53 $\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn\log(F)} + \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{2d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)}$$

```
[Out] 1/3*(d*x+c)^3/a^2/d-(d*x+c)^2/a^2/f/g/n/ln(F)+(d*x+c)^2/a/f/(a+b*(F^(g*(f*x
+e)))^n)/g/n/ln(F)+2*d*(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/
ln(F)^2-(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/ln(F)+2*d^2*polylog
(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3-2*d*(d*x+c)*polylog(2,-b
*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/ln(F)^2+2*d^2*polylog(3,-b*(F^(g*(f*x
+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438}

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = -\frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{2d(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^2fgn\log(F)} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{(c+dx)^2}{a^2fgn\log(F)} + \frac{(c+dx)^3}{3a^2d} + \frac{(c+dx)^2}{afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}$$

```
[In] Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

```
[Out] (c + d*x)^3/(3*a^2*d) - (c + d*x)^2/(a^2*f*g*n*Log[F]) + (c + d*x)^2/(a*f*(
a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F]) + (2*d*(c + d*x)*Log[1 + (b*(F^(g*(e
 + f*x)))^n)/a])/(a^2*f^2*g^2*n^2*Log[F]^2) - ((c + d*x)^2*Log[1 + (b*(F^(g
*(e + f*x)))^n)/a])/(a^2*f*g*n*Log[F]) + (2*d^2*PolyLog[2, -((b*(F^(g*(e +
f*x)))^n)/a)])/(a^2*f^3*g^3*n^3*Log[F]^3) - (2*d*(c + d*x)*PolyLog[2, -((b*
(F^(g*(e + f*x)))^n)/a)])/(a^2*f^2*g^2*n^2*Log[F]^2) + (2*d^2*PolyLog[3, -(
(b*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^3*g^3*n^3*Log[F]^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
```

```
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a}$$

$$= \frac{(c+dx)^3}{3a^2d} + \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2} - \frac{(2d)\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{afgn\log(F)}$$

$$= \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn\log(F)} + \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{(2d)\int (c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2fgn\log(F)} + \frac{(2bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2fgn\log(F)}$$

$$= \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn\log(F)} + \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{2d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{2d(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(2d^2)\int \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2f^2g^2n^2\log^2(F)} + \frac{(2d^2)\int \text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^2f^2g^2n^2\log^2(F)}$$

$$
\begin{aligned}
&= \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn\log(F)} + \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\
&+ \frac{2d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} \\
&- \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(2d^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^2f^3g^3n^3\log^3(F)} \\
&+ \frac{(2d^2)\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(-\frac{bx^n}{a}\right)}{x}dx, x, F^{g(e+fx)}\right)}{a^2f^3g^3n^2\log^3(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn\log(F)} + \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\
&+ \frac{2d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} \\
&+ \frac{2d^2\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} - \frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{2d^2\mathrm{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)}
\end{aligned}
$$

Mathematica [F]

$$
\int\frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}dx = \int\frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}dx
$$

```
[In] Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2,x]
[Out] Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. 2(292) = 584.

Time = 0.31 (sec) , antiderivative size = 1650, normalized size of antiderivative = 5.61

method	result	size
risch	Expression too large to display	1650

```
[In] int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
[Out] -2/a^2/ln(F)/f/g/n*c*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x
+2/a^2/ln(F)/f/g/n*c*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x+2/a
^2/ln(F)^2/f^2/g^2/n*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)
```

```
*ln(F^(g*(f*x+e)))*x-2/a^2/ln(F)^2/f^2/g^2/n*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x
)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x-2/a^2/ln(F)^2/f^2/g^2/n*d^2*ln(F^(
g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-2/a^2/ln
(F)^2/f^2/g^2/n*c*d*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g
*(f*x+e)))^n/a)+2/a^2/ln(F)^2/f^2/g^2/n*c*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*
x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))-2/a^2/ln(F)^2/f^2/g^2/n*c*d*ln(F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-2/3/a^2/ln(F)^3/f^3
/g^3*d^2*ln(F^(g*(f*x+e)))^3+1/n/g/f/ln(F)/a*(d^2*x^2+2*c*d*x+c^2)/(a+b*(F^
(g*(f*x+e)))^n)+2/a^2/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(
F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))+2/a^2/ln(F)^2/f^2/g^2/n^2*c*d*ln((F^(g*
(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-2/a^2/ln(F)^2/f^2/g^2/n^2*c*d*ln(
F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-2/a^2/ln(F)^2/f^2/g^2/n^2*c*d*p
olylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/a^2/ln(F)^3/f^3
/g^3/n*d^2*ln(F^(g*(f*x+e)))^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e
)))^n/a)+2/a^2/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)
*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/a^2/ln(F)/f/g/n*d^2*ln((F^(g*(f*x+e)))
^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2-1/a^2/ln(F)^3/f^3/g^3/n*d^2*ln((F^(g*(
f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+1/a^2/ln(F)/f/
g/n*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2+1/a^2/ln(F)^3/f^
3/g^3/n*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e))
)^2-1/a^2/ln(F)/f/g/n*c^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a
)+1/a^2/ln(F)/f/g/n*c^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+2/a^
2/ln(F)^3/f^3/g^3/n^3*d^2*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+
e)))^n/a)-1/a^2/ln(F)^3/f^3/g^3/n*d^2*ln(F^(g*(f*x+e)))^2+2/a^2/ln(F)^3/f^3
/g^3/n^3*d^2*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/a
^2/ln(F)^2/f^2/g^2*c*d*ln(F^(g*(f*x+e)))^2+1/a^2/ln(F)^2/f^2/g^2*d^2*ln(F^(
g*(f*x+e)))^2*x-2/a^2/ln(F)^2/f^2/g^2/n^2*d^2*polylog(2,-b*F^(n*g*f*x)*F^(-
n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x+2/a^2/ln(F)^2/f^2/g^2/n^2*d^2*ln((F^(g*(f*x
+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x-2/a^2/ln(F)^3/f^3/g^3/n^2*d^2*ln((F
^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))-2/a^2/ln(F)
^2/f^2/g^2/n^2*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(290) = 580.

Time = 0.26 (sec) , antiderivative size = 834, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$= \frac{3\,(ad^2e^2 - 2\,acdef + ac^2f^2)g^2n^2\log(F)^2 + (ad^2f^3g^3n^3x^3 + 3\,acdf^3g^3n^3x^2 + 3\,ac^2f^3g^3n^3x + (ad^2e^3 - 3\,ac}{}$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*g^2*n^2*log(F)^2 + (a*d^2*f^3*
g^3*n^3*x^3 + 3*a*c*d*f^3*g^3*n^3*x^2 + 3*a*c^2*f^3*g^3*n^3*x + (a*d^2*e^3
- 3*a*c*d*e^2*f + 3*a*c^2*e*f^2)*g^3*n^3)*log(F)^3 + ((b*d^2*f^3*g^3*n^3*x^
3 + 3*b*c*d*f^3*g^3*n^3*x^2 + 3*b*c^2*f^3*g^3*n^3*x + (b*d^2*e^3 - 3*b*c*d*
e^2*f + 3*b*c^2*e*f^2)*g^3*n^3)*log(F)^3 - 3*(b*d^2*f^2*g^2*n^2*x^2 + 2*b*c
*d*f^2*g^2*n^2*x - (b*d^2*e^2 - 2*b*c*d*e*f)*g^2*n^2)*log(F)^2)*F^(f*g*n*x
+ e*g*n) + 6*(a*d^2 + (b*d^2 - (b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))*F^(f*g
*n*x + e*g*n) - (a*d^2*f*g*n*x + a*c*d*f*g*n)*log(F))*dilog(-(F^(f*g*n*x +
e*g*n)*b + a)/a + 1) - 3*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*g^2*n^2*log
(F)^2 + 2*(a*d^2*e - a*c*d*f)*g*n*log(F) + ((b*d^2*e^2 - 2*b*c*d*e*f + b*c^
2*f^2)*g^2*n^2*log(F)^2 + 2*(b*d^2*e - b*c*d*f)*g*n*log(F))*F^(f*g*n*x + e*
g*n))*log(F^(f*g*n*x + e*g*n)*b + a) - 3*((a*d^2*f^2*g^2*n^2*x^2 + 2*a*c*d*
f^2*g^2*n^2*x - (a*d^2*e^2 - 2*a*c*d*e*f)*g^2*n^2)*log(F)^2 + ((b*d^2*f^2*g
^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*x - (b*d^2*e^2 - 2*b*c*d*e*f)*g^2*n^2)*log
(F)^2 - 2*(b*d^2*f*g*n*x + b*d^2*e*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 2*(a*
d^2*f*g*n*x + a*d^2*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 6*(
F^(f*g*n*x + e*g*n)*b*d^2 + a*d^2)*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(F
^(f*g*n*x + e*g*n)*a^2*b*f^3*g^3*n^3*log(F)^3 + a^3*f^3*g^3*n^3*log(F)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)**2/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$= c^2\left(\frac{fgnx+egn}{a^2fgn}+\frac{1}{(F^{fgnx+egn}ab+a^2)fgn\log(F)}-\frac{\log\left(F^{fgnx+egn}b+a\right)}{a^2fgn\log(F)}\right)$$
$$+\frac{d^2x^2+2\,cdx}{F^{fgnx}F^{egn}abfgn\log(F)+a^2fgn\log(F)}-\frac{2\,cdx}{a^2fgn\log(F)}+\frac{2\,cd\log\left(F^{fgnx}F^{egn}b+a\right)}{a^2f^2g^2n^2\log(F)^2}$$
$$-\frac{\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3(-\frac{F^{fgnx}F^{egn}b}{a})\right)d^2}{a^2f^3g^3n^3\log(F)^3}$$
$$-\frac{2\,(cdfgn\log(F)-d^2)\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)}{a^2f^3g^3n^3\log(F)^3}$$
$$+\frac{d^2f^3g^3n^3x^3\log(F)^3+3\,(cdfgn\log(F)-d^2)f^2g^2n^2x^2\log(F)^2}{3\,a^2f^3g^3n^3\log(F)^3}$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

```
[Out] c^2*((f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a*b + a^2)*f*g
*n*log(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))) + (d^2*x^2
+ 2*c*d*x)/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*g*n*log(F)) - 2*
c*d*x/(a^2*f*g*n*log(F)) + 2*c*d*log(F^(f*g*n*x)*F^(e*g*n)*b + a)/(a^2*f^2*
g^2*n^2*log(F)^2) - (f^2*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log
(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -
F^(f*g*n*x)*F^(e*g*n)*b/a))*d^2/(a^2*f^3*g^3*n^3*log(F)^3) - 2*(c*d*f*g*n*l
og(F) - d^2)*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^
(f*g*n*x)*F^(e*g*n)*b/a))/(a^2*f^3*g^3*n^3*log(F)^3) + 1/3*(d^2*f^3*g^3*n^3
*x^3*log(F)^3 + 3*(c*d*f*g*n*log(F) - d^2)*f^2*g^2*n^2*x^2*log(F)^2)/(a^2*f
^3*g^3*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(dx+c)^2}{\left(\left(F^{(fx+e)g}\right)^nb+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{{\left(c+d\,x\right)}^2}{{\left(a+b\,{\left(F^{g\,\left(e+f\,x\right)}\right)}^n\right)}^2}\,dx$$

```
[In] int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2,x)
[Out] int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

3.54 $\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx = \frac{(c+dx)^2}{2a^2d} - \frac{dx}{a^2fgn\log(F)} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{d\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)}$$

```
[Out] 1/2*(d*x+c)^2/a^2/d-d*x/a^2/f/g/n/ln(F)+(d*x+c)/a/f/(a+b*(F^(g*(f*x+e)))^n)
/g/n/ln(F)+d*ln(a+b*(F^(g*(f*x+e)))^n)/a^2/f^2/g^2/n^2/ln(F)^2-(d*x+c)*ln(1
+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/ln(F)-d*polylog(2,-b*(F^(g*(f*x+e)))^n/a)
/a^2/f^2/g^2/n^2/ln(F)^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2216, 2215, 2221, 2317, 2438, 2222, 2320, 272, 36, 29, 31}

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx = -\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^2fgn\log(F)} - \frac{d\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{(c+dx)^2}{2a^2d} - \frac{dx}{a^2fgn\log(F)} + \frac{c+dx}{afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}$$

```
[In] Int[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

```
[Out] (c + d*x)^2/(2*a^2*d) - (d*x)/(a^2*f*g*n*Log[F]) + (c + d*x)/(a*f*(a + b*(F
^(g*(e + f*x)))^n)*g*n*Log[F]) + (d*Log[a + b*(F^(g*(e + f*x)))^n])/(a^2*f^
2*g^2*n^2*Log[F]^2) - ((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a^2*f
*g*n*Log[F]) - (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^2*f^2*g^2*n^
2*Log[F]^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a}\\
&= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2} - \frac{d\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{afgn\log(F)}\\
&= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)}\\
&\quad - \frac{d\,\text{Subst}\left(\int \frac{1}{x(a+bx^n)}\,dx, x, F^{g(e+fx)}\right)}{af^2g^2n\log^2(F)} + \frac{d\int \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^2fgn\log(F)}
\end{aligned}$$

$$= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} + \frac{d\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{d\text{Subst}\left(\int\frac{1}{x(a+bx)}dx, x, \left(F^{g(e+fx)}\right)^n\right)}{af^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^2}{2a^2d} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{d\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{d\text{Subst}\left(\int\frac{1}{x}dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} + \frac{(bd)\text{Subst}\left(\int\frac{1}{a+bx}dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^2}{2a^2d} - \frac{dx}{a^2fgn\log(F)} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{d\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)}$$

Mathematica [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}dx = \int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}dx$$

```
[In] Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2,x]
[Out] Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(189) = 378$.

Time = 0.13 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.09

method	result
risch	$\frac{dx+c}{af\left(a+b\left(F^{g(fx+e)}\right)^n\right)gn\ln(F)} + \frac{d\ln\left(F^{g(fx+e)}\right)^2}{2a^2g^2f^2\ln(F)^2} - \frac{d\ln\left(F^{g(fx+e)}\right)\ln\left(1+\frac{b\,F^{ngfx}F^{-ngfx}\left(F^{g(fx+e)}\right)^n}{a}\right)}{a^2n\,g^2f^2\ln(F)^2} - \frac{d\,\text{Li}_2\left(-\frac{b\,F^{ngfx}}{\ldots}\right.}{a^2n^2g\ldots}$

```
[In] int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
[Out] (d*x+c)/a/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)+1/2/a^2/g^2/f^2/ln(F)^2*d*ln(
F^(g*(f*x+e)))^2-1/a^2/n/g^2/f^2/ln(F)^2*d*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*
```

```
f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/a^2/n^2/g^2/f^2/ln(F)^2*d*polylog(
2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/a^2/n/g/f/ln(F)*c*ln((
F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/a^2/n/g/f/ln(F)*c*ln(F^(n*
g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/a^2/n^2/g^2/f^2/ln(F)^2*d*ln((F^(g
*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-1/a^2/n^2/g^2/f^2/ln(F)^2*d*ln(F
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-1/a^2/n/g/f/ln(F)*d*ln((F^(g*(f*
x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+1/a^2/n/g^2/f^2/ln(F)^2*d*ln((F^(g
*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+1/a^2/n/g/f/ln
(F)*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-1/a^2/n/g^2/f^2/ln(F
)^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(188) = 376$.

Time = 0.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx =$$

$$-\frac{2\,(ade-acf)gn\log(F)-(adf^2g^2n^2x^2+2\,acf^2g^2n^2x-(ade^2-2\,acef)g^2n^2)\log(F)^2-((bdf^2g^2n^2x^2+\cdots}{\cdots}$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*d*e - a*c*f)*g*n*log(F) - (a*d*f^2*g^2*n^2*x^2 + 2*a*c*f^2*g^2*n
^2*x - (a*d*e^2 - 2*a*c*e*f)*g^2*n^2)*log(F)^2 - ((b*d*f^2*g^2*n^2*x^2 + 2*
b*c*f^2*g^2*n^2*x - (b*d*e^2 - 2*b*c*e*f)*g^2*n^2)*log(F)^2 - 2*(b*d*f*g*n*
x + b*d*e*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 2*(F^(f*g*n*x + e*g*n)*b*d + a
*d)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 2*((a*d*e - a*c*f)*g*n*log(
F) + ((b*d*e - b*c*f)*g*n*log(F) + b*d)*F^(f*g*n*x + e*g*n) + a*d)*log(F^(f
*g*n*x + e*g*n)*b + a) + 2*((b*d*f*g*n*x + b*d*e*g*n)*F^(f*g*n*x + e*g*n)*l
og(F) + (a*d*f*g*n*x + a*d*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a
))/(F^(f*g*n*x + e*g*n)*a^2*b*f^2*g^2*n^2*log(F)^2 + a^3*f^2*g^2*n^2*log(F)
^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

```
[Out] d*(x/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*g*n*log(F)) + integrat
e((f*g*n*x*log(F) - 1)/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*g*n*
log(F)), x)) + c*((f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a
*b + a^2)*f*g*n*log(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))
)
```

Giac [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{c+d\,x}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

```
[In] int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2,x)
```

```
[Out] int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

3.55 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx = \frac{x}{a^2} + \frac{1}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2fgn\log(F)}$$

```
[Out] x/a^2+1/a/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)-ln(a+b*(F^(g*(f*x+e)))^n)/a^2
/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2320, 272, 46}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2} dx = -\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2fgn\log(F)} + \frac{x}{a^2} + \frac{1}{afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^(-2),x]
```

```
[Out] x/a^2 + 1/(a*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F]) - Log[a + b*(F^(g*(e
 + f*x)))^n]/(a^2*f*g*n*Log[F])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^n)^2}\,dx, x, F^{g(e+fx)}\right)}{fg\log(F)} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)} \\
&= \frac{x}{a^2}+\frac{1}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$
\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{\frac{1}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn}+\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^2fgn}-\frac{\log\left(a^3fgn\log(F)+a^2bf\left(F^{g(e+fx)}\right)^ngn\log(F)\right)}{a^2fgn}}{\log(F)}
$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-2),x]
```

```
[Out] (1/(a*f*(a + b*(F^(g*(e + f*x)))^n)*g*n) + Log[(F^(g*(e + f*x)))^n]/(a^2*f*
g*n) - Log[a^3*f*g*n*Log[F] + a^2*b*f*(F^(g*(e + f*x)))^n*g*n*Log[F]]/(a^2*
f*g*n))/Log[F]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}{a^{2}}+\frac{1}{a\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{a^{2}}}{gf\ln(F)n}$
default	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}{a^{2}}+\frac{1}{a\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{a^{2}}}{gf\ln(F)n}$
risch	$\frac{\ln\left(F^{g(fx+e)}\right)}{\ln(F)fg\,a^{2}}+\frac{1}{af\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)gn\ln(F)}-\frac{\ln\left(\left(F^{g(fx+e)}\right)^{n}+\frac{a}{b}\right)}{\ln(F)fgn\,a^{2}}$
parallelrisch	$\frac{b^{2}\left(F^{g(fx+e)}\right)^{n}x\ln(F)fgn+x\ln(F)abfgn-\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)\left(F^{g(fx+e)}\right)^{n}b^{2}-\ln\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)ab+ab}{\ln(F)a^{2}bfgn\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}$
norman	$\frac{-\frac{b\,e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{\ln(F)fgn\,a^{2}}+\frac{b\ln\left(e^{g(fx+e)\ln(F)}\right)e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{a^{2}\ln(F)fg}+\frac{\ln\left(e^{g(fx+e)\ln(F)}\right)}{\ln(F)afg}}{a+b\,e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}-\frac{\ln\left(a+b\,e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}\right)}{\ln(F)fgn\,a^{2}}$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/g/f/ln(F)/n*(-1/a^2*ln(a+b*(F^(g*(f*x+e)))^n)+1/a/(a+b*(F^(g*(f*x+e)))^n)
+1/a^2*ln((F^(g*(f*x+e)))^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{2}}\,dx = \frac{F^{fgnx+egn}bfgnx\log(F)+afgnx\log(F)-\left(F^{fgnx+egn}b+a\right)\log\left(F^{fgnx+egn}b+a\right)+a}{F^{fgnx+egn}a^{2}bfgn\log(F)+a^{3}fgn\log(F)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

```
[Out] (F^(f*g*n*x + e*g*n)*b*f*g*n*x*log(F) + a*f*g*n*x*log(F) - (F^(f*g*n*x + e*
g*n)*b + a)*log(F^(f*g*n*x + e*g*n)*b + a) + a)/(F^(f*g*n*x + e*g*n)*a^2*b*
f*g*n*log(F) + a^3*f*g*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

```
[Out] Integral((a + b*(F**(g*(e + f*x)))**n)**(-2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\frac{fgnx+egn}{a^2fgn}+\frac{1}{(F^{fgnx+egn}ab+a^2)fgn\log(F)}-\frac{\log\left(F^{fgnx+egn}b+a\right)}{a^2fgn\log(F)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

```
[Out] (f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a*b + a^2)*f*g*n*lo
g(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\frac{\log\left(|F|^{fgnx}|F|^{egn}\right)}{a^2fgn\log(F)}-\frac{\log\left(\left|F^{fgnx}F^{egn}b+a\right|\right)}{a^2fgn\log(F)}+\frac{1}{(F^{fgnx}F^{egn}b+a)afgn\log(F)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

```
[Out] log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))/(a^2*f*g*n*log(F)) - log(abs(F^(f*g*n*
x)*F^(e*g*n)*b + a))/(a^2*f*g*n*log(F)) + 1/((F^(f*g*n*x)*F^(e*g*n)*b + a)*
a*f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$=\frac{x}{a^2}+\frac{1}{a\,f\,g\,n\,\ln(F)\,\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)}-\frac{\ln\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)}{a^2\,f\,g\,n\,\ln(F)}$$

```
[In] int(1/(a + b*(F^(g*(e + f*x)))^n)^2,x)
[Out] x/a^2 + 1/(a*f*g*n*log(F)*(a + b*(F^(f*g*x)*F^(e*g))^n)) - log(a + b*(F^(f*
g*x)*F^(e*g))^n)/(a^2*f*g*n*log(F))
```

3.56 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)}\,dx = \operatorname{Int}\left(\frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*(F^(f*g*x+e*g))^n)^2/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)}\,dx$$

```
[In] Int[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)),x]
```

```
[Out] Defer[Int][1/((a + b*(F^(e*g + f*g*x))^n)^2*(c + d*x)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx$$

```
[In] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)),x]
[Out] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)), x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2(dx+c)}dx$$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
[Out] int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*(F^(f*g*x + e*g))^(2*n) + 2
*(a*b*d*x + a*b*c)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2(c+dx)}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c),x)
[Out] Integral(1/((a + b*(F**(e*g + f*g*x))**n)**2*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 7.68

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="maxima")
[Out] 1/(a^2*d*f*g*n*x*log(F) + a^2*c*f*g*n*log(F) + (F^(e*g*n)*a*b*d*f*g*n*x*log
(F) + F^(e*g*n)*a*b*c*f*g*n*log(F))*F^(f*g*n*x)) + integrate((d*f*g*n*x*log
(F) + c*f*g*n*log(F) + d)/(a^2*d^2*f*g*n*x^2*log(F) + 2*a^2*c*d*f*g*n*x*log
(F) + a^2*c^2*f*g*n*log(F) + (F^(e*g*n)*a*b*d^2*f*g*n*x^2*log(F) + 2*F^(e*g
*n)*a*b*c*d*f*g*n*x*log(F) + F^(e*g*n)*a*b*c^2*f*g*n*log(F))*F^(f*g*n*x)),
x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(1/(((F^((f*x + e)*g))^n*b + a)^2*(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2\,(c+d\,x)}\,dx$$

```
[In] int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)),x)
[Out] int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)), x)
```

3.57 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2}\,dx=\text{Int}\left(\frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2(c+dx)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*(F^(f*g*x+e*g))^n)^2/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2}\,dx$$

```
[In] Int[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2),x]
```

```
[Out] Defer[Int][1/((a + b*(F^(e*g + f*g*x))^n)^2*(c + d*x)^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2(c+dx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx$$

```
[In] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2),x]
[Out] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2 (dx+c)^2}dx$$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
[Out] int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.32

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*(F^(f*g*x + e*g))^(2*n) + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c
^2)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 23.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)^2} dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c)**2,x)
[Out] Integral(1/((a + b*(F**(e*g + f*g*x))**n)**2*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 280, normalized size of antiderivative = 11.20

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^2} dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] 1/(a^2*d^2*f*g*n*x^2*log(F) + 2*a^2*c*d*f*g*n*x*log(F) + a^2*c^2*f*g*n*log(
F) + (F^(e*g*n)*a*b*d^2*f*g*n*x^2*log(F) + 2*F^(e*g*n)*a*b*c*d*f*g*n*x*log(
F) + F^(e*g*n)*a*b*c^2*f*g*n*log(F))*F^(f*g*n*x)) + integrate((d*f*g*n*x*lo
g(F) + c*f*g*n*log(F) + 2*d)/(a^2*d^3*f*g*n*x^3*log(F) + 3*a^2*c*d^2*f*g*n*
x^2*log(F) + 3*a^2*c^2*d*f*g*n*x*log(F) + a^2*c^3*f*g*n*log(F) + (F^(e*g*n)
*a*b*d^3*f*g*n*x^3*log(F) + 3*F^(e*g*n)*a*b*c*d^2*f*g*n*x^2*log(F) + 3*F^(e
*g*n)*a*b*c^2*d*f*g*n*x*log(F) + F^(e*g*n)*a*b*c^3*f*g*n*log(F))*F^(f*g*n*x
)), x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2} dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^2} dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(1/(((F^((f*x + e)*g))^n*b + a)^2*(d*x + c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2\,\left(c+d\,x\right)^2}\,dx$$

```
[In] int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2),x)
[Out] int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2), x)
```

3.58 $\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 594

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{(c+dx)^4}{4a^3d}+\frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn\log(F)}+\frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} + \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{3d^2(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{9d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{2a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{3d^3\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} + \frac{9d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{9d^3\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{6d^3\,\mathrm{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)}$$

```
[Out] 1/4*(d*x+c)^4/a^3/d+3/2*d*(d*x+c)^2/a^3/f^2/g^2/n^2/ln(F)^2-3/2*d*(d*x+c)^2
/a^2/f^2/(a+b*(F^(g*(f*x+e)))^n)/g^2/n^2/ln(F)^2-3/2*(d*x+c)^3/a^3/f/g/n/ln
(F)+1/2*(d*x+c)^3/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+(d*x+c)^3/a^2/f/(
a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)-3*d^2*(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/
a^3/f^3/g^3/n^3/ln(F)^3+9/2*d*(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f^2
```

```
/g^2/n^2/ln(F)^2-(d*x+c)^3*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f/g/n/ln(F)-3*d^
3*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^4/g^4/n^4/ln(F)^4+9*d^2*(d*x+c)*p
olylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^3/g^3/n^3/ln(F)^3-3*d*(d*x+c)^2*poly
log(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^2/g^2/n^2/ln(F)^2-9*d^3*polylog(3,-b*(F
^(g*(f*x+e)))^n/a)/a^3/f^4/g^4/n^4/ln(F)^4+6*d^2*(d*x+c)*polylog(3,-b*(F^(g
*(f*x+e)))^n/a)/a^3/f^3/g^3/n^3/ln(F)^3-6*d^3*polylog(4,-b*(F^(g*(f*x+e)))^
n/a)/a^3/f^4/g^4/n^4/ln(F)^4
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules

used = {2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222, 2317, 2438}

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{9d^2(c+dx)\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{6d^2(c+dx)\,\text{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{3d^2(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{9d(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{2a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^3fgn\log(F)} - \frac{3d^3\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} - \frac{9d^3\,\text{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} - \frac{6d^3\,\text{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^4}{4a^3d} - \frac{3d(c+dx)^2}{2a^2f^2g^2n^2\log^2(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)} + \frac{(c+dx)^3}{a^2fgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)} + \frac{(c+dx)^3}{2afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}$$

```
[In] Int[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

```
[Out] (c + d*x)^4/(4*a^3*d) + (3*d*(c + d*x)^2)/(2*a^3*f^2*g^2*n^2*Log[F]^2) - (3
*d*(c + d*x)^2)/(2*a^2*f^2*(a + b*(F^(g*(e + f*x)))^n)*g^2*n^2*Log[F]^2) -
(3*(c + d*x)^3)/(2*a^3*f*g*n*Log[F]) + (c + d*x)^3/(2*a*f*(a + b*(F^(g*(e +
 f*x)))^n)^2*g*n*Log[F]) + (c + d*x)^3/(a^2*f*(a + b*(F^(g*(e + f*x)))^n)*g
*n*Log[F]) - (3*d^2*(c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a^3*f^3*
```

```
g^3*n^3*Log[F]^3) + (9*d*(c + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(2
*a^3*f^2*g^2*n^2*Log[F]^2) - ((c + d*x)^3*Log[1 + (b*(F^(g*(e + f*x)))^n)/a
])/(a^3*f*g*n*Log[F]) - (3*d^3*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a
^3*f^4*g^4*n^4*Log[F]^4) + (9*d^2*(c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)
))^n)/a)])/(a^3*f^3*g^3*n^3*Log[F]^3) - (3*d*(c + d*x)^2*PolyLog[2, -((b*(F
^(g*(e + f*x)))^n)/a)])/(a^3*f^2*g^2*n^2*Log[F]^2) - (9*d^3*PolyLog[3, -((b
*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^4*g^4*n^4*Log[F]^4) + (6*d^2*(c + d*x)*Po
lyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^3*g^3*n^3*Log[F]^3) - (6*d^3
*PolyLog[4, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^4*g^4*n^4*Log[F]^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx}{a}$$

$$= \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a^2} - \frac{(3d)\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{2afgn\log(F)}$$

$$= \frac{(c+dx)^4}{4a^3d} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3} - \frac{(3d)\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{2a^2fgn\log(F)} - \frac{(3d)\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2fgn\log(F)} + \frac{(3bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{2a^2fgn\log(F)}$$

$$= \frac{(c+dx)^4}{4a^3d} - \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right) g^2n^2\log^2(F)} - \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} + \frac{(3d^2)\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2f^2g^2n^2\log^2(F)} + \frac{(3d)\int (c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3fgn\log(F)} + \frac{(3bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{2a^3fgn\log(F)} + \frac{(3bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3fgn\log(F)}$$

$$
\begin{aligned}
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\
&\quad - \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\
&\quad + \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{9d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{2a^3f^2g^2n^2\log^2(F)} \\
&\quad - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\
&\quad - \frac{(3d^2)\int(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^2g^2n^2\log^2(F)} \\
&\quad - \frac{(6d^2)\int(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^2g^2n^2\log^2(F)} \\
&\quad + \frac{(6d^2)\int(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^2g^2n^2\log^2(F)} - \frac{(3bd^2)\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3f^2g^2n^2\log^2(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\
&\quad - \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\
&\quad + \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{3d^2(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\
&\quad + \frac{9d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{2a^3f^2g^2n^2\log^2(F)} \\
&\quad - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} + \frac{9d^2(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\
&\quad - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\
&\quad + \frac{(3d^3)\int\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^3g^3n^3\log^3(F)} - \frac{(3d^3)\int\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^3g^3n^3\log^3(F)} \\
&\quad - \frac{(6d^3)\int\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^3g^3n^3\log^3(F)} - \frac{(6d^3)\int\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3f^3g^3n^3\log^3(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\
&\quad - \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\
&\quad + \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{3d^2(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\
&\quad + \frac{9d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{2a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\
&\quad + \frac{9d^2(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\
&\quad + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{(3d^3)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^3f^4g^4n^4\log^4(F)} \\
&\quad - \frac{(3d^3)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx^n}{a}\right)}{x}\,dx, x, F^{g(e+fx)}\right)}{a^3f^4g^4n^3\log^4(F)} \\
&\quad - \frac{(6d^3)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx^n}{a}\right)}{x}\,dx, x, F^{g(e+fx)}\right)}{a^3f^4g^4n^3\log^4(F)} \\
&\quad - \frac{(6d^3)\,\text{Subst}\left(\int\frac{\text{Li}_3\left(-\frac{bx^n}{a}\right)}{x}\,dx, x, F^{g(e+fx)}\right)}{a^3f^4g^4n^3\log^4(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} - \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\
&- \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\
&+ \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{3d^2(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\
&+ \frac{9d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{2a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\
&- \frac{3d^3\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} + \frac{9d^2(c+dx)\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\
&- \frac{3d(c+dx)^2\mathrm{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{9d^3\mathrm{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} \\
&+ \frac{6d^2(c+dx)\mathrm{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{6d^3\mathrm{Li}_4\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx
$$

```
[In] Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3,x]
[Out] Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4004 vs. $2(582) = 1164$.

Time = 0.44 (sec) , antiderivative size = 4005, normalized size of antiderivative = 6.74

method	result	size
risch	Expression too large to display	4005

```
[In] int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
[Out] -9/n^2/g^3/f^3/ln(F)^3/a^3*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f
*x)*b+a)*ln(F^(g*(f*x+e)))-9/n^2/g^2/f^2/ln(F)^2/a^3*c*d^2*ln(F^(n*g*f*x)*F
^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x+9/n^2/g^3/f^3/ln(F)^3/a^3*c*d^2*ln(F^(n*g*
f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))+9/n^2/g^3/f^3/ln(F)^
```

3/a^3*c*d^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))+9/n^2/g^3/f^3/ln(F)^3/a^3*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))*x+1/2*(2*ln(F)*b*d^3*f*g*n*x^3*(F^(g*(f*x+e)))^n+3*ln(F)*a*d^3*f*g*n*x^3+6*ln(F)*b*c*d^2*f*g*n*x^2*(F^(g*(f*x+e)))^n+9*ln(F)*a*c*d^2*f*g*n*x^2+6*ln(F)*b*c^2*d*f*g*n*x*(F^(g*(f*x+e)))^n+9*ln(F)*a*c^2*d*f*g*n*x+2*ln(F)*b*c^3*f*g*n*(F^(g*(f*x+e)))^n+3*ln(F)*a*c^3*f*g*n-3*b*d^3*x^2*(F^(g*(f*x+e)))^n-3*a*d^3*x^2-6*b*c*d^2*x*(F^(g*(f*x+e)))^n-6*a*c*d^2*x-3*b*c^2*d*(F^(g*(f*x+e)))^n-3*a*d*c^2)/n^2/g^2/f^2/ln(F)^2/a^2/(a+b*(F^(g*(f*x+e)))^n)^2+3/g^2/f^2/ln(F)^2/a^3*c*d^2*ln(F^(g*(f*x+e)))^2*x+6/n^3/g^3/f^3/ln(F)^3/a^3*c*d^2*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-9/2/n/g^3/f^3/ln(F)^3/a^3*d^3*ln(F^(g*(f*x+e)))^2*x-9/2/n^2/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2-9/2/n/g^3/f^3/ln(F)^3/a^3*c*d^2*ln(F^(g*(f*x+e)))^2-9/2/n^2/g^4/f^4/ln(F)^4/a^3*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))^2+9/2/n^2/g^2/f^2/ln(F)^2/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2+9/2/n^2/g^4/f^4/ln(F)^4/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2-9/2/n^2/g^2/f^2/ln(F)^2/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2-3/n^2/g^2/f^2/ln(F)^2/a^3*d^3*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x^2-3/n^2/g^2/f^2/ln(F)^2/a^3*c^2*d*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+9/2/n^2/g^2/f^2/ln(F)^2/a^3*c^2*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-9/2/n^2/g^2/f^2/ln(F)^2/a^3*c^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-3/n^3/g^3/f^3/ln(F)^3/a^3*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+3/n^3/g^3/f^3/ln(F)^3/a^3*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+9/n^3/g^3/f^3/ln(F)^3/a^3*c*d^2*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-3/n^3/g^4/f^4/ln(F)^4/a^3*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))+3/n^3/g^3/f^3/ln(F)^3/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-3/n^3/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-3/n^3/g^3/f^3/ln(F)^3/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+3/n^3/g^4/f^4/ln(F)^4/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+1/n/g/f/ln(F)/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^3-1/n/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^3+3/4/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(g*(f*x+e)))^4+3/2/g^2/f^2/ln(F)^2/a^3*d^3*ln(F^(g*(f*x+e)))^2*x^2-2/g^3/f^3/ln(F)^3/a^3*d^3*ln(F^(g*(f*x+e)))^3*x-6/n^4/g^4/f^4/ln(F)^4/a^3*d^3*polylog(4,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/n/g/f/ln(F)/a^3*c^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/n/g/f/ln(F)/a^3*c^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-9/n^4/g^4/f^4/ln(F)^4/a^3*d^3*polylog(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-3/n^4/g^4/f^4/ln(F)^4/a^3*d^3*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+3/2/n^2/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(g*(f*x+e)))^2+3/n/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(g*(f*x+e)))^3-2/g^3/f^3/ln(F)^3/a^3*c*d^2*ln(F^(g*(f*x+e)))^3+3/2/g^2/f^2/ln(F)^2/a^3*d*c^2*ln(F^(g*(f*x+e)))^2-6/n/g^2/f^2/ln(F)^2/a^3*c*d^2*ln(1+b*F^(n*g*f*

```
x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))*x+6/n/g^2/f^2/ln(F)^
2/a^3*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x
+e)))*x-6/n/g^2/f^2/ln(F)^2/a^3*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*
x+e)))^n)*ln(F^(g*(f*x+e)))*x-1/n/g^4/f^4/ln(F)^4/a^3*d^3*ln(1+b*F^(n*g*f*x
)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))^3-1/n/g/f/ln(F)/a^3*d
^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^3+1/n/g^4/f^4/ln(F)
^4/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+
e)))^3+9/n^3/g^3/f^3/ln(F)^3/a^3*d^3*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*
(F^(g*(f*x+e)))^n/a)*x+6/n^3/g^3/f^3/ln(F)^3/a^3*d^3*polylog(3,-b*F^(n*g*f*
x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-9/n^2/g^3/f^3/ln(F)^3/a^3*d^3*ln((F^
(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))*x+9/n^2/g^3/
f^3/ln(F)^3/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*
(f*x+e)))*x-6/n^2/g^2/f^2/ln(F)^2/a^3*c*d^2*polylog(2,-b*F^(n*g*f*x)*F^(-n*
g*f*x)*(F^(g*(f*x+e)))^n/a)*x+3/n/g^2/f^2/ln(F)^2/a^3*d^3*ln((F^(g*(f*x+e))
)^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))*x^2-3/n/g^3/f^3/ln(F)^3
/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)
))^2*x+3/n/g^3/f^3/ln(F)^3/a^3*c*d^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*
(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))^2-3/n/g/f/ln(F)/a^3*c*d^2*ln((F^(g*(f*x+e)
))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2-3/n/g^3/f^3/ln(F)^3/a^3*c*d^2*ln((F^
(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+3/n/g/f/ln
(F)/a^3*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2-3/n/g/f/ln
(F)/a^3*c^2*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+3/n/g/f/
ln(F)/a^3*c^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-3/n/g^2/f^
2/ln(F)^2/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f
*x+e)))*x^2+3/n/g^3/f^3/ln(F)^3/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(
f*x+e)))^n)*ln(F^(g*(f*x+e)))^2*x-3/n/g^2/f^2/ln(F)^2/a^3*d^3*ln(1+b*F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))*x^2+3/n/g^3/f^3/l
n(F)^3/a^3*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g
*(f*x+e)))^2*x-3/n/g^2/f^2/ln(F)^2/a^3*c^2*d*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x
)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))+3/n/g^2/f^2/ln(F)^2/a^3*c^2*d*ln((
F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))-3/n/g^2/f^
2/ln(F)^2/a^3*c^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*
(f*x+e)))+3/n/g^3/f^3/ln(F)^3/a^3*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(
f*x+e)))^n)*ln(F^(g*(f*x+e)))^2+9/n^2/g^2/f^2/ln(F)^2/a^3*c*d^2*ln((F^(g*(f
*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2704 vs. $2(579) = 1158$.

Time = 0.32 (sec) , antiderivative size = 2704, normalized size of antiderivative = 4.55

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)
*g^3*n^3*log(F)^3 + 6*(a^2*d^3*e^2 - 2*a^2*c*d^2*e*f + a^2*c^2*d*f^2)*g^2*n
^2*log(F)^2 - (a^2*d^3*f^4*g^4*n^4*x^4 + 4*a^2*c*d^2*f^4*g^4*n^4*x^3 + 6*a^
2*c^2*d*f^4*g^4*n^4*x^2 + 4*a^2*c^3*f^4*g^4*n^4*x - (a^2*d^3*e^4 - 4*a^2*c*
d^2*e^3*f + 6*a^2*c^2*d*e^2*f^2 - 4*a^2*c^3*e*f^3)*g^4*n^4)*log(F)^4 - ((b^
2*d^3*f^4*g^4*n^4*x^4 + 4*b^2*c*d^2*f^4*g^4*n^4*x^3 + 6*b^2*c^2*d*f^4*g^4*n
^4*x^2 + 4*b^2*c^3*f^4*g^4*n^4*x - (b^2*d^3*e^4 - 4*b^2*c*d^2*e^3*f + 6*b^2
*c^2*d*e^2*f^2 - 4*b^2*c^3*e*f^3)*g^4*n^4)*log(F)^4 - 6*(b^2*d^3*f^3*g^3*n^
3*x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3*x + (b^2*d^3*
e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2)*g^3*n^3)*log(F)^3 + 6*(b^2*d^3
*f^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x - (b^2*d^3*e^2 - 2*b^2*c*d^2*e
*f)*g^2*n^2)*log(F)^2)*F^(2*f*g*n*x + 2*e*g*n) - 2*((a*b*d^3*f^4*g^4*n^4*x^
4 + 4*a*b*c*d^2*f^4*g^4*n^4*x^3 + 6*a*b*c^2*d*f^4*g^4*n^4*x^2 + 4*a*b*c^3*f
^4*g^4*n^4*x - (a*b*d^3*e^4 - 4*a*b*c*d^2*e^3*f + 6*a*b*c^2*d*e^2*f^2 - 4*a
*b*c^3*e*f^3)*g^4*n^4)*log(F)^4 - 2*(2*a*b*d^3*f^3*g^3*n^3*x^3 + 6*a*b*c*d^
2*f^3*g^3*n^3*x^2 + 6*a*b*c^2*d*f^3*g^3*n^3*x + (3*a*b*d^3*e^3 - 9*a*b*c*d^
2*e^2*f + 9*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*g^3*n^3)*log(F)^3 + 3*(a*b*d^3*f
^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2*g^2*n^2*x - (2*a*b*d^3*e^2 - 4*a*b*c*d^2*e
*f + a*b*c^2*d*f^2)*g^2*n^2)*log(F)^2)*F^(f*g*n*x + e*g*n) + 12*(a^2*d^3 +
(a^2*d^3*f^2*g^2*n^2*x^2 + 2*a^2*c*d^2*f^2*g^2*n^2*x + a^2*c^2*d*f^2*g^2*n^
2)*log(F)^2 + (b^2*d^3 + (b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2
*x + b^2*c^2*d*f^2*g^2*n^2)*log(F)^2 - 3*(b^2*d^3*f*g*n*x + b^2*c*d^2*f*g*n
)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(a*b*d^3 + (a*b*d^3*f^2*g^2*n^2*x^2 +
 2*a*b*c*d^2*f^2*g^2*n^2*x + a*b*c^2*d*f^2*g^2*n^2)*log(F)^2 - 3*(a*b*d^3*f
*g*n*x + a*b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 3*(a^2*d^3*f*g*n*x
+ a^2*c*d^2*f*g*n)*log(F))*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 2*(2
*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*g^3*n^
3*log(F)^3 + 9*(a^2*d^3*e^2 - 2*a^2*c*d^2*e*f + a^2*c^2*d*f^2)*g^2*n^2*log(
F)^2 + 6*(a^2*d^3*e - a^2*c*d^2*f)*g*n*log(F) + (2*(b^2*d^3*e^3 - 3*b^2*c*d
^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*g^3*n^3*log(F)^3 + 9*(b^2*d^3*e
^2 - 2*b^2*c*d^2*e*f + b^2*c^2*d*f^2)*g^2*n^2*log(F)^2 + 6*(b^2*d^3*e - b^2
*c*d^2*f)*g*n*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(2*(a*b*d^3*e^3 - 3*a*b*c
*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*g^3*n^3*log(F)^3 + 9*(a*b*d^3
*e^2 - 2*a*b*c*d^2*e*f + a*b*c^2*d*f^2)*g^2*n^2*log(F)^2 + 6*(a*b*d^3*e - a
*b*c*d^2*f)*g*n*log(F))*F^(f*g*n*x + e*g*n))*log(F^(f*g*n*x + e*g*n)*b + a)
 + 2*(2*(a^2*d^3*f^3*g^3*n^3*x^3 + 3*a^2*c*d^2*f^3*g^3*n^3*x^2 + 3*a^2*c^2*
d*f^3*g^3*n^3*x + (a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2)*g^3
*n^3)*log(F)^3 - 9*(a^2*d^3*f^2*g^2*n^2*x^2 + 2*a^2*c*d^2*f^2*g^2*n^2*x - (
a^2*d^3*e^2 - 2*a^2*c*d^2*e*f)*g^2*n^2)*log(F)^2 + (2*(b^2*d^3*f^3*g^3*n^3*
x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3*x + (b^2*d^3*e^
3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2)*g^3*n^3)*log(F)^3 - 9*(b^2*d^3*f
^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x - (b^2*d^3*e^2 - 2*b^2*c*d^2*e*f
)*g^2*n^2)*log(F)^2 + 6*(b^2*d^3*f*g*n*x + b^2*d^3*e*g*n)*log(F))*F^(2*f*g*
n*x + 2*e*g*n) + 2*(2*(a*b*d^3*f^3*g^3*n^3*x^3 + 3*a*b*c*d^2*f^3*g^3*n^3*x^
2 + 3*a*b*c^2*d*f^3*g^3*n^3*x + (a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^
```

```
2*d*e*f^2)*g^3*n^3)*log(F)^3 - 9*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2
*g^2*n^2*x - (a*b*d^3*e^2 - 2*a*b*c*d^2*e*f)*g^2*n^2)*log(F)^2 + 6*(a*b*d^3
*f*g*n*x + a*b*d^3*e*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 6*(a^2*d^3*f*g*n*x
+ a^2*d^3*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 24*(2*F^(f*g*
n*x + e*g*n)*a*b*d^3 + F^(2*f*g*n*x + 2*e*g*n)*b^2*d^3 + a^2*d^3)*polylog(4
, -F^(f*g*n*x + e*g*n)*b/a) + 12*(3*a^2*d^3 + (3*b^2*d^3 - 2*(b^2*d^3*f*g*n
*x + b^2*c*d^2*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(3*a*b*d^3 - 2*(a
*b*d^3*f*g*n*x + a*b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 2*(a^2*d^3*
f*g*n*x + a^2*c*d^2*f*g*n)*log(F))*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(2
*F^(f*g*n*x + e*g*n)*a^4*b*f^4*g^4*n^4*log(F)^4 + F^(2*f*g*n*x + 2*e*g*n)*a
^3*b^2*f^4*g^4*n^4*log(F)^4 + a^5*f^4*g^4*n^4*log(F)^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{\left(c+dx\right)^3}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}\,dx$$

```
[In] integrate((d*x+c)**3/(a+b*(F**(g*(f*x+e)))**n)**3,x)
[Out] Integral((c + d*x)**3/(a + b*(F**(e*g + f*g*x))**n)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$= \frac{1}{2}c^3\left(\frac{2F^{fgnx+egn}b+3a}{(2F^{fgnx+egn}a^3b+F^{2fgnx+2egn}a^2b^2+a^4)fgn\log(F)}+\frac{2(fgnx+egn)}{a^3fgn}-\frac{2\log\left(F^{fgnx+egn}b+a\right)}{a^3fgn\log(F)}\right)$$
$$+\frac{3ad^3fgnx^3\log(F)-3ac^2d+3(3acd^2fgn\log(F)-ad^3)x^2+(2F^{egn}bd^3fgnx^3\log(F)-3F^{egn}bc^2d+}{2\left(2F^{fgnx}F^{egn}a^3bf^2g^2n^2\log(F)^2+\right.}$$
$$-\frac{3(3c^2dfgn\log(F)-2cd^2)x}{2a^3f^2g^2n^2\log(F)^2}+\frac{3(3c^2dfgn\log(F)-2cd^2)\log\left(F^{fgnx}F^{egn}b+a\right)}{2a^3f^3g^3n^3\log(F)^3}$$
$$-\frac{\left(f^3g^3n^3x^3\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^3+3f^2g^2n^2x^2\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)^2-6fgnx\log(F)\mathrm{Li}_3(-\right.}{a^3f^4g^4n^4\log(F)^4}$$
$$-\frac{3\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\mathrm{Li}_3(-\frac{F^{fgnx}F^{egn}b}{a})\right)(2c}{2a^3f^4g^4n^4\log(F)^4}$$
$$-\frac{3\left(c^2df^2g^2n^2\log(F)^2-3cd^2fgn\log(F)+d^3\right)\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right.}{a^3f^4g^4n^4\log(F)^4}$$
$$+\frac{d^3f^4g^4n^4x^4\log(F)^4+2(2cd^2fgn\log(F)-3d^3)f^3g^3n^3x^3\log(F)^3+6\left(c^2df^2g^2n^2\log(F)^2-3cd^2fgn\right.}{4a^3f^4g^4n^4\log(F)^4}$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*((2*F^(f*g*n*x + e*g*n)*b + 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F^
(2*f*g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*g*n*log(F)) + 2*(f*g*n*x + e*g*n)/(a
^3*f*g*n) - 2*log(F^(f*g*n*x + e*g*n)*b + a)/(a^3*f*g*n*log(F))) + 1/2*(3*a
*d^3*f*g*n*x^3*log(F) - 3*a*c^2*d + 3*(3*a*c*d^2*f*g*n*log(F) - a*d^3)*x^2
+ (2*F^(e*g*n)*b*d^3*f*g*n*x^3*log(F) - 3*F^(e*g*n)*b*c^2*d + 3*(2*F^(e*g*n
)*b*c*d^2*f*g*n*log(F) - F^(e*g*n)*b*d^3)*x^2 + 6*(F^(e*g*n)*b*c^2*d*f*g*n*
log(F) - F^(e*g*n)*b*c*d^2)*x)*F^(f*g*n*x) + 3*(3*a*c^2*d*f*g*n*log(F) - 2*
a*c*d^2)*x)/(2*F^(f*g*n*x)*F^(e*g*n)*a^3*b*f^2*g^2*n^2*log(F)^2 + F^(2*f*g*
n*x)*F^(2*e*g*n)*a^2*b^2*f^2*g^2*n^2*log(F)^2 + a^4*f^2*g^2*n^2*log(F)^2) -
 3/2*(3*c^2*d*f*g*n*log(F) - 2*c*d^2)*x/(a^3*f^2*g^2*n^2*log(F)^2) + 3/2*(3
*c^2*d*f*g*n*log(F) - 2*c*d^2)*log(F^(f*g*n*x)*F^(e*g*n)*b + a)/(a^3*f^3*g^
3*n^3*log(F)^3) - (f^3*g^3*n^3*x^3*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F
)^3 + 3*f^2*g^2*n^2*x^2*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F)^2 - 6*f*g*
n*x*log(F)*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a) + 6*polylog(4, -F^(f*g*n*
x)*F^(e*g*n)*b/a))*d^3/(a^3*f^4*g^4*n^4*log(F)^4) - 3/2*(f^2*g^2*n^2*x^2*lo
g(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*x)*F^
(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*(2*c*d^2*f*
g*n*log(F) - 3*d^3)/(a^3*f^4*g^4*n^4*log(F)^4) - 3*(c^2*d*f^2*g^2*n^2*log(F
)^2 - 3*c*d^2*f*g*n*log(F) + d^3)*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a +
```

```
1)*log(F) + dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a))/(a^3*f^4*g^4*n^4*log(F)^4) +
 1/4*(d^3*f^4*g^4*n^4*x^4*log(F)^4 + 2*(2*c*d^2*f*g*n*log(F) - 3*d^3)*f^3*g
^3*n^3*x^3*log(F)^3 + 6*(c^2*d*f^2*g^2*n^2*log(F)^2 - 3*c*d^2*f*g*n*log(F)
+ d^3)*f^2*g^2*n^2*x^2*log(F)^2)/(a^3*f^4*g^4*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{\left(dx+c\right)^3}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

```
[In] integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^3/((F^((f*x + e)*g))^n*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{\left(c+d\,x\right)^3}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}\,dx$$

```
[In] int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3,x)
[Out] int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3, x)
```

3.59 $\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 439

$$\begin{aligned}\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3} dx &= \frac{(c+dx)^3}{3a^3d} + \frac{d^2x}{a^3f^2g^2n^2\log^2(F)} \\ &- \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\ &- \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\ &+ \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\ &- \frac{d^2\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{3d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\ &- \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\ &+ \frac{3d^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\ &- \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\ &+ \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)}\end{aligned}$$

```
[Out] 1/3*(d*x+c)^3/a^3/d+d^2*x/a^3/f^2/g^2/n^2/ln(F)^2-d*(d*x+c)/a^2/f^2/(a+b*(F
^(g*(f*x+e)))^n)/g^2/n^2/ln(F)^2-3/2*(d*x+c)^2/a^3/f/g/n/ln(F)+1/2*(d*x+c)^
```

```
2/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+(d*x+c)^2/a^2/f/(a+b*(F^(g*(f*x+e
)))^n)/g/n/ln(F)-d^2*ln(a+b*(F^(g*(f*x+e)))^n)/a^3/f^3/g^3/n^3/ln(F)^3+3*d*
(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f^2/g^2/n^2/ln(F)^2-(d*x+c)^2*ln(1+
b*(F^(g*(f*x+e)))^n/a)/a^3/f/g/n/ln(F)+3*d^2*polylog(2,-b*(F^(g*(f*x+e)))^n
/a)/a^3/f^3/g^3/n^3/ln(F)^3-2*d*(d*x+c)*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a
^3/f^2/g^2/n^2/ln(F)^2+2*d^2*polylog(3,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^3/g^3/
n^3/ln(F)^3
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438, 272, 36, 29, 31}

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = -\frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{3d(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^3fgn\log(F)} + \frac{3d^2\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{d^2\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{3a^3d} + \frac{d^2x}{a^3f^2g^2n^2\log^2(F)} - \frac{d(c+dx)}{a^2f^2g^2n^2\log^2(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)} + \frac{(c+dx)^2}{a^2fgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)} + \frac{(c+dx)^2}{2afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}$$

```
[In] Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

```
[Out] (c + d*x)^3/(3*a^3*d) + (d^2*x)/(a^3*f^2*g^2*n^2*Log[F]^2) - (d*(c + d*x))/
(a^2*f^2*(a + b*(F^(g*(e + f*x)))^n)*g^2*n^2*Log[F]^2) - (3*(c + d*x)^2)/(2
*a^3*f*g*n*Log[F]) + (c + d*x)^2/(2*a*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*n*L
og[F]) + (c + d*x)^2/(a^2*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F]) - (d^2*
```

```
Log[a + b*(F^(g*(e + f*x)))^n])/(a^3*f^3*g^3*n^3*Log[F]^3) + (3*d*(c + d*x)
*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a^3*f^2*g^2*n^2*Log[F]^2) - ((c + d*x
)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a^3*f*g*n*Log[F]) + (3*d^2*PolyLog
[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^3*g^3*n^3*Log[F]^3) - (2*d*(c + d
*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^2*g^2*n^2*Log[F]^2) +
(2*d^2*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^3*g^3*n^3*Log[F]^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx}{a}$$

$$= \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a^2} - \frac{d\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{afgn\log(F)}$$

$$= \frac{(c+dx)^3}{3a^3d} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3} - \frac{d\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2fgn\log(F)} - \frac{(2d)\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2fgn\log(F)} + \frac{(bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a^2fgn\log(F)}$$

$$= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right) g^2n^2\log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} + \frac{d^2\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2f^2g^2n^2\log^2(F)} + \frac{(2d)\int (c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)\,dx}{a^3fgn\log(F)} + \frac{(bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3fgn\log(F)} + \frac{(2bd)\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3fgn\log(F)}$$

$$= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} + \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{2d(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{d^2\text{Subst}\left(\int\frac{1}{x(a+bx^n)}\,dx,x,F^{g(e+fx)}\right)}{a^2f^3g^3n^2\log^3(F)} - \frac{d^2\int\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^3f^2g^2n^2\log^2(F)} - \frac{(2d^2)\int\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^3f^2g^2n^2\log^2(F)} + \frac{(2d^2)\int\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^3f^2g^2n^2\log^2(F)}$$

$$= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} + \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{2d(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{d^2\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx,x,\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{(2d^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx,x,\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{d^2\text{Subst}\left(\int\frac{1}{x(a+bx)}\,dx,x,\left(F^{g(e+fx)}\right)^n\right)}{a^2f^3g^3n^3\log^3(F)} + \frac{(2d^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx^n}{a}\right)}{x}\,dx,x,F^{g(e+fx)}\right)}{a^3f^3g^3n^2\log^3(F)}$$

$$= \frac{(c+dx)^3}{3a^3d} - \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} + \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{3d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} + \frac{3d^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{2d(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{2d^2\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{d^2\text{Subst}\left(\int\frac{1}{x}\,dx,x,\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{(bd^2)\,\text{Subst}\left(\int\frac{1}{a+bx}\,dx,x,\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)}$$

$$= \frac{(c+dx)^3}{3a^3d} + \frac{d^2x}{a^3f^2g^2n^2\log^2(F)} - \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} - \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} + \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{d^2\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} + \frac{3d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} + \frac{3d^2\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} - \frac{2d(c+dx)\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{2d^2\text{Li}_3\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)}$$

Mathematica [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

```
[In] Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3,x]
[Out] Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1886 vs. 2(433) = 866.

Time = 0.43 (sec) , antiderivative size = 1887, normalized size of antiderivative = 4.30

method	result	size
risch	Expression too large to display	1887

```
[In] int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^3/ln(F)^3/f^3/g^3*d^2*ln(F^(g*(f*x+e)))^3-1/a^3/ln(F)/f/g/n*c^2*ln((
F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/a^3/ln(F)/f/g/n*c^2*ln(F^(
n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+2/a^3/ln(F)^3/f^3/g^3/n^3*d^2*poly
log(3,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+3/a^3/ln(F)^3/f^3/g^
3/n^3*d^2*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/a^3/
ln(F)^3/f^3/g^3/n^3*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+
1/a^3/ln(F)^3/f^3/g^3/n^3*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n
)-3/2/a^3/ln(F)^3/f^3/g^3/n*d^2*ln(F^(g*(f*x+e)))^2+1/a^3/ln(F)^2/f^2/g^2*c
*d*ln(F^(g*(f*x+e)))^2+1/a^3/ln(F)^2/f^2/g^2*d^2*ln(F^(g*(f*x+e)))^2*x+1/2*
(2*ln(F)*b*d^2*f*g*n*x^2*(F^(g*(f*x+e)))^n+3*ln(F)*a*d^2*f*g*n*x^2+4*ln(F)*
b*c*d*f*g*n*x*(F^(g*(f*x+e)))^n+6*ln(F)*a*c*d*f*g*n*x+2*ln(F)*b*c^2*f*g*n*(
F^(g*(f*x+e)))^n+3*ln(F)*a*c^2*f*g*n-2*b*d^2*x*(F^(g*(f*x+e)))^n-2*a*d^2*x-
2*b*c*d*(F^(g*(f*x+e)))^n-2*a*c*d)/n^2/g^2/f^2/ln(F)^2/a^2/(a+b*(F^(g*(f*x+
e)))^n)^2-3/a^3/ln(F)^2/f^2/g^2/n^2*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(
f*x+e)))^n)*x-3/a^3/ln(F)^3/f^3/g^3/n^2*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*
x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+3/a^3/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(n
*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))+3/a^3/ln(F)^3/f^3
/g^3/n^2*d^2*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e
)))^n/a)-1/a^3/ln(F)^3/f^3/g^3/n*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n
*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+1/a^3/ln(F)^3/f^3/g^3/n*d^2*ln(F^(n*g*f*x)
*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2+1/a^3/ln(F)^3/f^3/g^3/
n*d^2*ln(F^(g*(f*x+e)))^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n
/a)+3/a^3/ln(F)^2/f^2/g^2/n^2*c*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*
f*x)*b+a)-3/a^3/ln(F)^2/f^2/g^2/n^2*c*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(
f*x+e)))^n)-2/a^3/ln(F)^2/f^2/g^2/n^2*c*d*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*
f*x)*(F^(g*(f*x+e)))^n/a)-2/a^3/ln(F)^2/f^2/g^2/n^2*d^2*polylog(2,-b*F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-1/a^3/ln(F)/f/g/n*d^2*ln((F^(g*(f
*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2+1/a^3/ln(F)/f/g/n*d^2*ln(F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2+3/a^3/ln(F)^2/f^2/g^2/n^2*d^2*ln(
(F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x-2/a^3/ln(F)^2/f^2/g^2/n*d
^2*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x
+2/a^3/ln(F)^2/f^2/g^2/n*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*
b+a)*ln(F^(g*(f*x+e)))*x-2/a^3/ln(F)^2/f^2/g^2/n*d^2*ln(F^(n*g*f*x)*F^(-n*g
*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x-2/a^3/ln(F)/f/g/n*c*d*ln((F^(g
*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+2/a^3/ln(F)^2/f^2/g^2/n*c*d*ln
```

```
((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+2/a^3/ln
(F)/f/g/n*c*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-2/a^3/ln(F)^
2/f^2/g^2/n*c*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x
+e)))-2/a^3/ln(F)^2/f^2/g^2/n*c*d*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-
n*g*f*x)*(F^(g*(f*x+e)))^n/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(431) = 862$.

Time = 0.30 (sec) , antiderivative size = 1518, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(9*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*g^2*n^2*log(F)^2 + 6*(a^
2*d^2*e - a^2*c*d*f)*g*n*log(F) + 2*(a^2*d^2*f^3*g^3*n^3*x^3 + 3*a^2*c*d*f^
3*g^3*n^3*x^2 + 3*a^2*c^2*f^3*g^3*n^3*x + (a^2*d^2*e^3 - 3*a^2*c*d*e^2*f +
3*a^2*c^2*e*f^2)*g^3*n^3)*log(F)^3 + (2*(b^2*d^2*f^3*g^3*n^3*x^3 + 3*b^2*c*
d*f^3*g^3*n^3*x^2 + 3*b^2*c^2*f^3*g^3*n^3*x + (b^2*d^2*e^3 - 3*b^2*c*d*e^2*
f + 3*b^2*c^2*e*f^2)*g^3*n^3)*log(F)^3 - 9*(b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2
*c*d*f^2*g^2*n^2*x - (b^2*d^2*e^2 - 2*b^2*c*d*e*f)*g^2*n^2)*log(F)^2 + 6*(b
^2*d^2*f*g*n*x + b^2*d^2*e*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*(2*(a*b
*d^2*f^3*g^3*n^3*x^3 + 3*a*b*c*d*f^3*g^3*n^3*x^2 + 3*a*b*c^2*f^3*g^3*n^3*x
+ (a*b*d^2*e^3 - 3*a*b*c*d*e^2*f + 3*a*b*c^2*e*f^2)*g^3*n^3)*log(F)^3 - 3*(
2*a*b*d^2*f^2*g^2*n^2*x^2 + 4*a*b*c*d*f^2*g^2*n^2*x - (3*a*b*d^2*e^2 - 6*a*
b*c*d*e*f + a*b*c^2*f^2)*g^2*n^2)*log(F)^2 + 3*(a*b*d^2*f*g*n*x + (2*a*b*d^
2*e - a*b*c*d*f)*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 6*(3*a^2*d^2 + (3*b^2*d
^2 - 2*(b^2*d^2*f*g*n*x + b^2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) +
2*(3*a*b*d^2 - 2*(a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g
*n) - 2*(a^2*d^2*f*g*n*x + a^2*c*d*f*g*n)*log(F))*dilog(-(F^(f*g*n*x + e*g*
n)*b + a)/a + 1) - 6*((a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*g^2*n^2*l
og(F)^2 + a^2*d^2 + 3*(a^2*d^2*e - a^2*c*d*f)*g*n*log(F) + ((b^2*d^2*e^2 -
2*b^2*c*d*e*f + b^2*c^2*f^2)*g^2*n^2*log(F)^2 + b^2*d^2 + 3*(b^2*d^2*e - b^
2*c*d*f)*g*n*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*((a*b*d^2*e^2 - 2*a*b*c*d*
e*f + a*b*c^2*f^2)*g^2*n^2*log(F)^2 + a*b*d^2 + 3*(a*b*d^2*e - a*b*c*d*f)*g
*n*log(F))*F^(f*g*n*x + e*g*n))*log(F^(f*g*n*x + e*g*n)*b + a) - 6*((a^2*d^
2*f^2*g^2*n^2*x^2 + 2*a^2*c*d*f^2*g^2*n^2*x - (a^2*d^2*e^2 - 2*a^2*c*d*e*f)
*g^2*n^2)*log(F)^2 + ((b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2*c*d*f^2*g^2*n^2*x -
(b^2*d^2*e^2 - 2*b^2*c*d*e*f)*g^2*n^2)*log(F)^2 - 3*(b^2*d^2*f*g*n*x + b^2*
d^2*e*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*((a*b*d^2*f^2*g^2*n^2*x^2 +
2*a*b*c*d*f^2*g^2*n^2*x - (a*b*d^2*e^2 - 2*a*b*c*d*e*f)*g^2*n^2)*log(F)^2 -
 3*(a*b*d^2*f*g*n*x + a*b*d^2*e*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 3*(a^2*d
```

```
^2*f*g*n*x + a^2*d^2*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 12
*(2*F^(f*g*n*x + e*g*n)*a*b*d^2 + F^(2*f*g*n*x + 2*e*g*n)*b^2*d^2 + a^2*d^2
)*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(2*F^(f*g*n*x + e*g*n)*a^4*b*f^3*g^
3*n^3*log(F)^3 + F^(2*f*g*n*x + 2*e*g*n)*a^3*b^2*f^3*g^3*n^3*log(F)^3 + a^5
*f^3*g^3*n^3*log(F)^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+dx)^2}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}\,dx$$

```
[In] integrate((d*x+c)**2/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

```
[Out] Integral((c + d*x)**2/(a + b*(F**(e*g + f*g*x))**n)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$
$$= \frac{1}{2}c^2\left(\frac{2F^{fgnx+egn}b+3a}{(2F^{fgnx+egn}a^3b+F^{2fgnx+2egn}a^2b^2+a^4)fgn\log(F)} + \frac{2(fgnx+egn)}{a^3fgn} - \frac{2\log\left(F^{fgnx+egn}b+a\right)}{a^3fgn\log(F)}\right)$$
$$+ \frac{3ad^2fgnx^2\log(F) - 2acd + 2(F^{egn}bd^2fgnx^2\log(F) - F^{egn}bcd + (2F^{egn}bcdfgn\log(F) - F^{egn}bd^2)x)F}{2\left(2F^{fgnx}F^{egn}a^3bf^2g^2n^2\log(F)^2 + F^{2fgnx}F^{2egn}a^2b^2f^2g^2n^2\log(F)^2 + a^4f^2g^2n\right.}$$
$$- \frac{(3cdfgn\log(F) - d^2)x}{a^3f^2g^2n^2\log(F)^2}$$
$$- \frac{\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2 + 2fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F) - 2\mathrm{Li}_3\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)d^2}{a^3f^3g^3n^3\log(F)^3}$$
$$- \frac{(2cdfgn\log(F) - 3d^2)\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)}{a^3f^3g^3n^3\log(F)^3}$$
$$+ \frac{(3cdfgn\log(F) - d^2)\log\left(F^{fgnx}F^{egn}b+a\right)}{a^3f^3g^3n^3\log(F)^3}$$
$$+ \frac{2d^2f^3g^3n^3x^3\log(F)^3 + 3(2cdfgn\log(F) - 3d^2)f^2g^2n^2x^2\log(F)^2}{6a^3f^3g^3n^3\log(F)^3}$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*((2*F^(f*g*n*x + e*g*n)*b + 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F^
(2*f*g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*g*n*log(F)) + 2*(f*g*n*x + e*g*n)/(a
^3*f*g*n) - 2*log(F^(f*g*n*x + e*g*n)*b + a)/(a^3*f*g*n*log(F))) + 1/2*(3*a
*d^2*f*g*n*x^2*log(F) - 2*a*c*d + 2*(F^(e*g*n)*b*d^2*f*g*n*x^2*log(F) - F^(
e*g*n)*b*c*d + (2*F^(e*g*n)*b*c*d*f*g*n*log(F) - F^(e*g*n)*b*d^2)*x)*F^(f*g
*n*x) + 2*(3*a*c*d*f*g*n*log(F) - a*d^2)*x)/(2*F^(f*g*n*x)*F^(e*g*n)*a^3*b*
f^2*g^2*n^2*log(F)^2 + F^(2*f*g*n*x)*F^(2*e*g*n)*a^2*b^2*f^2*g^2*n^2*log(F)
^2 + a^4*f^2*g^2*n^2*log(F)^2) - (3*c*d*f*g*n*log(F) - d^2)*x/(a^3*f^2*g^2*
n^2*log(F)^2) - (f^2*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^
2 + 2*f*g*n*x*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f
*g*n*x)*F^(e*g*n)*b/a))*d^2/(a^3*f^3*g^3*n^3*log(F)^3) - (2*c*d*f*g*n*log(F
) - 3*d^2)*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^(f
*g*n*x)*F^(e*g*n)*b/a))/(a^3*f^3*g^3*n^3*log(F)^3) + (3*c*d*f*g*n*log(F) -
d^2)*log(F^(f*g*n*x)*F^(e*g*n)*b + a)/(a^3*f^3*g^3*n^3*log(F)^3) + 1/6*(2*d
^2*f^3*g^3*n^3*x^3*log(F)^3 + 3*(2*c*d*f*g*n*log(F) - 3*d^2)*f^2*g^2*n^2*x^
2*log(F)^2)/(a^3*f^3*g^3*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx=\int \frac{(dx+c)^2}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

```
[In] integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^2/((F^((f*x + e)*g))^n*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx=\int \frac{(c+d\,x)^2}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}\,dx$$

```
[In] int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3,x)
[Out] int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3, x)
```

3.60 $\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\begin{aligned}\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx &= \frac{(c+dx)^2}{2a^3d}-\frac{d}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)}\\
&-\frac{3dx}{2a^3fgn\log(F)}+\frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)}\\
&+\frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)}+\frac{3d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{2a^3f^2g^2n^2\log^2(F)}\\
&-\frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)}-\frac{d\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)}\end{aligned}$$

```
[Out] 1/2*(d*x+c)^2/a^3/d-1/2*d/a^2/f^2/(a+b*(F^(g*(f*x+e)))^n)/g^2/n^2/ln(F)^2-3
/2*d*x/a^3/f/g/n/ln(F)+1/2*(d*x+c)/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+
(d*x+c)/a^2/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)+3/2*d*ln(a+b*(F^(g*(f*x+e))
)^n)/a^3/f^2/g^2/n^2/ln(F)^2-(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f/g/n/
ln(F)-d*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^2/g^2/n^2/ln(F)^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2216, 2215, 2221, 2317, 2438, 2222, 2320, 272, 36, 29, 31, 46}

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = -\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{a^3fgn\log(F)}-\frac{d\,\text{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)}+\frac{3d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{2a^3f^2g^2n^2\log^2(F)}+\frac{(c+dx)^2}{2a^3d}-\frac{3dx}{2a^3fgn\log(F)}+\frac{c+dx}{a^2fgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}-\frac{d}{2a^2f^2g^2n^2\log^2(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}+\frac{c+dx}{2afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}$$

```
[In] Int[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

```
[Out] (c + d*x)^2/(2*a^3*d) - d/(2*a^2*f^2*(a + b*(F^(g*(e + f*x)))^n)*g^2*n^2*Lo
g[F]^2) - (3*d*x)/(2*a^3*f*g*n*Log[F]) + (c + d*x)/(2*a*f*(a + b*(F^(g*(e +
 f*x)))^n)^2*g*n*Log[F]) + (c + d*x)/(a^2*f*(a + b*(F^(g*(e + f*x)))^n)*g*n
*Log[F]) + (3*d*Log[a + b*(F^(g*(e + f*x)))^n])/(2*a^3*f^2*g^2*n^2*Log[F]^2
) - ((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(a^3*f*g*n*Log[F]) - (d*
PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(a^3*f^2*g^2*n^2*Log[F]^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx}{a}\\
&= \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2}\\
&\quad - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{a^2} - \frac{d\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx}{2afgn\log(F)}\\
&= \frac{(c+dx)^2}{2a^3d} + \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)}\\
&\quad - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^3} - \frac{d\text{Subst}\left(\int \frac{1}{x(a+bx^n)^2}\,dx, x, F^{g(e+fx)}\right)}{2af^2g^2n\log^2(F)} - \frac{d\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx}{a^2fgn\log(F)}\\
&= \frac{(c+dx)^2}{2a^3d} + \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)}\\
&\quad - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{d\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{2af^2g^2n^2\log^2(F)}\\
&\quad - \frac{d\text{Subst}\left(\int \frac{1}{x(a+bx^n)}\,dx, x, F^{g(e+fx)}\right)}{a^2f^2g^2n\log^2(F)} + \frac{d\int \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{a^3fgn\log(F)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(c+dx)^2}{2a^3d} + \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} \\
&+ \frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\
&+ \frac{d\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^3f^2g^2n^2\log^2(F)} - \frac{d\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} \\
&- \frac{d\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{2af^2g^2n^2\log^2(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c+dx)^2}{2a^3d} - \frac{d}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right) g^2n^2\log^2(F)} - \frac{dx}{2a^3fgn\log(F)} \\
&+ \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} \\
&+ \frac{d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{2a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{d\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\
&- \frac{d\text{Subst}\left(\int \frac{1}{x}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^3f^2g^2n^2\log^2(F)} + \frac{(bd)\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{a^3f^2g^2n^2\log^2(F)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(c+dx)^2}{2a^3d} - \frac{d}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right) g^2n^2\log^2(F)} - \frac{3dx}{2a^3fgn\log(F)} \\
&+ \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} \\
&+ \frac{3d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{2a^3f^2g^2n^2\log^2(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} - \frac{d\text{Li}_2\left(-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx
$$

```
[In] Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3,x]
[Out] Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(266) = 532.

Time = 0.28 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.42

method	result
risch	$\frac{2\left(F^{g(fx+e)}\right)^{n}\ln(F)bdfgnx+3\ln(F)adfgnx+2\left(F^{g(fx+e)}\right)^{n}\ln(F)bcfgn+3c\ln(F)afgn-\left(F^{g(fx+e)}\right)^{n}bd-ad}{2n^{2}g^{2}f^{2}\ln(F)^{2}a^{2}\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^{2}}+\frac{3d\ln\left(\left(F^{g(fx+e)}\right)^{n}\right.}{2\ln(F)^{2}}$

```
[In] int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*(F^(g*(f*x+e)))^n*ln(F)*b*d*f*g*n*x+3*ln(F)*a*d*f*g*n*x+2*(F^(g*(f*x
+e)))^n*ln(F)*b*c*f*g*n+3*c*ln(F)*a*f*g*n-(F^(g*(f*x+e)))^n*b*d-a*d)/n^2/g^
2/f^2/ln(F)^2/a^2/(a+b*(F^(g*(f*x+e)))^n)^2+3/2/ln(F)^2/f^2/g^2/n^2/a^3*d*l
n((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-3/2/ln(F)^2/f^2/g^2/n^2/a
^3*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/2/ln(F)^2/f^2/g^2/a^3
*d*ln(F^(g*(f*x+e)))^2-1/ln(F)^2/f^2/g^2/n/a^3*d*ln(F^(g*(f*x+e)))*ln(1+b*F
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F)^2/f^2/g^2/n^2/a^3*d*po
lylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F)/f/g/n/a^3*
d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+1/ln(F)^2/f^2/g^2/n/
a^3*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+
1/ln(F)/f/g/n/a^3*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-1/ln(F
)^2/f^2/g^2/n/a^3*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*
(f*x+e)))-1/ln(F)/f/g/n/a^3*c*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)
*b+a)+1/ln(F)/f/g/n/a^3*c*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. 2(265) = 530.

Time = 0.26 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.52

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}}\,dx =$$

$$-\frac{3\left(a^{2}de-a^{2}cf\right)gn\log\left(F\right)+a^{2}d-\left(a^{2}df^{2}g^{2}n^{2}x^{2}+2\,a^{2}cf^{2}g^{2}n^{2}x-\left(a^{2}de^{2}-2\,a^{2}cef\right)g^{2}n^{2}\right)\log\left(F\right)^{2}-\left(\right.}{}$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*(a^2*d*e - a^2*c*f)*g*n*log(F) + a^2*d - (a^2*d*f^2*g^2*n^2*x^2 + 2
*a^2*c*f^2*g^2*n^2*x - (a^2*d*e^2 - 2*a^2*c*e*f)*g^2*n^2)*log(F)^2 - ((b^2*
d*f^2*g^2*n^2*x^2 + 2*b^2*c*f^2*g^2*n^2*x - (b^2*d*e^2 - 2*b^2*c*e*f)*g^2*n
^2)*log(F)^2 - 3*(b^2*d*f*g*n*x + b^2*d*e*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g
*n) + (a*b*d - 2*(a*b*d*f^2*g^2*n^2*x^2 + 2*a*b*c*f^2*g^2*n^2*x - (a*b*d*e^
```

```
2 - 2*a*b*c*e*f)*g^2*n^2)*log(F)^2 + 2*(2*a*b*d*f*g*n*x + (3*a*b*d*e - a*b*
c*f)*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 2*(2*F^(f*g*n*x + e*g*n)*a*b*d + F^
(2*f*g*n*x + 2*e*g*n)*b^2*d + a^2*d)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a +
 1) - (2*(a^2*d*e - a^2*c*f)*g*n*log(F) + 3*a^2*d + (2*(b^2*d*e - b^2*c*f)*
g*n*log(F) + 3*b^2*d)*F^(2*f*g*n*x + 2*e*g*n) + 2*(2*(a*b*d*e - a*b*c*f)*g*
n*log(F) + 3*a*b*d)*F^(f*g*n*x + e*g*n))*log(F^(f*g*n*x + e*g*n)*b + a) + 2
*((b^2*d*f*g*n*x + b^2*d*e*g*n)*F^(2*f*g*n*x + 2*e*g*n)*log(F) + 2*(a*b*d*f
*g*n*x + a*b*d*e*g*n)*F^(f*g*n*x + e*g*n)*log(F) + (a^2*d*f*g*n*x + a^2*d*e
*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a))/(2*F^(f*g*n*x + e*g*n)*a^
4*b*f^2*g^2*n^2*log(F)^2 + F^(2*f*g*n*x + 2*e*g*n)*a^3*b^2*f^2*g^2*n^2*log(
F)^2 + a^5*f^2*g^2*n^2*log(F)^2)
```

Sympy [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{c+dx}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

```
[Out] Integral((c + d*x)/(a + b*(F**(e*g + f*g*x))**n)**3, x)
```

Maxima [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*d*((3*a*f*g*n*x*log(F) + (2*F^(e*g*n)*b*f*g*n*x*log(F) - F^(e*g*n)*b)*F
^(f*g*n*x) - a)/(2*F^(f*g*n*x)*F^(e*g*n)*a^3*b*f^2*g^2*n^2*log(F)^2 + F^(2*
f*g*n*x)*F^(2*e*g*n)*a^2*b^2*f^2*g^2*n^2*log(F)^2 + a^4*f^2*g^2*n^2*log(F)^
2) + 2*integrate(1/2*(2*f*g*n*x*log(F) - 3)/(F^(f*g*n*x)*F^(e*g*n)*a^2*b*f*
g*n*log(F) + a^3*f*g*n*log(F)), x)) + 1/2*c*((2*F^(f*g*n*x + e*g*n)*b + 3*a
)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F^(2*f*g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*
g*n*log(F)) + 2*(f*g*n*x + e*g*n)/(a^3*f*g*n) - 2*log(F^(f*g*n*x + e*g*n)*b
 + a)/(a^3*f*g*n*log(F)))
```

Giac [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

```
[In] integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
[Out] integrate((d*x + c)/((F^((f*x + e)*g))^n*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{c+d\,x}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}\,dx$$

```
[In] int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3,x)
[Out] int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3, x)
```

3.61 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3} dx = \frac{x}{a^3} + \frac{1}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{1}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3fgn\log(F)}$$

```
[Out] x/a^3+1/2/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+1/a^2/f/(a+b*(F^(g*(f*x+e
)))^n)/g/n/ln(F)-ln(a+b*(F^(g*(f*x+e)))^n)/a^3/f/g/n/ln(F)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2320, 272, 46}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3} dx = -\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3fgn\log(F)} + \frac{x}{a^3} + \frac{1}{a^2fgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)} + \frac{1}{2afgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^(-3),x]
```

```
[Out] x/a^3 + 1/(2*a*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*n*Log[F]) + 1/(a^2*f*(a +
b*(F^(g*(e + f*x)))^n)*g*n*Log[F]) - Log[a + b*(F^(g*(e + f*x)))^n]/(a^3*f*
g*n*Log[F])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^n)^3}\,dx, x, F^{g(e+fx)}\right)}{fg\log(F)}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right)\,dx, x, \left(F^{g(e+fx)}\right)^n\right)}{fgn\log(F)}\\
&= \frac{x}{a^3} + \frac{1}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)}\\
&\quad + \frac{1}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3fgn\log(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{\frac{3a+2b\left(F^{g(e+fx)}\right)^n}{2a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn}+\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^3fgn}-\frac{\log\left(a^4fgn\log(F)+a^3bf\left(F^{g(e+fx)}\right)^ngn\log(F)\right)}{a^3fgn}}{\log(F)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-3),x]
```

```
[Out] ((3*a + 2*b*(F^(g*(e + f*x)))^n)/(2*a^2*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*n
) + Log[(F^(g*(e + f*x)))^n]/(a^3*f*g*n) - Log[a^4*f*g*n*Log[F] + a^3*b*f*(
F^(g*(e + f*x)))^n*g*n*Log[F]]/(a^3*f*g*n))/Log[F]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a^3}+\frac{1}{a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)}+\frac{1}{2a\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a^3}}{gf\ln(F)n}$
default	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a^3}+\frac{1}{a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)}+\frac{1}{2a\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a^3}}{gf\ln(F)n}$
risch	$\frac{\ln\left(F^{g(fx+e)}\right)}{\ln(F)a^3fg}+\frac{2b\left(F^{g(fx+e)}\right)^n+3a}{2\ln(F)fgn\,a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}-\frac{\ln\left(\left(F^{g(fx+e)}\right)^n+\frac{a}{b}\right)}{\ln(F)a^3fgn}$
norman	$\frac{\frac{b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{\ln(F)fgn\,a^2}+\frac{b^2x\,\mathrm{e}^{2n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{a^3}+\frac{x}{a}+\frac{2bx\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{a^2}+\frac{3}{2\ln(F)afgn}}{\left(a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}\right)^2}-\frac{\ln\left(a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)}\right.}\right.}{\ln(F)a^3fgn}$
parallelrisch	$\frac{2b^4\left(F^{g(fx+e)}\right)^{2n}x\ln(F)fgn+4b^3\left(F^{g(fx+e)}\right)^nx\ln(F)afgn+2x\ln(F)a^2b^2fgn-2\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)\left(F^{g(fx+e)}\right)^{2n}b^4-}{2\ln(F)a^3b^2fgn\left(a+b\left(F^{g(fx+\right.\right.}}$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/g/f/ln(F)/n*(-1/a^3*ln(a+b*(F^(g*(f*x+e)))^n)+1/a^2/(a+b*(F^(g*(f*x+e)))^
n)+1/2/a/(a+b*(F^(g*(f*x+e)))^n)^2+1/a^3*ln((F^(g*(f*x+e)))^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$
$$= \frac{2\,F^{2\,fgnx+2\,egn}b^2fgnx\log\left(F\right)+2\,a^2fgnx\log\left(F\right)+2\,(2\,abfgnx\log\left(F\right)+ab)F^{fgnx+egn}+3\,a^2-2\,(2\,F^{fgn}}{2\,(2\,F^{fgnx+egn}a^4bfgn\log\left(F\right)+F^{2\,fgnx+2\,egn}a^3b^2fgn\log\left(F\right)+a^5}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*F^(2*f*g*n*x + 2*e*g*n)*b^2*f*g*n*x*log(F) + 2*a^2*f*g*n*x*log(F) +
2*(2*a*b*f*g*n*x*log(F) + a*b)*F^(f*g*n*x + e*g*n) + 3*a^2 - 2*(2*F^(f*g*n*
x + e*g*n)*a*b + F^(2*f*g*n*x + 2*e*g*n)*b^2 + a^2)*log(F^(f*g*n*x + e*g*n)
*b + a))/(2*F^(f*g*n*x + e*g*n)*a^4*b*f*g*n*log(F) + F^(2*f*g*n*x + 2*e*g*n
)*a^3*b^2*f*g*n*log(F) + a^5*f*g*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

```
[Out] Integral((a + b*(F**(g*(e + f*x)))**n)**(-3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{2\,F^{fgnx+egn}b+3\,a}{2\,(2\,F^{fgnx+egn}a^3b+F^{2\,fgnx+2\,egn}a^2b^2+a^4)fgn\log\left(F\right)} + \frac{fgnx+egn}{a^3fgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{a^3fgn\log\left(F\right)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*F^(f*g*n*x + e*g*n)*b + 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F^(2*f*
g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*g*n*log(F)) + (f*g*n*x + e*g*n)/(a^3*f*g*
n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^3*f*g*n*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{\log\left(|F|^{fgnx}|F|^{egn}\right)}{a^3fgn\log(F)} - \frac{\log\left(\left|F^{fgnx}F^{egn}b+a\right|\right)}{a^3fgn\log(F)} + \frac{2\,F^{fgnx}F^{egn}ab+3\,a^2}{2\left(F^{fgnx}F^{egn}b+a\right)^2a^3fgn\log(F)}$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

```
[Out] log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))/(a^3*f*g*n*log(F)) - log(abs(F^(f*g*n*
x)*F^(e*g*n)*b + a))/(a^3*f*g*n*log(F)) + 1/2*(2*F^(f*g*n*x)*F^(e*g*n)*a*b
+ 3*a^2)/((F^(f*g*n*x)*F^(e*g*n)*b + a)^2*a^3*f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{x}{a^3} + \frac{1}{2\,a\,f\,g\,n\,\ln(F)\,\left(a^2+b^2\left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}+2\,a\,b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)} + \frac{1}{a^2\,f\,g\,n\,\ln(F)\,\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)} - \frac{\ln\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)}{a^3\,f\,g\,n\,\ln(F)}$$

```
[In] int(1/(a + b*(F^(g*(e + f*x)))^n)^3,x)
```

```
[Out] x/a^3 + 1/(2*a*f*g*n*log(F)*(a^2 + b^2*(F^(f*g*x)*F^(e*g))^(2*n) + 2*a*b*(F
^(f*g*x)*F^(e*g))^n)) + 1/(a^2*f*g*n*log(F)*(a + b*(F^(f*g*x)*F^(e*g))^n))
- log(a + b*(F^(f*g*x)*F^(e*g))^n)/(a^3*f*g*n*log(F))
```

3.62 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx=\operatorname{Int}\left(\frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3(c+dx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*(F^(f*g*x+e*g))^n)^3/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx$$

```
[In] Int[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)),x]
```

```
[Out] Defer[Int][1/((a + b*(F^(e*g + f*g*x))^n)^3*(c + d*x)), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3(c+dx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx$$

```
[In] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)),x]
[Out] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3(dx+c)}dx$$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
[Out] int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3(dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(1/(a^3*d*x + a^3*c + (b^3*d*x + b^3*c)*(F^(f*g*x + e*g))^(3*n) + 3
*(a*b^2*d*x + a*b^2*c)*(F^(f*g*x + e*g))^(2*n) + 3*(a^2*b*d*x + a^2*b*c)*(F
^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 12.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3 (c+dx)}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c),x)
[Out] Integral(1/((a + b*(F**(e*g + f*g*x))**n)**3*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 651, normalized size of antiderivative = 26.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="maxima")
[Out] 1/2*(3*a*d*f*g*n*x*log(F) + 3*a*c*f*g*n*log(F) + (2*F^(e*g*n)*b*d*f*g*n*x*l
og(F) + 2*F^(e*g*n)*b*c*f*g*n*log(F) + F^(e*g*n)*b*d)*F^(f*g*n*x) + a*d)/(a
^4*d^2*f^2*g^2*n^2*x^2*log(F)^2 + 2*a^4*c*d*f^2*g^2*n^2*x*log(F)^2 + a^4*c^
2*f^2*g^2*n^2*log(F)^2 + (F^(2*e*g*n)*a^2*b^2*d^2*f^2*g^2*n^2*x^2*log(F)^2
+ 2*F^(2*e*g*n)*a^2*b^2*c*d*f^2*g^2*n^2*x*log(F)^2 + F^(2*e*g*n)*a^2*b^2*c^
2*f^2*g^2*n^2*log(F)^2)*F^(2*f*g*n*x) + 2*(F^(e*g*n)*a^3*b*d^2*f^2*g^2*n^2*
x^2*log(F)^2 + 2*F^(e*g*n)*a^3*b*c*d*f^2*g^2*n^2*x*log(F)^2 + F^(e*g*n)*a^3
*b*c^2*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)) + integrate(1/2*(2*d^2*f^2*g^2*n^
2*x^2*log(F)^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 + 3*c*d*f*g*n*log(F) + 2*d^2 +
(4*c*d*f^2*g^2*n^2*log(F)^2 + 3*d^2*f*g*n*log(F))*x)/(a^3*d^3*f^2*g^2*n^2*x
^3*log(F)^2 + 3*a^3*c*d^2*f^2*g^2*n^2*x^2*log(F)^2 + 3*a^3*c^2*d*f^2*g^2*n^
2*x*log(F)^2 + a^3*c^3*f^2*g^2*n^2*log(F)^2 + (F^(e*g*n)*a^2*b*d^3*f^2*g^2*
n^2*x^3*log(F)^2 + 3*F^(e*g*n)*a^2*b*c*d^2*f^2*g^2*n^2*x^2*log(F)^2 + 3*F^(
e*g*n)*a^2*b*c^2*d*f^2*g^2*n^2*x*log(F)^2 + F^(e*g*n)*a^2*b*c^3*f^2*g^2*n^2
*log(F)^2)*F^(f*g*n*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(1/(((F^((f*x + e)*g))^n*b + a)^3*(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx=\int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3\,\left(c+d\,x\right)}\,dx$$

```
[In] int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)),x)
[Out] int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)), x)
```

3.63 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx=\text{Int}\left(\frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3(c+dx)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*(F^(f*g*x+e*g))^n)^3/(d*x+c)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx$$

```
[In] Int[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2),x]
```

```
[Out] Defer[Int][1/((a + b*(F^(e*g + f*g*x))^n)^3*(c + d*x)^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3(c+dx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx$$

```
[In] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2),x]
[Out] Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3 (dx+c)^2}dx$$

```
[In] int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
[Out] int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 6.36

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(1/(a^3*d^2*x^2 + 2*a^3*c*d*x + a^3*c^2 + (b^3*d^2*x^2 + 2*b^3*c*d*
x + b^3*c^2)*(F^(f*g*x + e*g))^(3*n) + 3*(a*b^2*d^2*x^2 + 2*a*b^2*c*d*x + a
*b^2*c^2)*(F^(f*g*x + e*g))^(2*n) + 3*(a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + a^2*
b*c^2)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 138.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3(c+dx)^2}\,dx$$

```
[In] integrate(1/(a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c)**2,x)
[Out] Integral(1/((a + b*(F**(e*g + f*g*x))**n)**3*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 807, normalized size of antiderivative = 32.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^nb+a\right)^3(dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] 1/2*(3*a*d*f*g*n*x*log(F) + 3*a*c*f*g*n*log(F) + 2*(F^(e*g*n)*b*d*f*g*n*x*l
og(F) + F^(e*g*n)*b*c*f*g*n*log(F) + F^(e*g*n)*b*d)*F^(f*g*n*x) + 2*a*d)/(a
^4*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*a^4*c*d^2*f^2*g^2*n^2*x^2*log(F)^2 + 3*
a^4*c^2*d*f^2*g^2*n^2*x*log(F)^2 + a^4*c^3*f^2*g^2*n^2*log(F)^2 + (F^(2*e*g
*n)*a^2*b^2*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*F^(2*e*g*n)*a^2*b^2*c*d^2*f^2*
g^2*n^2*x^2*log(F)^2 + 3*F^(2*e*g*n)*a^2*b^2*c^2*d*f^2*g^2*n^2*x*log(F)^2 +
 F^(2*e*g*n)*a^2*b^2*c^3*f^2*g^2*n^2*log(F)^2)*F^(2*f*g*n*x) + 2*(F^(e*g*n)
*a^3*b*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*F^(e*g*n)*a^3*b*c*d^2*f^2*g^2*n^2*x
^2*log(F)^2 + 3*F^(e*g*n)*a^3*b*c^2*d*f^2*g^2*n^2*x*log(F)^2 + F^(e*g*n)*a^
3*b*c^3*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)) + integrate((d^2*f^2*g^2*n^2*x^2
*log(F)^2 + c^2*f^2*g^2*n^2*log(F)^2 + 3*c*d*f*g*n*log(F) + 3*d^2 + (2*c*d*
f^2*g^2*n^2*log(F)^2 + 3*d^2*f*g*n*log(F))*x)/(a^3*d^4*f^2*g^2*n^2*x^4*log(
F)^2 + 4*a^3*c*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 6*a^3*c^2*d^2*f^2*g^2*n^2*x^2
*log(F)^2 + 4*a^3*c^3*d*f^2*g^2*n^2*x*log(F)^2 + a^3*c^4*f^2*g^2*n^2*log(F)
^2 + (F^(e*g*n)*a^2*b*d^4*f^2*g^2*n^2*x^4*log(F)^2 + 4*F^(e*g*n)*a^2*b*c*d^
3*f^2*g^2*n^2*x^3*log(F)^2 + 6*F^(e*g*n)*a^2*b*c^2*d^2*f^2*g^2*n^2*x^2*log(
F)^2 + 4*F^(e*g*n)*a^2*b*c^3*d*f^2*g^2*n^2*x*log(F)^2 + F^(e*g*n)*a^2*b*c^4
*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx=\int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3(dx+c)^2}\,dx$$

```
[In] integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(1/(((F^((f*x + e)*g))^n*b + a)^3*(d*x + c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)^2}\,dx=\int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3\,\left(c+d\,x\right)^2}\,dx$$

```
[In] int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2),x)
[Out] int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2), x)
```

3.64 $\int (a+be^x)\sqrt{c+dx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int (a+be^x)\sqrt{c+dx}\,dx = be^x\sqrt{c+dx}+\frac{2a(c+dx)^{3/2}}{3d}-\frac{1}{2}b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)$$

```
[Out] 2/3*a*(d*x+c)^(3/2)/d-1/2*b*erfi((d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/ex
p(c/d)+b*exp(x)*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2214, 2207, 2211, 2235}

$$\int (a+be^x)\sqrt{c+dx}\,dx = \frac{2a(c+dx)^{3/2}}{3d}-\frac{1}{2}\sqrt{\pi}b\sqrt{d}e^{-\frac{c}{d}}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)+be^x\sqrt{c+dx}$$

```
[In] Int[(a + b*E^x)*Sqrt[c + d*x],x]
```

```
[Out] b*E^x*Sqrt[c + d*x] + (2*a*(c + d*x)^(3/2))/(3*d) - (b*Sqrt[d]*Sqrt[Pi]*Erf
i[Sqrt[c + d*x]/Sqrt[d]])/(2*E^(c/d))
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a\sqrt{c+dx}+be^x\sqrt{c+dx}\right)\,dx\\
&= \frac{2a(c+dx)^{3/2}}{3d}+b\int e^x\sqrt{c+dx}\,dx\\
&= be^x\sqrt{c+dx}+\frac{2a(c+dx)^{3/2}}{3d}-\frac{1}{2}(bd)\int\frac{e^x}{\sqrt{c+dx}}\,dx\\
&= be^x\sqrt{c+dx}+\frac{2a(c+dx)^{3/2}}{3d}-b\text{Subst}\left(\int e^{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)\\
&= be^x\sqrt{c+dx}+\frac{2a(c+dx)^{3/2}}{3d}-\frac{1}{2}b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$
\int (a+be^x)\sqrt{c+dx}\,dx = be^x\sqrt{c+dx}+\frac{2a(c+dx)^{3/2}}{3d}-\frac{1}{2}b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)
$$

```
[In] Integrate[(a + b*E^x)*Sqrt[c + d*x],x]
```

```
[Out] b*E^x*Sqrt[c + d*x] + (2*a*(c + d*x)^(3/2))/(3*d) - (b*Sqrt[d]*Sqrt[Pi]*Erf
i[Sqrt[c + d*x]/Sqrt[d]])/(2*E^(c/d))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\frac{2a(dx+c)^{\frac{3}{2}}}{3}+2b\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\,\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right)}{d}$	77
default	$\frac{\frac{2a(dx+c)^{\frac{3}{2}}}{3}+2b\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\,\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right)}{d}$	77
parts	$\frac{2a(dx+c)^{\frac{3}{2}}}{3d}+\frac{2b\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\,\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right)}{d}$	79

```
[In] int((a+b*exp(x))*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(1/3*a*(d*x+c)^(3/2)+b/exp(1/d*c)*(1/2*(d*x+c)^(1/2)*exp(1/d*(d*x+c))*d
-1/4*d*Pi^(1/2)/(-1/d)^(1/2)*erf((-1/d)^(1/2)*(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int (a+be^x)\sqrt{c+dx}\,dx = \frac{3\sqrt{\pi}bd^2\sqrt{-\frac{1}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}+2\left(2\,adx+3\,bde^x+2\,ac\right)\sqrt{dx+c}}{6\,d}$$

```
[In] integrate((a+b*exp(x))*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 1/6*(3*sqrt(pi)*b*d^2*sqrt(-1/d)*erf(sqrt(d*x + c)*sqrt(-1/d))*e^(-c/d) + 2
*(2*a*d*x + 3*b*d*e^x + 2*a*c)*sqrt(d*x + c))/d
```

Sympy [F]

$$\int (a+be^x)\sqrt{c+dx}\,dx = \int (a+be^x)\sqrt{c+dx}\,dx$$

```
[In] integrate((a+b*exp(x))*(d*x+c)**(1/2),x)
[Out] Integral((a + b*exp(x))*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int (a+be^x)\sqrt{c+dx}\,dx = \frac{4\,(dx+c)^{\frac{3}{2}}a - 3\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx+c}\,de^{\left(\frac{dx+c}{d}-\frac{c}{d}\right)}\right)b}{6\,d}$$

```
[In] integrate((a+b*exp(x))*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*(d*x + c)^(3/2)*a - 3*(sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-1/d))*e^(-
c/d)/sqrt(-1/d) - 2*sqrt(d*x + c)*d*e^((d*x + c)/d - c/d))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(53) = 106.

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

$$\int (a+be^x)\sqrt{c+dx}\,dx = -\frac{\frac{6\sqrt{\pi}bcd\,\mathrm{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} - 12\sqrt{dx+c}\,ac - 4\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)a - 3\left(\frac{\sqrt{\pi}(2\,c+d)d\,\mathrm{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e}{\sqrt{-d}}\right.}{6\,d}$$

```
[In] integrate((a+b*exp(x))*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(6*sqrt(pi)*b*c*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) - 1
2*sqrt(d*x + c)*a*c - 4*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a - 3*(sqrt(p
i)*(2*c + d)*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) + 2*sqrt(d*
x + c)*d*e^x)*b)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+be^x)\sqrt{c+dx}\,dx = \int (a+b\,e^x)\,\sqrt{c+d\,x}\,dx$$

```
[In] int((a + b*exp(x))*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*exp(x))*(c + d*x)^(1/2), x)
```

3.65 $\int (a+be^x)^2 \sqrt{c+dx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int (a+be^x)^2 \sqrt{c+dx} \, dx = 2abe^x\sqrt{c+dx} + \frac{1}{2}b^2e^{2x}\sqrt{c+dx} + \frac{2a^2(c+dx)^{3/2}}{3d} - ab\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) - \frac{1}{4}b^2\sqrt{d}e^{-\frac{2c}{d}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)$$

```
[Out] 2/3*a^2*(d*x+c)^(3/2)/d-1/8*b^2*erfi(2^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)
*2^(1/2)*Pi^(1/2)/exp(2*c/d)-a*b*erfi((d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/
2)/exp(c/d)+2*a*b*exp(x)*(d*x+c)^(1/2)+1/2*b^2*exp(2*x)*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2214, 2207, 2211, 2235}

$$\int (a+be^x)^2 \sqrt{c+dx} \, dx = \frac{2a^2(c+dx)^{3/2}}{3d} - \sqrt{\pi}ab\sqrt{d}e^{-\frac{c}{d}}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) + 2abe^x\sqrt{c+dx} - \frac{1}{4}\sqrt{\frac{\pi}{2}}b^2\sqrt{d}e^{-\frac{2c}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{1}{2}b^2e^{2x}\sqrt{c+dx}$$

```
[In] Int[(a + b*E^x)^2*Sqrt[c + d*x],x]
```

```
[Out] 2*a*b*E^x*Sqrt[c + d*x] + (b^2*E^(2*x)*Sqrt[c + d*x])/2 + (2*a^2*(c + d*x)^
(3/2))/(3*d) - (a*b*Sqrt[d]*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/E^(c/d) -
```

```
(b^2*Sqrt[d]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/Sqrt[d]])/(4*E^((2*c)
/d))
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2\sqrt{c+dx}+2abe^x\sqrt{c+dx}+b^2e^{2x}\sqrt{c+dx}\right)\,dx\\
&= \frac{2a^2(c+dx)^{3/2}}{3d}+(2ab)\int e^x\sqrt{c+dx}\,dx+b^2\int e^{2x}\sqrt{c+dx}\,dx\\
&= 2abe^x\sqrt{c+dx}+\frac{1}{2}b^2e^{2x}\sqrt{c+dx}+\frac{2a^2(c+dx)^{3/2}}{3d}\\
&\quad-(abd)\int\frac{e^x}{\sqrt{c+dx}}\,dx-\frac{1}{4}\left(b^2d\right)\int\frac{e^{2x}}{\sqrt{c+dx}}\,dx\\
&= 2abe^x\sqrt{c+dx}+\frac{1}{2}b^2e^{2x}\sqrt{c+dx}+\frac{2a^2(c+dx)^{3/2}}{3d}\\
&\quad-(2ab)\text{Subst}\left(\int e^{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)\\
&\quad-\frac{1}{2}b^2\text{Subst}\left(\int e^{-\frac{2c}{d}+\frac{2x^2}{d}}\,dx,x,\sqrt{c+dx}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2abe^x\sqrt{c+dx} + \frac{1}{2}b^2e^{2x}\sqrt{c+dx} + \frac{2a^2(c+dx)^{3/2}}{3d} \\
&\quad - ab\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) - \frac{1}{4}b^2\sqrt{d}e^{-\frac{2c}{d}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$
\begin{aligned}
&\int (a+be^x)^2\sqrt{c+dx}\,dx \\
&= -ab\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) \\
&\quad + \frac{4\sqrt{c+dx}(12abde^x+3b^2de^{2x}+4a^2(c+dx)) - 3b^2d^{3/2}e^{-\frac{2c}{d}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)}{24d}
\end{aligned}
$$

```
[In] Integrate[(a + b*E^x)^2*Sqrt[c + d*x],x]
```

```
[Out] -((a*b*Sqrt[d]*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/E^(c/d)) + (4*Sqrt[c +
 d*x]*(12*a*b*d*E^x + 3*b^2*d*E^(2*x) + 4*a^2*(c + d*x)) - (3*b^2*d^(3/2)*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/Sqrt[d]])/E^((2*c)/d))/(24*d)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{2a^2(dx+c)^{\frac{3}{2}}}{3}+2b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\,\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}}\right)+4ab\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}}{2}d-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\,\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right.}{d}$
default	$\frac{\frac{2a^2(dx+c)^{\frac{3}{2}}}{3}+2b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\,\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}}\right)+4ab\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}}{2}d-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\,\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right.}{d}$
parts	$\frac{2a^2(dx+c)^{\frac{3}{2}}}{3d}+\frac{2b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\,\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}}\right)}{d}+\frac{4ab\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}}{2}d-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\right.}{4\sqrt{-\frac{1}{d}}}\right.}{d}$

```
[In] int((a+b*exp(x))^2*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*a^2*(d*x+c)^(3/2)+b^2/exp(1/d*c)^2*(1/4*d*(d*x+c)^(1/2)*exp(2/d*(d
*x+c))-1/8*d*Pi^(1/2)/(-2/d)^(1/2)*erf((-2/d)^(1/2)*(d*x+c)^(1/2)))+2*a*b/e
xp(1/d*c)*(1/2*(d*x+c)^(1/2)*exp(1/d*(d*x+c))*d-1/4*d*Pi^(1/2)/(-1/d)^(1/2)
*erf((-1/d)^(1/2)*(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int (a+be^x)^2\sqrt{c+dx}\,dx$$
$$= \frac{3\sqrt{2}\sqrt{\pi}b^2d^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)}+24\sqrt{\pi}abd^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}+4\left(4a^2dx\right.}{24d}$$

```
[In] integrate((a+b*exp(x))^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*sqrt(pi)*b^2*d^2*sqrt(-1/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
-1/d))*e^(-2*c/d) + 24*sqrt(pi)*a*b*d^2*sqrt(-1/d)*erf(sqrt(d*x + c)*sqrt(-
1/d))*e^(-c/d) + 4*(4*a^2*d*x + 3*b^2*d*e^(2*x) + 12*a*b*d*e^x + 4*a^2*c)*s
qrt(d*x + c))/d
```

Sympy [F]

$$\int (a+be^x)^2\sqrt{c+dx}\,dx=\int (a+be^x)^2\sqrt{c+dx}\,dx$$

```
[In] integrate((a+b*exp(x))**2*(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*exp(x))**2*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int (a+be^x)^2\sqrt{c+dx}\,dx$$
$$= \frac{16(dx+c)^{\frac{3}{2}}a^2-24\left(\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}}-2\sqrt{dx+c}de^{\left(\frac{dx+c}{d}-\frac{c}{d}\right)}\right)ab-3\left(\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-\frac{1}{d}}}\right.}{24d}$$

```
[In] integrate((a+b*exp(x))^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(16*(d*x + c)^(3/2)*a^2 - 24*(sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-1/d))
*e^(-c/d)/sqrt(-1/d) - 2*sqrt(d*x + c)*d*e^((d*x + c)/d - c/d))*a*b - 3*(sq
rt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-1/d))*e^(-2*c/d)/sqrt(-1/d
) - 4*sqrt(d*x + c)*d*e^(2*(d*x + c)/d - 2*c/d))*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(110) = 220.

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.72

$$\int (a+be^x)^2\sqrt{c+dx}\,dx = -\frac{\frac{12\sqrt{2}\sqrt{\pi}b^2cd\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-d}}+\frac{48\sqrt{\pi}abcd\,\mathrm{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}}-48\sqrt{dx+c}a^2c-16\left((dx+c)^{\frac{3}{2}}-3\sqrt{\cdots}\right.}{\cdots}$$

```
[In] integrate((a+b*exp(x))^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(12*sqrt(2)*sqrt(pi)*b^2*c*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d)*e
^(-2*c/d)/sqrt(-d) + 48*sqrt(pi)*a*b*c*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e^(
-c/d)/sqrt(-d) - 48*sqrt(d*x + c)*a^2*c - 16*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^2 - 24*(sqrt(pi)*(2*c + d)*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e^(-c
/d)/sqrt(-d) + 2*sqrt(d*x + c)*d*e^x)*a*b - 3*(sqrt(2)*sqrt(pi)*(4*c + d)*d
*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d)*e^(-2*c/d)/sqrt(-d) + 4*sqrt(d*x +
c)*d*e^(2*x))*b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+be^x)^2\sqrt{c+dx}\,dx = \int (a+b\,e^x)^2\sqrt{c+d\,x}\,dx$$

```
[In] int((a + b*exp(x))^2*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*exp(x))^2*(c + d*x)^(1/2), x)
```

3.66 $\int (a+be^x)^3 \sqrt{c+dx}\, dx$

Optimal result

Integrand size = 19, antiderivative size = 224

$$\int (a+be^x)^3 \sqrt{c+dx}\, dx = 3a^2be^x\sqrt{c+dx} + \frac{3}{2}ab^2e^{2x}\sqrt{c+dx} + \frac{1}{3}b^3e^{3x}\sqrt{c+dx} + \frac{2a^3(c+dx)^{3/2}}{3d} - \frac{3}{2}a^2b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) - \frac{3}{4}ab^2\sqrt{d}e^{-\frac{2c}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) - \frac{1}{6}b^3\sqrt{d}e^{-\frac{3c}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{c+dx}}{\sqrt{d}}\right)$$

```
[Out] 2/3*a^3*(d*x+c)^(3/2)/d-1/18*b^3*erfi(3^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2
)*3^(1/2)*Pi^(1/2)/exp(3*c/d)-3/8*a*b^2*erfi(2^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*d^(1/2)*2^(1/2)*Pi^(1/2)/exp(2*c/d)-3/2*a^2*b*erfi((d*x+c)^(1/2)/d^(1/2))*
d^(1/2)*Pi^(1/2)/exp(c/d)+3*a^2*b*exp(x)*(d*x+c)^(1/2)+3/2*a*b^2*exp(2*x)*(
d*x+c)^(1/2)+1/3*b^3*exp(3*x)*(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

= {2214, 2207, 2211, 2235}

$$\int (a+be^x)^3\sqrt{c+dx}\,dx = \frac{2a^3(c+dx)^{3/2}}{3d} - \frac{3}{2}\sqrt{\pi}a^2b\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) + 3a^2be^x\sqrt{c+dx} - \frac{3}{4}\sqrt{\frac{\pi}{2}}ab^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{3}{2}ab^2e^{2x}\sqrt{c+dx} - \frac{1}{6}\sqrt{\frac{\pi}{3}}b^3\sqrt{d}e^{-\frac{3c}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{1}{3}b^3e^{3x}\sqrt{c+dx}$$

```
[In] Int[(a + b*E^x)^3*Sqrt[c + d*x],x]
```

```
[Out] 3*a^2*b*E^x*Sqrt[c + d*x] + (3*a*b^2*E^(2*x)*Sqrt[c + d*x])/2 + (b^3*E^(3*x
)*Sqrt[c + d*x])/3 + (2*a^3*(c + d*x)^(3/2))/(3*d) - (3*a^2*b*Sqrt[d]*Sqrt[
Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/(2*E^(c/d)) - (3*a*b^2*Sqrt[d]*Sqrt[Pi/2]*
Erfi[(Sqrt[2]*Sqrt[c + d*x])/Sqrt[d]])/(4*E^((2*c)/d)) - (b^3*Sqrt[d]*Sqrt[
Pi/3]*Erfi[(Sqrt[3]*Sqrt[c + d*x])/Sqrt[d]])/(6*E^((3*c)/d))
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^3\sqrt{c+dx}+3a^2be^x\sqrt{c+dx}+3ab^2e^{2x}\sqrt{c+dx}+b^3e^{3x}\sqrt{c+dx}\right)\,dx\\
&= \frac{2a^3(c+dx)^{3/2}}{3d}+\left(3a^2b\right)\int e^x\sqrt{c+dx}\,dx+\left(3ab^2\right)\int e^{2x}\sqrt{c+dx}\,dx+b^3\int e^{3x}\sqrt{c+dx}\,dx\\
&= 3a^2be^x\sqrt{c+dx}+\frac{3}{2}ab^2e^{2x}\sqrt{c+dx}+\frac{1}{3}b^3e^{3x}\sqrt{c+dx}+\frac{2a^3(c+dx)^{3/2}}{3d}\\
&\quad-\frac{1}{2}\left(3a^2bd\right)\int\frac{e^x}{\sqrt{c+dx}}\,dx-\frac{1}{4}\left(3ab^2d\right)\int\frac{e^{2x}}{\sqrt{c+dx}}\,dx-\frac{1}{6}\left(b^3d\right)\int\frac{e^{3x}}{\sqrt{c+dx}}\,dx\\
&= 3a^2be^x\sqrt{c+dx}+\frac{3}{2}ab^2e^{2x}\sqrt{c+dx}+\frac{1}{3}b^3e^{3x}\sqrt{c+dx}\\
&\quad+\frac{2a^3(c+dx)^{3/2}}{3d}-\left(3a^2b\right)\text{Subst}\left(\int e^{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)\\
&\quad-\frac{1}{2}\left(3ab^2\right)\text{Subst}\left(\int e^{-\frac{2c}{d}+\frac{2x^2}{d}}\,dx,x,\sqrt{c+dx}\right)\\
&\quad-\frac{1}{3}b^3\text{Subst}\left(\int e^{-\frac{3c}{d}+\frac{3x^2}{d}}\,dx,x,\sqrt{c+dx}\right)\\
&= 3a^2be^x\sqrt{c+dx}+\frac{3}{2}ab^2e^{2x}\sqrt{c+dx}+\frac{1}{3}b^3e^{3x}\sqrt{c+dx}\\
&\quad+\frac{2a^3(c+dx)^{3/2}}{3d}-\frac{3}{2}a^2b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)\\
&\quad-\frac{3}{4}ab^2\sqrt{d}e^{-\frac{2c}{d}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)-\frac{1}{6}b^3\sqrt{d}e^{-\frac{3c}{d}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{c+dx}}{\sqrt{d}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$
\int (a+be^x)^3\sqrt{c+dx}\,dx =
$$

$$
-\frac{-12\sqrt{c+dx}\left(18a^2bde^x+9ab^2de^{2x}+2b^3de^{3x}+4a^3(c+dx)\right)+108a^2bd^{3/2}e^{-\frac{c}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)+27ab^2d^{3/}}{72d}
$$

```
[In] Integrate[(a + b*E^x)^3*Sqrt[c + d*x],x]
```

```
[Out] -1/72*(-12*Sqrt[c + d*x]*(18*a^2*b*d*E^x + 9*a*b^2*d*E^(2*x) + 2*b^3*d*E^(3
*x) + 4*a^3*(c + d*x)) + (108*a^2*b*d^(3/2)*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqr
t[d]])/E^(c/d) + (27*a*b^2*d^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/
Sqrt[d]])/E^((2*c)/d) + (4*b^3*d^(3/2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[c + d*
x])/Sqrt[d]])/E^((3*c)/d))/d
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{2a^3(dx+c)^{\frac{3}{2}}}{3}+2b^3\mathrm{e}^{-\frac{3c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{3dx+3c}{d}}}{6}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{3}{d}}\,\sqrt{dx+c}\right)}{12\sqrt{-\frac{3}{d}}}\right)+6a\,b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\,\sqrt{dx}\right.}{8\sqrt{-\frac{2}{d}}}\right.}{d}$
default	$\frac{\frac{2a^3(dx+c)^{\frac{3}{2}}}{3}+2b^3\mathrm{e}^{-\frac{3c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{3dx+3c}{d}}}{6}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{3}{d}}\,\sqrt{dx+c}\right)}{12\sqrt{-\frac{3}{d}}}\right)+6a\,b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\,\sqrt{dx}\right.}{8\sqrt{-\frac{2}{d}}}\right.}{d}$
parts	$\frac{2a^3(dx+c)^{\frac{3}{2}}}{3d}+\frac{2b^3\mathrm{e}^{-\frac{3c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{3dx+3c}{d}}}{6}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{3}{d}}\,\sqrt{dx+c}\right)}{12\sqrt{-\frac{3}{d}}}\right)}{d}+\frac{6a\,b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-}\right.}{8\sqrt{-}}\right.}{d}$

```
[In] int((a+b*exp(x))^3*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*a^3*(d*x+c)^(3/2)+b^3/exp(1/d*c)^3*(1/6*d*(d*x+c)^(1/2)*exp(3/d*(d
*x+c))-1/12*d*Pi^(1/2)/(-3/d)^(1/2)*erf((-3/d)^(1/2)*(d*x+c)^(1/2)))+3*a*b^
2/exp(1/d*c)^2*(1/4*d*(d*x+c)^(1/2)*exp(2/d*(d*x+c))-1/8*d*Pi^(1/2)/(-2/d)^
(1/2)*erf((-2/d)^(1/2)*(d*x+c)^(1/2)))+3*a^2*b/exp(1/d*c)*(1/2*(d*x+c)^(1/2
)*exp(1/d*(d*x+c))*d-1/4*d*Pi^(1/2)/(-1/d)^(1/2)*erf((-1/d)^(1/2)*(d*x+c)^(
1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int (a+be^x)^3\sqrt{c+dx}\,dx$$
$$=\frac{27\sqrt{2}\sqrt{\pi}ab^2d^2\sqrt{-\frac{1}{d}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)}+4\sqrt{3}\sqrt{\pi}b^3d^2\sqrt{-\frac{1}{d}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{3c}{d}\right)}+}{}$$

```
[In] integrate((a+b*exp(x))^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*(27*sqrt(2)*sqrt(pi)*a*b^2*d^2*sqrt(-1/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-1/d))*e^(-2*c/d) + 4*sqrt(3)*sqrt(pi)*b^3*d^2*sqrt(-1/d)*erf(sqrt(3)*sq
rt(d*x + c)*sqrt(-1/d))*e^(-3*c/d) + 108*sqrt(pi)*a^2*b*d^2*sqrt(-1/d)*erf(
sqrt(d*x + c)*sqrt(-1/d))*e^(-c/d) + 12*(4*a^3*d*x + 2*b^3*d*e^(3*x) + 9*a*
b^2*d*e^(2*x) + 18*a^2*b*d*e^x + 4*a^3*c)*sqrt(d*x + c))/d
```

Sympy [F]

$$\int (a+be^x)^3 \sqrt{c+dx}\,dx = \int (a+be^x)^3 \sqrt{c+dx}\,dx$$

```
[In] integrate((a+b*exp(x))**3*(d*x+c)**(1/2),x)
[Out] Integral((a + b*exp(x))**3*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

$$\int (a+be^x)^3 \sqrt{c+dx}\,dx = \frac{48\,(dx+c)^{\frac{3}{2}}a^3 - 108\left(\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx+c}de^{\left(\frac{dx+c}{d}-\frac{c}{d}\right)}\right)a^2b - 27\left(\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(\right.}}{\sqrt{-\frac{1}{d}}}\right.}{72\,d}$$

```
[In] integrate((a+b*exp(x))^3*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/72*(48*(d*x + c)^(3/2)*a^3 - 108*(sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-1/d)
)*e^(-c/d)/sqrt(-1/d) - 2*sqrt(d*x + c)*d*e^((d*x + c)/d - c/d))*a^2*b - 27
*(sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-1/d))*e^(-2*c/d)/sqrt(
-1/d) - 4*sqrt(d*x + c)*d*e^(2*(d*x + c)/d - 2*c/d))*a*b^2 - 4*(sqrt(3)*sqr
t(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-1/d))*e^(-3*c/d)/sqrt(-1/d) - 6*sqr
t(d*x + c)*d*e^(3*(d*x + c)/d - 3*c/d))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(168) = 336.

Time = 0.36 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.64

$$\int (a+be^x)^3 \sqrt{c+dx}\,dx = -\frac{\frac{108\sqrt{2}\sqrt{\pi}ab^2cd\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-d}} + \frac{24\sqrt{3}\sqrt{\pi}b^3cd\operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{3c}{d}\right)}}{\sqrt{-d}} + \frac{216\sqrt{\pi}a^2bcd\operatorname{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} - \cdots}{\cdots}$$

```
[In] integrate((a+b*exp(x))^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/72*(108*sqrt(2)*sqrt(pi)*a*b^2*c*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d
)*e^(-2*c/d)/sqrt(-d) + 24*sqrt(3)*sqrt(pi)*b^3*c*d*erf(-sqrt(3)*sqrt(d*x +
 c)*sqrt(-d)/d)*e^(-3*c/d)/sqrt(-d) + 216*sqrt(pi)*a^2*b*c*d*erf(-sqrt(d*x
+ c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) - 144*sqrt(d*x + c)*a^3*c - 48*((d*x + c
)^(3/2) - 3*sqrt(d*x + c)*c)*a^3 - 108*(sqrt(pi)*(2*c + d)*d*erf(-sqrt(d*x
+ c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) + 2*sqrt(d*x + c)*d*e^x)*a^2*b - 27*(sqr
t(2)*sqrt(pi)*(4*c + d)*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d)*e^(-2*c/d)
/sqrt(-d) + 4*sqrt(d*x + c)*d*e^(2*x))*a*b^2 - 4*(sqrt(3)*sqrt(pi)*(6*c + d
)*d*erf(-sqrt(3)*sqrt(d*x + c)*sqrt(-d)/d)*e^(-3*c/d)/sqrt(-d) + 6*sqrt(d*x
 + c)*d*e^(3*x))*b^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+be^x)^3 \sqrt{c+dx}\,dx = \int (a+b\,\mathrm{e}^x)^3 \sqrt{c+d\,x}\,dx$$

```
[In] int((a + b*exp(x))^3*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*exp(x))^3*(c + d*x)^(1/2), x)
```

3.67 $\int \frac{\sqrt{c+dx}}{a+be^x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\sqrt{c+dx}}{a+be^x} \, dx = \operatorname{Int}\left(\frac{\sqrt{c+dx}}{a+be^x}, x\right)$$

```
[Out] Unintegrable((d*x+c)^(1/2)/(a+b*exp(x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{c+dx}}{a+be^x} \, dx = \int \frac{\sqrt{c+dx}}{a+be^x} \, dx$$

```
[In] Int[Sqrt[c + d*x]/(a + b*E^x),x]
```

```
[Out] Defer[Int][Sqrt[c + d*x]/(a + b*E^x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{c+dx}}{a+be^x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}}{a+be^x} \, dx = \int \frac{\sqrt{c+dx}}{a+be^x} \, dx$$

```
[In] Integrate[Sqrt[c + d*x]/(a + b*E^x),x]
[Out] Integrate[Sqrt[c + d*x]/(a + b*E^x), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{dx+c}}{a+b\,\mathrm{e}^{x}}dx$$

```
[In] int((d*x+c)^(1/2)/(a+b*exp(x)),x)
[Out] int((d*x+c)^(1/2)/(a+b*exp(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x} \, dx = \int \frac{\sqrt{dx+c}}{be^x+a} \, dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x)),x, algorithm="fricas")
[Out] integral(sqrt(d*x + c)/(b*e^x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+dx}}{a+be^x} \, dx = \int \frac{\sqrt{c+dx}}{a+be^x} \, dx$$

```
[In] integrate((d*x+c)**(1/2)/(a+b*exp(x)),x)
[Out] Integral(sqrt(c + d*x)/(a + b*exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{dx+c}}{be^x+a}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x)),x, algorithm="maxima")
[Out] integrate(sqrt(d*x + c)/(b*e^x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{dx+c}}{be^x+a}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)/(b*e^x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{c+d\,x}}{a+b\,\mathrm{e}^x}\,dx$$

```
[In] int((c + d*x)^(1/2)/(a + b*exp(x)),x)
[Out] int((c + d*x)^(1/2)/(a + b*exp(x)), x)
```

3.68 $\int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx = \operatorname{Int}\left(\frac{\sqrt{c+dx}}{(a+be^x)^2}, x\right)$$

```
[Out] Unintegrable((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$$

```
[In] Int[Sqrt[c + d*x]/(a + b*E^x)^2,x]
[Out] Defer[Int][Sqrt[c + d*x]/(a + b*E^x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{c+dx}}{(a+be^x)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx$$

```
[In] Integrate[Sqrt[c + d*x]/(a + b*E^x)^2,x]
[Out] Integrate[Sqrt[c + d*x]/(a + b*E^x)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{dx+c}}{(a+b\,\mathrm{e}^{x})^2}dx$$

```
[In] int((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
[Out] int((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^2}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x))^2,x, algorithm="fricas")
[Out] integral(sqrt(d*x + c)/(b^2*e^(2*x) + 2*a*b*e^x + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx$$

```
[In] integrate((d*x+c)**(1/2)/(a+b*exp(x))**2,x)
[Out] Integral(sqrt(c + d*x)/(a + b*exp(x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^2}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x))^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x + c)/(b*e^x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^2}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)/(b*e^x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{c+d\,x}}{\left(a+b\,\mathrm{e}^x\right)^2}\,dx$$

```
[In] int((c + d*x)^(1/2)/(a + b*exp(x))^2,x)
[Out] int((c + d*x)^(1/2)/(a + b*exp(x))^2, x)
```

3.69 $\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \text{Int}\left(\frac{\sqrt{c+dx}}{(a+be^x)^3},x\right)$$

```
[Out] Unintegrable((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$

```
[In] Int[Sqrt[c + d*x]/(a + b*E^x)^3,x]
[Out] Defer[Int][Sqrt[c + d*x]/(a + b*E^x)^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$

```
[In] Integrate[Sqrt[c + d*x]/(a + b*E^x)^3,x]
[Out] Integrate[Sqrt[c + d*x]/(a + b*E^x)^3, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{dx+c}}{(a+b\,\mathrm{e}^{x})^{3}}dx$$

```
[In] int((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
[Out] int((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^3}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x))^3,x, algorithm="fricas")
[Out] integral(sqrt(d*x + c)/(b^3*e^(3*x) + 3*a*b^2*e^(2*x) + 3*a^2*b*e^x + a^3),
 x)
```

Sympy [N/A]

Not integrable

Time = 6.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$

```
[In] integrate((d*x+c)**(1/2)/(a+b*exp(x))**3,x)
[Out] Integral(sqrt(c + d*x)/(a + b*exp(x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^3}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x))^3,x, algorithm="maxima")
[Out] integrate(sqrt(d*x + c)/(b*e^x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^3}\,dx$$

```
[In] integrate((d*x+c)^(1/2)/(a+b*exp(x))^3,x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)/(b*e^x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx=\int \frac{\sqrt{c+d\,x}}{{\left(a+b\,\mathrm{e}^x\right)}^3}\,dx$$

```
[In] int((c + d*x)^(1/2)/(a + b*exp(x))^3,x)
[Out] int((c + d*x)^(1/2)/(a + b*exp(x))^3, x)
```

3.70 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 340

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \frac{a^3(c+dx)^{1+m}}{d(1+m)}$$
$$+\frac{3^{-1-m}b^3F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n}(c+dx)^m\Gamma\left(1+m,-\frac{3fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$
$$+\frac{3\ 2^{-1-m}ab^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\Gamma\left(1+m,-\frac{2fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$
$$+\frac{3a^2bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^{n}(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

```
[Out] a^3*(d*x+c)^(1+m)/d/(1+m)+3^(-1-m)*b^3*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F
^(f*g*x+e*g))^(3*n)*(d*x+c)^m*GAMMA(1+m,-3*f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln(
F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)+3*2^(-1-m)*a*b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(
f*x+e))*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^m*GAMMA(1+m,-2*f*g*n*(d*x+c)*ln(F)/d)
/f/g/n/ln(F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)+3*a^2*b*F^((e-c*f/d)*g*n-g*n*(f*x
+e))*(F^(f*g*x+e*g))^n*(d*x+c)^m*GAMMA(1+m,-f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln
(F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2214, 2213, 2212}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \frac{a^3(c+dx)^{m+1}}{d(m+1)} + \frac{3a^2b(c+dx)^m \left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)} + \frac{3ab^2 2^{-m-1}(c+dx)^m \left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{2fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)} + \frac{b^3 3^{-m-1}(c+dx)^m \left(F^{eg+fgx}\right)^{3n} F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{3fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m,x]
```

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3^(-1 - m)*b^3*F^(3*(e - (c*f)/d)*g*
n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*(c + d*x)^m*Gamma[1 + m, (-3*f
*g*n*(c + d*x)*Log[F])/d])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m)
 + (3*2^(-1 - m)*a*b^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g +
f*g*x))^(2*n)*(c + d*x)^m*Gamma[1 + m, (-2*f*g*n*(c + d*x)*Log[F])/d])/(f*g
*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m) + (3*a^2*b*F^((e - (c*f)/d)*g*
n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*(c + d*x)^m*Gamma[1 + m, -((f*g*n*(c
 + d*x)*Log[F])/d)])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
```

```
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^3(c+dx)^m + 3a^2b\left(F^{eg+fgx}\right)^n (c+dx)^m + 3ab^2\left(F^{eg+fgx}\right)^{2n} (c+dx)^m + b^3\left(F^{eg+fgx}\right)^{3n} (c+dx)^m\right) dx$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int \left(F^{eg+fgx}\right)^n (c+dx)^m\,dx + \left(3ab^2\right)\int \left(F^{eg+fgx}\right)^{2n} (c+dx)^m\,dx + b^3\int \left(F^{eg+fgx}\right)^{3n} (c+dx)^m\,dx$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \left(3a^2bF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n\right)\int F^{n(eg+fgx)}(c+dx)^m\,dx + \left(3ab^2F^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n}\right)\int F^{2n(eg+fgx)}(c+dx)^m\,dx + \left(b^3F^{-3n(eg+fgx)}\left(F^{eg+fgx}\right)^{3n}\right)\int F^{3n(eg+fgx)}(c+dx)^m\,dx$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3^{-1-m}b^3F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n}(c+dx)^m\Gamma\left(1+m,-\frac{3fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right.}{fgn\log(F)} + \frac{3\ 2^{-1-m}ab^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\Gamma\left(1+m,-\frac{2fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)}{d}\right.}{fgn\log(F)} + \frac{3a^2bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

Mathematica [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m,x]
[Out] Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m, x]
```

Maple [F]

$$\int \left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^{3}(dx+c)^{m}\,dx$$

[In] int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x)

[Out] int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.79

$$\int \left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}(c+dx)^{m}\,dx = \frac{18\,(a^2bdm+a^2bd)e^{\left(\frac{(de-cf)gn\log(F)-dm\log\left(-\frac{fgn\log(F)}{d}\right)}{d}\right)}\Gamma\left(m+1,-\frac{(dfgnx+cfgn)\log(F)}{d}\right)+9\,(ab^2dm+ab^2d)e^{\left(\frac{2\,(de}{\ldots}\right.}}{}$$

[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x, algorithm="fricas")

[Out] 1/6*(18*(a^2*b*d*m + a^2*b*d)*e^(((d*e - c*f)*g*n*log(F) - d*m*log(-f*g*n*log(F)/d))/d)*gamma(m + 1, -(d*f*g*n*x + c*f*g*n)*log(F)/d) + 9*(a*b^2*d*m + a*b^2*d)*e^((2*(d*e - c*f)*g*n*log(F) - d*m*log(-2*f*g*n*log(F)/d))/d)*gamma(m + 1, -2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 2*(b^3*d*m + b^3*d)*e^((3*(d*e - c*f)*g*n*log(F) - d*m*log(-3*f*g*n*log(F)/d))/d)*gamma(m + 1, -3*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 6*(a^3*d*f*g*n*x + a^3*c*f*g*n)*(d*x + c)^m*log(F))/((d*f*g*m + d*f*g)*n*log(F))

Sympy [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}(c+dx)^{m}\,dx = \int \left(a+b\left(F^{eg+fgx}\right)^{n}\right)^{3}(c+dx)^{m}\,dx$$

[In] integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c)**m,x)

[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**3*(c + d*x)**m, x)

Maxima [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^3/(d*(m + 1)) + integrate((3*F^(f*g*n*x)*F^(e*g*n)*a^2*
b + 3*F^(2*f*g*n*x)*F^(2*e*g*n)*a*b^2 + F^(3*f*g*n*x)*F^(3*e*g*n)*b^3)*(d*x
 + c)^m, x)
```

Giac [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3 (c+d\,x)^m\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m,x)
```

```
[Out] int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m, x)
```

3.71 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 228

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{2^{-1-m}b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\Gamma\left(1+m,-\frac{2fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)} + \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^{n}(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

```
[Out] a^2*(d*x+c)^(1+m)/d/(1+m)+2^(-1-m)*b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F
^(f*g*x+e*g))^(2*n)*(d*x+c)^m*GAMMA(1+m,-2*f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln(
F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)+2*a*b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g
*x+e*g))^n*(d*x+c)^m*GAMMA(1+m,-f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln(F)/((-f*g*n
*(d*x+c)*ln(F)/d)^m)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

= {2214, 2213, 2212}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \frac{a^2(c+dx)^{m+1}}{d(m+1)} + \frac{2ab(c+dx)^m\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)} + \frac{b^2 2^{-m-1}(c+dx)^m\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m,x]
```

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (2^(-1 - m)*b^2*F^(2*(e - (c*f)/d)*g*
n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*(c + d*x)^m*Gamma[1 + m, (-2*f
*g*n*(c + d*x)*Log[F])/d])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m)
 + (2*a*b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*(c + d*
x)^m*Gamma[1 + m, -((f*g*n*(c + d*x)*Log[F])/d)])/(f*g*n*Log[F]*(-((f*g*n*(
c + d*x)*Log[F])/d))^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2(c+dx)^m + 2ab\left(F^{eg+fgx}\right)^n (c+dx)^m + b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\right)dx \\ &= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int \left(F^{eg+fgx}\right)^n (c+dx)^m\,dx + b^2\int \left(F^{eg+fgx}\right)^{2n}(c+dx)^m\,dx\end{aligned}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \left(2abF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n\right)\int F^{n(eg+fgx)}(c+dx)^m\,dx + \left(b^2F^{-2n(eg+fgx)}\left(F^{eg+fgx}\right)^{2n}\right)\int F^{2n(eg+fgx)}(c+dx)^m\,dx$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{2^{-1-m}b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\Gamma\left(1+m,-\frac{2fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right.}{fgn\log(F)} + \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

Mathematica [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^m\,dx$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m,x]
[Out] Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m, x]
```

Maple [F]

$$\int \left(a+b\left(F^{g(fx+e)}\right)^n\right)^2(dx+c)^m\,dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x)
[Out] int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)^m\,dx$$

$$= \frac{4(abdm+abd)e^{\left(\frac{(de-cf)gn\log(F)-dm\log\left(-\frac{fgn\log(F)}{d}\right)}{d}\right)}\Gamma\left(m+1,-\frac{(dfgnx+cfgn)\log(F)}{d}\right) + (b^2dm+b^2d)e^{\left(\frac{2(de-cf)gn\log\ldots}{\ldots}\right.}}{2(dfgm+dfg)n\,\mathrm{l}\ldots}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a*b*d*m + a*b*d)*e^(((d*e - c*f)*g*n*log(F) - d*m*log(-f*g*n*log(F)
/d))/d)*gamma(m + 1, -(d*f*g*n*x + c*f*g*n)*log(F)/d) + (b^2*d*m + b^2*d)*e
^((2*(d*e - c*f)*g*n*log(F) - d*m*log(-2*f*g*n*log(F)/d))/d)*gamma(m + 1, -
2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 2*(a^2*d*f*g*n*x + a^2*c*f*g*n)*(d*x +
c)^m*log(F))/((d*f*g*m + d*f*g)*n*log(F))
```

Sympy [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)^m\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**m,x)
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)**2*(c + d*x)**m, x)
```

Maxima [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x, algorithm="maxima")
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate((2*F^(f*g*n*x)*F^(e*g*n)*a*b
+ F^(2*f*g*n*x)*F^(2*e*g*n)*b^2)*(d*x + c)^m, x)
```

Giac [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2 (c+d\,x)^m\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m, x)
```

3.72 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx=\frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)+b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*(
d*x+c)^m*GAMMA(1+m,-f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln(F)/((-f*g*n*(d*x+c)*ln(
F)/d)^m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2214, 2213, 2212}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx=\frac{a(c+dx)^{m+1}}{d(m+1)}+\frac{b(c+dx)^m\left(F^{eg+fgx}\right)^nF^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m,x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x)
)*(F^(e*g + f*g*x))^n*(c + d*x)^m*Gamma[1 + m, -((f*g*n*(c + d*x)*Log[F])/d
)])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2213

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)), Int[(c + d*x
)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(c+dx)^m + b\left(F^{eg+fgx}\right)^n (c+dx)^m\right) dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} + b\int \left(F^{eg+fgx}\right)^n (c+dx)^m\,dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} + \left(bF^{-n(eg+fgx)}\left(F^{eg+fgx}\right)^n\right)\int F^{n(eg+fgx)}(c+dx)^m\,dx \\
&= \frac{a(c+dx)^{1+m}}{d(1+m)} \\
&\quad + \frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n (c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$$
$$=\frac{(c+dx)^m\left(\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)-bF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n(c+dx)\Gamma\left(2+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\right.}{d(1+m)}$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m,x]
```

```
[Out] ((c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)*(c + d*x) - (b*(F^(g*(e + f*x)))^
n*(c + d*x)*Gamma[2 + m, -((f*g*n*(c + d*x)*Log[F])/d)]*(-((f*g*n*(c + d*x)
*Log[F])/d))^(-1 - m))/F^((f*g*n*(c + d*x))/d)))/(d*(1 + m))
```

Maple [F]

$$\int \left(a+b\left(F^{g(fx+e)}\right)^n\right)(dx+c)^m\,dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x)
```

```
[Out] int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$$
$$=\frac{(bdm+bd)e^{\left(\frac{(de-cf)gn\log(F)-dm\log\left(-\frac{fgn\log(F)}{d}\right)}{d}\right)}\Gamma\left(m+1,-\frac{(dfgnx+cfgn)\log(F)}{d}\right)+(adfgnx+acfgn)(dx+c)^m}{(dfgm+dfg)n\log(F)}$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] ((b*d*m + b*d)*e^(((d*e - c*f)*g*n*log(F) - d*m*log(-f*g*n*log(F)/d))/d)*ga
mma(m + 1, -(d*f*g*n*x + c*f*g*n)*log(F)/d) + (a*d*f*g*n*x + a*c*f*g*n)*(d*
x + c)^m*log(F))/((d*f*g*m + d*f*g)*n*log(F))
```

Sympy [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx=\int \left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^m\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**m,x)
[Out] Integral((a + b*(F**(e*g + f*g*x))**n)*(c + d*x)**m, x)
```

Maxima [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx=\int \left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x, algorithm="maxima")
[Out] F^(e*g*n)*b*integrate(e^(f*g*n*x*log(F) + m*log(d*x + c)), x) + (d*x + c)^(
m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx=\int \left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx=\int \left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)(c+d\,x)^m\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m, x)
```

3.73 $\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n} \, dx = \text{Int}\left(\frac{(c+dx)^m}{a+b\left(F^{eg+fgx}\right)^n},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*(F^(f*g*x+e*g))^n),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n} \, dx = \int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n} \, dx$$

```
[In] Int[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*(F^(e*g + f*g*x))^n), x]
```

Rubi steps

$$\text{integral} = \int \frac{(c+dx)^m}{a+b\left(F^{eg+fgx}\right)^n} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx=\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

```
[In] Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x]
[Out] Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+b\left(F^{g(fx+e)}\right)^n}dx$$

```
[In] int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x)
[Out] int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx=\int \frac{(dx+c)^m}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
[Out] integral((d*x + c)^m/((F^(f*g*x + e*g))^n*b + a), x)
```

Sympy [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx=\int \frac{(c+dx)^m}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

```
[In] integrate((d*x+c)**m/(a+b*(F**(g*(f*x+e)))**n),x)
[Out] Integral((c + d*x)**m/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx=\int \frac{(dx+c)^m}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(F^((f*x + e)*g*n)*b + a), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx=\int \frac{(dx+c)^m}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

```
[In] integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
[Out] integrate((d*x + c)^m/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx=\int \frac{(c+d\,x)^m}{a+b\,\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

```
[In] int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x)
[Out] int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n), x)
```

3.74 $\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\operatorname{Int}\left(\frac{(c+dx)^m}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*(F^(f*g*x+e*g))^n)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

```
[In] Int[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*(F^(e*g + f*g*x))^n)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{(c+dx)^m}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

```
[In] Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2,x]
[Out] Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}dx$$

```
[In] int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x)
[Out] int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(dx+c)^m}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m/(2*(F^(f*g*x + e*g))^n*a*b + (F^(f*g*x + e*g))^(2*n)*b
^2 + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 56.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(c+dx)^m}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}\,dx$$

```
[In] integrate((d*x+c)**m/(a+b*(F**(g*(f*x+e)))**n)**2,x)
[Out] Integral((c + d*x)**m/(a + b*(F**(e*g + f*g*x))**n)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(dx+c)^m}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(F^((f*x + e)*g*n)*b + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(dx+c)^m}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

```
[In] integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx=\int \frac{(c+d\,x)^m}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

```
[In] int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2,x)
[Out] int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

3.75 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx = \text{Int}\left(\left(a+b\left(F^{eg+fgx}\right)^n\right)^p (c+dx)^m, x\right)$$

```
[Out] Unintegrable((a+b*(F^(f*g*x+e*g))^n)^p*(d*x+c)^m,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx$$

```
[In] Int[(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m,x]
```

```
[Out] Defer[Int][(a + b*(F^(e*g + f*g*x))^n)^p*(c + d*x)^m, x]
```

Rubi steps

$$\text{integral} = \int \left(a+b\left(F^{eg+fgx}\right)^n\right)^p (c+dx)^m\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx=\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx$$

```
[In] Integrate[(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m,x]
[Out] Integrate[(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(F^{g(fx+e)}\right)^n\right)^p (dx+c)^m\,dx$$

```
[In] int((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x)
[Out] int((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx=\int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^p (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x, algorithm="fricas")
[Out] integral(((F^(f*g*x + e*g))^n*b + a)^p*(d*x + c)^m, x)
```

Sympy [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx=\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx$$

```
[In] integrate((a+b*(F**(g*(f*x+e)))**n)**p*(d*x+c)**m,x)
[Out] integrate((a+b*(F**(g*(f*x+e)))**n)**p*(d*x+c)**m,x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx=\int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^p (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((F^((f*x + e)*g*n)*b + a)^p*(d*x + c)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx=\int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^p (dx+c)^m\,dx$$

```
[In] integrate((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x, algorithm="giac")
[Out] integrate(((F^((f*x + e)*g))^n*b + a)^p*(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx=\int \left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^p (c+d\,x)^m\,dx$$

```
[In] int((a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m,x)
[Out] int((a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m, x)
```

3.76 $\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{x^3\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x\operatorname{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6\operatorname{PolyLog}\left(4,-\frac{bF^{c+dx}}{a}\right)}{bd^4\log^4(F)}$$

```
[Out] x^3*ln(1+b*F^(d*x+c)/a)/b/d/ln(F)+3*x^2*polylog(2,-b*F^(d*x+c)/a)/b/d^2/ln(
F)^2-6*x*polylog(3,-b*F^(d*x+c)/a)/b/d^3/ln(F)^3+6*polylog(4,-b*F^(d*x+c)/a
)/b/d^4/ln(F)^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2221, 2611, 6744, 2320, 6724}

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{6\operatorname{PolyLog}\left(4,-\frac{bF^{c+dx}}{a}\right)}{bd^4\log^4(F)} - \frac{6x\operatorname{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} + \frac{x^3\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}$$

```
[In] Int[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x)),x]
```

```
[Out] (x^3*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (3*x^2*PolyLog[2, -((b*F^(c
 + d*x))/a])/(b*d^2*Log[F]^2) - (6*x*PolyLog[3, -((b*F^(c + d*x))/a])/(b*
d^3*Log[F]^3) + (6*PolyLog[4, -((b*F^(c + d*x))/a])/(b*d^4*Log[F]^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{x^3\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}-\frac{3\int x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)\,dx}{bd\log(F)}\\&=\frac{x^3\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{3x^2\mathrm{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}-\frac{6\int x\mathrm{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)\,dx}{bd^2\log^2(F)}\end{aligned}$$

$$= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2 \mathrm{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x\mathrm{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6\int \mathrm{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)\,dx}{bd^3\log^3(F)}$$

$$= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2 \mathrm{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x\mathrm{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6\mathrm{Subst}\left(\int \frac{\mathrm{Li}_3\left(-\frac{bx}{a}\right)}{x}\,dx, x, F^{c+dx}\right)}{bd^4\log^4(F)}$$

$$= \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2 \mathrm{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x\mathrm{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6\mathrm{Li}_4\left(-\frac{bF^{c+dx}}{a}\right)}{bd^4\log^4(F)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{x^3 \log\left(1 + \frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2 \operatorname{PolyLog}\left(2, -\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x \operatorname{PolyLog}\left(3, -\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6 \operatorname{PolyLog}\left(4, -\frac{bF^{c+dx}}{a}\right)}{bd^4\log^4(F)}$$

```
[In] Integrate[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x)),x]
```

```
[Out] (x^3*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (3*x^2*PolyLog[2, -((b*F^(c
 + d*x))/a)])/(b*d^2*Log[F]^2) - (6*x*PolyLog[3, -((b*F^(c + d*x))/a)])/(b*
d^3*Log[F]^3) + (6*PolyLog[4, -((b*F^(c + d*x))/a)])/(b*d^4*Log[F]^4)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.96

method	result
risch	$\frac{6\,\mathrm{Li}_4\left(-\frac{b\,F^{dx}F^{c}}{a}\right)}{d^4\ln(F)^4 b} + \frac{\ln\left(1+\frac{b\,F^{dx}F^{c}}{a}\right)c^3}{d^4\ln(F) b} - \frac{c^3\ln\left(F^{c}F^{dx}b+a\right)}{d^4\ln(F) b} + \frac{c^3\ln\left(F^{dx}F^{c}\right)}{d^4\ln(F) b} - \frac{c^3 x}{d^3 b} + \frac{\ln\left(1+\frac{b\,F^{dx}F^{c}}{a}\right)x^3}{d\ln(F) b} +$

```
[In] int(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/d^4/ln(F)^4/b*polylog(4,-b*F^(d*x)*F^c/a)+1/d^4/ln(F)/b*ln(1+b*F^(d*x)*F^
c/a)*c^3-1/d^4/ln(F)/b*c^3*ln(F^c*F^(d*x)*b+a)+1/d^4/ln(F)/b*c^3*ln(F^(d*x)
*F^c)-1/d^3/b*c^3*x+1/d/ln(F)/b*ln(1+b*F^(d*x)*F^c/a)*x^3+3/d^2/ln(F)^2/b*p
olylog(2,-b*F^(d*x)*F^c/a)*x^2-6/d^3/ln(F)^3/b*polylog(3,-b*F^(d*x)*F^c/a)*
x-3/4/d^4/b*c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{3\,d^2x^2\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)\log\left(F\right)^2 - c^3\log\left(F^{dx+c}b+a\right)\log\left(F\right)^3 + \left(d^3x^3+c^3\right)\log\left(F\right)^3\log\left(\frac{F^{dx+c}b+a}{a}\right) - 6}{bd^4\log\left(F\right)^4}$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

```
[Out] (3*d^2*x^2*dilog(-(F^(d*x + c)*b + a)/a + 1)*log(F)^2 - c^3*log(F^(d*x + c)
*b + a)*log(F)^3 + (d^3*x^3 + c^3)*log(F)^3*log((F^(d*x + c)*b + a)/a) - 6*
d*x*log(F)*polylog(3, -F^(d*x + c)*b/a) + 6*polylog(4, -F^(d*x + c)*b/a))/(
b*d^4*log(F)^4)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+dx}x^3}{F^{c+dx}b+a}\,dx$$

```
[In] integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c)),x)
```

```
[Out] Integral(F**(c + d*x)*x**3/(F**(c + d*x)*b + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{d^3x^3\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log\left(F\right)^3 + 3\,d^2x^2\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log\left(F\right)^2 - 6\,dx\log\left(F\right)\mathrm{Li}_3\left(-\frac{F^{dx}F^cb}{a}\right) + 6\,\mathrm{Li}_4\left(-\frac{F^{dx}F^cb}{a}\right)}{bd^4\log\left(F\right)^4}$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d^3*x^3*log(F^(d*x)*F^c*b/a + 1)*log(F)^3 + 3*d^2*x^2*dilog(-F^(d*x)*F^c*b
/a)*log(F)^2 - 6*d*x*log(F)*polylog(3, -F^(d*x)*F^c*b/a) + 6*polylog(4, -F^
(d*x)*F^c*b/a))/(b*d^4*log(F)^4)
```

Giac [F]

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \int \frac{F^{dx+c}x^3}{F^{dx+c}b+a}\,dx$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="giac")
[Out] integrate(F^(d*x + c)*x^3/(F^(d*x + c)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+d\,x}\,x^3}{a+F^{c+d\,x}\,b}\,dx$$

```
[In] int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b),x)
[Out] int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b), x)
```

3.77 $\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx=\frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{2x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}-\frac{2\,\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)}$$

```
[Out] x^2*ln(1+b*F^(d*x+c)/a)/b/d/ln(F)+2*x*polylog(2,-b*F^(d*x+c)/a)/b/d^2/ln(F)
^2-2*polylog(3,-b*F^(d*x+c)/a)/b/d^3/ln(F)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2221, 2611, 2320, 6724}

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx$$
$$=-\frac{2\,\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)}+\frac{2x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}+\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}$$

```
[In] Int[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x)),x]
```

```
[Out] (x^2*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (2*x*PolyLog[2, -((b*F^(c +
 d*x))/a)])/(b*d^2*Log[F]^2) - (2*PolyLog[3, -((b*F^(c + d*x))/a)])/(b*d^3*
Log[F]^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}-\frac{2\int x\log\left(1+\frac{bF^{c+dx}}{a}\right)\,dx}{bd\log(F)}\\
&= \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{2x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}-\frac{2\int \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)\,dx}{bd^2\log^2(F)}\\
&= \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{2x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}-\frac{2\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx,x,F^{c+dx}\right)}{bd^3\log^3(F)}\\
&= \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{2x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}-\frac{2\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{2\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)}$$

```
[In] Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x)),x]
```

```
[Out] (x^2*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (2*x*PolyLog[2, -((b*F^(c +
 d*x))/a)])/(b*d^2*Log[F]^2) - (2*PolyLog[3, -((b*F^(c + d*x))/a)])/(b*d^3*
Log[F]^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(85) = 170$.

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.28

method	result
risch	$\frac{c^2x}{d^2b}+\frac{2c^3}{3d^3b}+\frac{\ln\left(1+\frac{b\,F^{dx}F^c}{a}\right)x^2}{d\ln(F)b}-\frac{\ln\left(1+\frac{b\,F^{dx}F^c}{a}\right)c^2}{d^3\ln(F)b}+\frac{2\,\mathrm{Li}_2\left(-\frac{b\,F^{dx}F^c}{a}\right)x}{d^2\ln(F)^2b}-\frac{2\,\mathrm{Li}_3\left(-\frac{b\,F^{dx}F^c}{a}\right)}{d^3\ln(F)^3b}+\frac{c^2\ln(F^cF^{dx}b+a)}{d^3\ln(F)b}-$

```
[In] int(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2/b*c^2*x+2/3/d^3/b*c^3+1/d/ln(F)/b*ln(1+b*F^(d*x)*F^c/a)*x^2-1/d^3/ln(
F)/b*ln(1+b*F^(d*x)*F^c/a)*c^2+2/d^2/ln(F)^2/b*polylog(2,-b*F^(d*x)*F^c/a)*
x-2/d^3/ln(F)^3/b*polylog(3,-b*F^(d*x)*F^c/a)+1/d^3/ln(F)/b*c^2*ln(F^c*F^(d
*x)*b+a)-1/d^3/ln(F)/b*c^2*ln(F^(d*x)*F^c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.27

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \frac{c^2\log\left(F^{dx+c}b+a\right)\log(F)^2 + 2\,dx\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)\log(F) + (d^2x^2-c^2)\log(F)^2\log\left(\frac{F^{dx+c}b+a}{a}\right) - 2\,\mathrm{po}}{bd^3\log(F)^3}$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

```
[Out] (c^2*log(F^(d*x + c)*b + a)*log(F)^2 + 2*d*x*dilog(-(F^(d*x + c)*b + a)/a +
 1)*log(F) + (d^2*x^2 - c^2)*log(F)^2*log((F^(d*x + c)*b + a)/a) - 2*polylo
g(3, -F^(d*x + c)*b/a))/(b*d^3*log(F)^3)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+dx}x^2}{F^{c+dx}b+a}\,dx$$

```
[In] integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c)),x)
```

```
[Out] Integral(F**(c + d*x)*x**2/(F**(c + d*x)*b + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \frac{d^2x^2\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F)^2+2\,dx\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log(F)-2\,\mathrm{Li}_3(-\frac{F^{dx}F^cb}{a})}{bd^3\log(F)^3}$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d^2*x^2*log(F^(d*x)*F^c*b/a + 1)*log(F)^2 + 2*d*x*dilog(-F^(d*x)*F^c*b/a)*
log(F) - 2*polylog(3, -F^(d*x)*F^c*b/a))/(b*d^3*log(F)^3)
```

Giac [F]

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \int \frac{F^{dx+c}x^2}{F^{dx+c}b+a}\,dx$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)*x^2/(F^(d*x + c)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+d\,x}\,x^2}{a+F^{c+d\,x}\,b}\,dx$$

```
[In] int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b),x)
```

```
[Out] int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b), x)
```

3.78 $\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx=\frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}$$

```
[Out] x*ln(1+b*F^(d*x+c)/a)/b/d/ln(F)+polylog(2,-b*F^(d*x+c)/a)/b/d^2/ln(F)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2221, 2317, 2438}

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx=\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}+\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}$$

```
[In] Int[(F^(c + d*x)*x)/(a + b*F^(c + d*x)),x]
```

```
[Out] (x*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + PolyLog[2, -((b*F^(c + d*x))/
a)]/(b*d^2*Log[F]^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}-\frac{\int\log\left(1+\frac{bF^{c+dx}}{a}\right)\,dx}{bd\log(F)}\\ &= \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}-\frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx,x,F^{c+dx}\right)}{bd^2\log^2(F)}\\ &= \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int\frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx=\frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)}+\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}$$

```
[In] Integrate[(F^(c + d*x)*x)/(a + b*F^(c + d*x)),x]
[Out] (x*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + PolyLog[2, -((b*F^(c + d*x))/
a)]/(b*d^2*Log[F]^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(54) = 108.

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.85

method	result	size
risch	$-\frac{cx}{db}-\frac{c^2}{2d^2b}+\frac{\ln\left(1+\frac{b\,F^{dx}\,F^{c}}{a}\right)x}{d\ln(F)b}+\frac{\ln\left(1+\frac{b\,F^{dx}\,F^{c}}{a}\right)c}{d^2\ln(F)b}+\frac{\mathrm{Li}_2\left(-\frac{b\,F^{dx}\,F^{c}}{a}\right)}{d^2\ln(F)^2b}-\frac{c\ln(F^{c}F^{dx}b+a)}{d^2\ln(F)b}+\frac{c\ln(F^{dx}F^{c})}{d^2\ln(F)b}$	154

```
[In] int(F^(d*x+c)*x/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/b*c*x-1/2/d^2/b*c^2+1/d/ln(F)/b*ln(1+b*F^(d*x)*F^c/a)*x+1/d^2/ln(F)/b*
ln(1+b*F^(d*x)*F^c/a)*c+1/d^2/ln(F)^2/b*polylog(2,-b*F^(d*x)*F^c/a)-1/d^2/l
n(F)/b*c*ln(F^c*F^(d*x)*b+a)+1/d^2/ln(F)/b*c*ln(F^(d*x)*F^c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.39

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = -\frac{c\log\left(F^{dx+c}b+a\right)\log\left(F\right)-\left(dx+c\right)\log\left(F\right)\log\left(\frac{F^{dx+c}b+a}{a}\right)-\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)}{bd^2\log\left(F\right)^2}$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(c*log(F^(d*x + c)*b + a)*log(F) - (d*x + c)*log(F)*log((F^(d*x + c)*b + a
)/a) - dilog(-(F^(d*x + c)*b + a)/a + 1))/(b*d^2*log(F)^2)
```

Sympy [F]

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+dx}x}{F^{c+dx}b+a}\,dx$$

```
[In] integrate(F**(d*x+c)*x/(a+b*F**(d*x+c)),x)
```

```
[Out] Integral(F**(c + d*x)*x/(F**(c + d*x)*b + a), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \frac{dx\log\left(\frac{F^{dx}F^{c}b}{a}+1\right)\log\left(F\right)+\mathrm{Li}_2\left(-\frac{F^{dx}F^{c}b}{a}\right)}{bd^2\log\left(F\right)^2}$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c)),x, algorithm="maxima")
[Out] (d*x*log(F^(d*x)*F^c*b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(b*d^2*log(
F)^2)
```

Giac [F]

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \int \frac{F^{dx+c}x}{F^{dx+c}b+a}\,dx$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c)),x, algorithm="giac")
[Out] integrate(F^(d*x + c)*x/(F^(d*x + c)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+d\,x}\,x}{a+F^{c+d\,x}\,b}\,dx$$

```
[In] int((F^(c + d*x)*x)/(a + F^(c + d*x)*b),x)
[Out] int((F^(c + d*x)*x)/(a + F^(c + d*x)*b), x)
```

3.79 $\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

```
[Out] ln(a+b*F^(d*x+c))/b/d/ln(F)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2278, 31}

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

```
[In] Int[F^(c + d*x)/(a + b*F^(c + d*x)),x]
```

```
[Out] Log[a + b*F^(c + d*x)]/(b*d*Log[F])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, F^{c+dx}\right)}{d\log(F)}\\ &= \frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

```
[In] Integrate[F^(c + d*x)/(a + b*F^(c + d*x)),x]
[Out] Log[a + b*F^(c + d*x)]/(b*d*Log[F])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\ln\left(a+b\,F^{dx+c}\right)}{bd\ln(F)}$	24
default	$\frac{\ln\left(a+b\,F^{dx+c}\right)}{bd\ln(F)}$	24
parallelrisch	$\frac{\ln\left(a+b\,F^{dx+c}\right)}{bd\ln(F)}$	24
norman	$\frac{\ln\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}{bd\ln(F)}$	26
risch	$-\frac{c}{db}+\frac{\ln\left(F^{dx+c}+\frac{a}{b}\right)}{bd\ln(F)}$	36

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*F^(d*x+c))/b/d/ln(F)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{dx+c}b+a\right)}{bd\log(F)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="fricas")
[Out] log(F^(d*x + c)*b + a)/(b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{bd\log(F)}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c)),x)
[Out] log(F**(c + d*x) + a/b)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{dx+c}b+a\right)}{bd\log(F)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="maxima")
[Out] log(F^(d*x + c)*b + a)/(b*d*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(\left|F^{dx+c}b+a\right|\right)}{bd\log(F)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="giac")
[Out] log(abs(F^(d*x + c)*b + a))/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx=\frac{\ln\left(a+F^{c+d\,x}\,b\right)}{b\,d\,\ln\left(F\right)}$$

```
[In] int(F^(c + d*x)/(a + F^(c + d*x)*b),x)
[Out] log(a + F^(c + d*x)*b)/(b*d*log(F))
```

3.80 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx = \text{Int}\left(\frac{F^{c+dx}}{(a+bF^{c+dx})x}, x\right)$$

```
[Out] Unintegrable(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$$

```
[In] Int[F^(c + d*x)/((a + b*F^(c + d*x))*x),x]
[Out] Defer[Int][F^(c + d*x)/((a + b*F^(c + d*x))*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{F^{c+dx}}{(a+bF^{c+dx})x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx$$

```
[In] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x),x]
[Out] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})\,x}dx$$

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
[Out] int(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x,x, algorithm="fricas")
[Out] integral(F^(d*x + c)/(F^(d*x + c)*b*x + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{c+dx}}{x\,(F^{c+dx}b+a)}\,dx$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))/x,x)
[Out] Integral(F**(c + d*x)/(x*(F**(c + d*x)*b + a)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x,x, algorithm="maxima")
[Out] -a*integrate(1/(F^(d*x)*F^c*b^2*x + a*b*x), x) + log(x)/b
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x,x, algorithm="giac")
[Out] integrate(F^(d*x + c)/((F^(d*x + c)*b + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{c+d\,x}}{x\,\left(a+F^{c+d\,x}\,b\right)}\,dx$$

```
[In] int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)),x)
[Out] int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)), x)
```

3.81 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \mathrm{Int}\left(\frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2},x\right)$$

```
[Out] Unintegrable(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx$$

```
[In] Int[F^(c + d*x)/((a + b*F^(c + d*x))*x^2),x]
```

```
[Out] Defer[Int][F^(c + d*x)/((a + b*F^(c + d*x))*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx$$

```
[In] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x^2),x]
[Out] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})\,x^2}dx$$

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
[Out] int(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x^2}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x, algorithm="fricas")
[Out] integral(F^(d*x + c)/(F^(d*x + c)*b*x^2 + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{c+dx}}{x^2\,(F^{c+dx}b+a)}\,dx$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))/x**2,x)
[Out] Integral(F**(c + d*x)/(x**2*(F**(c + d*x)*b + a)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx=\int \frac{F^{dx+c}}{(F^{dx+c}b+a)x^2}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x, algorithm="maxima")
[Out] -a*integrate(1/(F^(d*x)*F^c*b^2*x^2 + a*b*x^2), x) - 1/(b*x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx=\int \frac{F^{dx+c}}{(F^{dx+c}b+a)x^2}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x, algorithm="giac")
[Out] integrate(F^(d*x + c)/((F^(d*x + c)*b + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx=\int \frac{F^{c+d\,x}}{x^2\,\left(a+F^{c+d\,x}\,b\right)}\,dx$$

```
[In] int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)),x)
[Out] int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)), x)
```

3.82 $\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2} dx = \frac{x^3}{abd\log(F)} - \frac{x^3}{bd(a+bF^{c+dx})\log(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{6x\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} + \frac{6\operatorname{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{abd^4\log^4(F)}$$

```
[Out] x^3/a/b/d/ln(F)-x^3/b/d/(a+b*F^(d*x+c))/ln(F)-3*x^2*ln(1+b*F^(d*x+c)/a)/a/b
/d^2/ln(F)^2-6*x*polylog(2,-b*F^(d*x+c)/a)/a/b/d^3/ln(F)^3+6*polylog(3,-b*F
^(d*x+c)/a)/a/b/d^4/ln(F)^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2222, 2215, 2221, 2611, 2320, 6724}

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2} dx = \frac{6\operatorname{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{abd^4\log^4(F)} - \frac{6x\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} - \frac{3x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{abd^2\log^2(F)} - \frac{x^3}{bd\log(F)(a+bF^{c+dx})} + \frac{x^3}{abd\log(F)}$$

```
[In] Int[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^2,x]
```

```
[Out] x^3/(a*b*d*Log[F]) - x^3/(b*d*(a + b*F^(c + d*x))*Log[F]) - (3*x^2*Log[1 +
(b*F^(c + d*x))/a])/(a*b*d^2*Log[F]^2) - (6*x*PolyLog[2, -((b*F^(c + d*x))/
```

```
a)])/(a*b*d^3*Log[F]^3) + (6*PolyLog[3, -((b*F^(c + d*x))/a)])/(a*b*d^4*Log
[F]^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} + \frac{3\int \frac{x^2}{a+bF^{c+dx}}\,dx}{bd\log(F)}$$

$$= \frac{x^3}{abd\log(F)} - \frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{3\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx}{ad\log(F)}$$

$$= \frac{x^3}{abd\log(F)} - \frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} + \frac{6\int x\log\left(1+\frac{bF^{c+dx}}{a}\right)\,dx}{abd^2\log^2(F)}$$

$$= \frac{x^3}{abd\log(F)} - \frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{6x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} + \frac{6\int \text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)\,dx}{abd^3\log^3(F)}$$

$$= \frac{x^3}{abd\log(F)} - \frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{6x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} + \frac{6\text{Subst}\left(\int \frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx, x, F^{c+dx}\right)}{abd^4\log^4(F)}$$

$$= \frac{x^3}{abd\log(F)} - \frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{6x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} + \frac{6\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{abd^4\log^4(F)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} + \frac{3\left(\frac{x^3}{3a} - \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{ad\log(F)} - \frac{2x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{ad^2\log^2(F)} + \frac{2\,\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{ad^3\log^3(F)}\right)}{bd\log(F)}$$

```
[In] Integrate[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^2,x]
```

```
[Out] -(x^3/(b*d*(a + b*F^(c + d*x))*Log[F])) + (3*(x^3/(3*a) - (x^2*Log[1 + (b*F
^(c + d*x))/a])/(a*d*Log[F]) - (2*x*PolyLog[2, -((b*F^(c + d*x))/a)])/(a*d^
2*Log[F]^2) + (2*PolyLog[3, -((b*F^(c + d*x))/a)])/(a*d^3*Log[F]^3)))/(b*d*
Log[F])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{x^{3}}{bd\left(a+b\,F^{dx+c}\right)\ln\left(F\right)}+\frac{x^{3}}{abd\ln\left(F\right)}-\frac{3c^{2}x}{b\ln\left(F\right)d^{3}a}-\frac{2c^{3}}{b\ln\left(F\right)d^{4}a}-\frac{3\ln\left(1+\frac{b\,F^{dx}F^{c}}{a}\right)x^{2}}{b\ln\left(F\right)^{2}d^{2}a}+\frac{3\ln\left(1+\frac{b\,F^{dx}F^{c}}{a}\right)c^{2}}{b\ln\left(F\right)^{2}d^{4}a}-\frac{6\,\mathrm{Li}_{2}\left(-\right.}{b\ln(}$

```
[In] int(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^3/b/d/(a+b*F^(d*x+c))/ln(F)+x^3/a/b/d/ln(F)-3/b/ln(F)/d^3/a*c^2*x-2/b/ln
(F)/d^4/a*c^3-3/b/ln(F)^2/d^2/a*ln(1+b*F^(d*x)*F^c/a)*x^2+3/b/ln(F)^2/d^4/a
*ln(1+b*F^(d*x)*F^c/a)*c^2-6/b/ln(F)^3/d^3/a*polylog(2,-b*F^(d*x)*F^c/a)*x+
6/b/ln(F)^4/d^4/a*polylog(3,-b*F^(d*x)*F^c/a)-3/b/ln(F)^2/d^4*c^2/a*ln(F^c*
F^(d*x)*b+a)+3/b/ln(F)^2/d^4*c^2/a*ln(F^(d*x)*F^c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.76

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{ac^3\log\left(F\right)^3+\left(bd^3x^3+bc^3\right)F^{dx+c}\log\left(F\right)^3-6\left(F^{dx+c}bdx\log\left(F\right)+adx\log\left(F\right)\right)\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)-3}{}$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*c^3*log(F)^3 + (b*d^3*x^3 + b*c^3)*F^(d*x + c)*log(F)^3 - 6*(F^(d*x + c)
*b*d*x*log(F) + a*d*x*log(F))*dilog(-(F^(d*x + c)*b + a)/a + 1) - 3*(F^(d*x
 + c)*b*c^2*log(F)^2 + a*c^2*log(F)^2)*log(F^(d*x + c)*b + a) - 3*((b*d^2*x
^2 - b*c^2)*F^(d*x + c)*log(F)^2 + (a*d^2*x^2 - a*c^2)*log(F)^2)*log((F^(d*
x + c)*b + a)/a) + 6*(F^(d*x + c)*b + a)*polylog(3, -F^(d*x + c)*b/a))/(F^(
d*x + c)*a*b^2*d^4*log(F)^4 + a^2*b*d^4*log(F)^4)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx = -\frac{x^3}{F^{c+dx}b^2d\log{(F)} + abd\log{(F)}} + \frac{3\int \frac{x^2}{a+be^{c\log{(F)}}e^{dx\log{(F)}}}\,dx}{bd\log{(F)}}$$

```
[In] integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c))**2,x)
```

```
[Out] -x**3/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F)) + 3*Integral(x**2/(a + b*
exp(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx \\ &= -\frac{x^3}{F^{dx}F^cb^2d\log{(F)} + abd\log{(F)}} + \frac{x^3}{abd\log{(F)}} \\ &\quad - \frac{3\left(d^2x^2\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log{(F)}^2 + 2\,dx\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log{(F)} - 2\,\mathrm{Li}_3(-\frac{F^{dx}F^cb}{a})\right)}{abd^4\log{(F)}^4}\end{aligned}$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -x^3/(F^(d*x)*F^c*b^2*d*log(F) + a*b*d*log(F)) + x^3/(a*b*d*log(F)) - 3*(d^
2*x^2*log(F^(d*x)*F^c*b/a + 1)*log(F)^2 + 2*d*x*dilog(-F^(d*x)*F^c*b/a)*log
(F) - 2*polylog(3, -F^(d*x)*F^c*b/a))/(a*b*d^4*log(F)^4)
```

Giac [F]

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx = \int \frac{F^{dx+c}x^3}{(F^{dx+c}b+a)^2}\,dx$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)*x^3/(F^(d*x + c)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx=\int \frac{F^{c+d\,x}\,x^3}{\left(a+F^{c+d\,x}\,b\right)^2}\,dx$$

```
[In] int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^2,x)
[Out] int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^2, x)
```

3.83 $\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = \frac{x^2}{abd\log(F)} - \frac{x^2}{bd\,(a+bF^{c+dx})\log(F)} - \frac{2x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{2\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)}$$

```
[Out] x^2/a/b/d/ln(F)-x^2/b/d/(a+b*F^(d*x+c))/ln(F)-2*x*ln(1+b*F^(d*x+c)/a)/a/b/d
^2/ln(F)^2-2*polylog(2,-b*F^(d*x+c)/a)/a/b/d^3/ln(F)^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2222, 2215, 2221, 2317, 2438}

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = -\frac{2\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} - \frac{2x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{abd^2\log^2(F)} - \frac{x^2}{bd\log(F)\,(a+bF^{c+dx})} + \frac{x^2}{abd\log(F)}$$

```
[In] Int[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^2,x]
```

```
[Out] x^2/(a*b*d*Log[F]) - x^2/(b*d*(a + b*F^(c + d*x))*Log[F]) - (2*x*Log[1 + (b
*F^(c + d*x))/a])/(a*b*d^2*Log[F]^2) - (2*PolyLog[2, -((b*F^(c + d*x))/a)])
/(a*b*d^3*Log[F]^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{bd\left(a+bF^{c+dx}\right)\log(F)}+\frac{2\int\frac{x}{a+bF^{c+dx}}\,dx}{bd\log(F)}\\
&= \frac{x^2}{abd\log(F)}-\frac{x^2}{bd\left(a+bF^{c+dx}\right)\log(F)}-\frac{2\int\frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx}{ad\log(F)}\\
&= \frac{x^2}{abd\log(F)}-\frac{x^2}{bd\left(a+bF^{c+dx}\right)\log(F)}-\frac{2x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)}+\frac{2\int\log\left(1+\frac{bF^{c+dx}}{a}\right)\,dx}{abd^2\log^2(F)}
\end{aligned}$$

$$= \frac{x^2}{abd\log(F)} - \frac{x^2}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{2x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} + \frac{2\text{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, F^{c+dx}\right)}{abd^3\log^3(F)}$$

$$= \frac{x^2}{abd\log(F)} - \frac{x^2}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{2x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{2\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int\frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{dx\log(F)\left(bdF^{c+dx}x\log(F) - 2\left(a+bF^{c+dx}\right)\log\left(1+\frac{bF^{c+dx}}{a}\right)\right) - 2\left(a+bF^{c+dx}\right)\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\left(a+bF^{c+dx}\right)\log^3(F)}$$

```
[In] Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^2,x]
```

```
[Out] (d*x*Log[F]*(b*d*F^(c + d*x)*x*Log[F] - 2*(a + b*F^(c + d*x))*Log[1 + (b*F^
(c + d*x))/a]) - 2*(a + b*F^(c + d*x))*PolyLog[2, -((b*F^(c + d*x))/a)])/(a
*b*d^3*(a + b*F^(c + d*x))*Log[F]^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(107) = 214.

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{x^2}{bd\left(a+b\,F^{dx+c}\right)\ln(F)} + \frac{x^2}{abd\ln(F)} + \frac{2cx}{b\ln(F)d^2a} + \frac{c^2}{b\ln(F)d^3a} - \frac{2\ln\left(1+\frac{b\,F^{dx}F^{c}}{a}\right)x}{b\ln(F)^2d^2a} - \frac{2\ln\left(1+\frac{b\,F^{dx}F^{c}}{a}\right)c}{b\ln(F)^2d^3a} - \frac{2\,\text{Li}_2\left(-\frac{b\,F^{d}}{c}\right.}{b\ln(F)^3d}$

```
[In] int(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^2/b/d/(a+b*F^(d*x+c))/ln(F)+x^2/a/b/d/ln(F)+2/b/ln(F)/d^2/a*c*x+1/b/ln(F
)/d^3/a*c^2-2/b/ln(F)^2/d^2/a*ln(1+b*F^(d*x)*F^c/a)*x-2/b/ln(F)^2/d^3/a*ln(
1+b*F^(d*x)*F^c/a)*c-2/b/ln(F)^3/d^3/a*polylog(2,-b*F^(d*x)*F^c/a)+2/b/ln(F
)^2/d^3*c/a*ln(F^c*F^(d*x)*b+a)-2/b/ln(F)^2/d^3*c/a*ln(F^(d*x)*F^c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = -\frac{ac^2\log(F)^2 - (bd^2x^2 - bc^2)F^{dx+c}\log(F)^2 + 2\left(F^{dx+c}b + a\right)\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a} + 1\right) - 2\left(F^{dx+c}bc\log(F) - \right.}{F^{dx+c}ab^2d^3\log(F)^3 + }$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*log(F)^2 - (b*d^2*x^2 - b*c^2)*F^(d*x + c)*log(F)^2 + 2*(F^(d*x + c
)*b + a)*dilog(-(F^(d*x + c)*b + a)/a + 1) - 2*(F^(d*x + c)*b*c*log(F) + a*
c*log(F))*log(F^(d*x + c)*b + a) + 2*((b*d*x + b*c)*F^(d*x + c)*log(F) + (a
*d*x + a*c)*log(F))*log((F^(d*x + c)*b + a)/a))/(F^(d*x + c)*a*b^2*d^3*log(
F)^3 + a^2*b*d^3*log(F)^3)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = -\frac{x^2}{F^{c+dx}b^2d\log(F) + abd\log(F)} + \frac{2\int \frac{x}{a+be^{c\log(F)}e^{dx\log(F)}}\,dx}{bd\log(F)}$$

```
[In] integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c))**2,x)
```

```
[Out] -x**2/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F)) + 2*Integral(x/(a + b*exp
(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = -\frac{x^2}{F^{dx}F^cb^2d\log(F) + abd\log(F)} + \frac{x^2}{abd\log(F)} - \frac{2\left(dx\log\left(\frac{F^{dx}F^cb}{a} + 1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\right)}{abd^3\log(F)^3}$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -x^2/(F^(d*x)*F^c*b^2*d*log(F) + a*b*d*log(F)) + x^2/(a*b*d*log(F)) - 2*(d*
x*log(F^(d*x)*F^c*b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(a*b*d^3*log(F
)^3)
```

Giac [F]

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = \int \frac{F^{dx+c}x^2}{(F^{dx+c}b+a)^2}\,dx$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="giac")
[Out] integrate(F^(d*x + c)*x^2/(F^(d*x + c)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^2}\,dx = \int \frac{F^{c+d\,x}\,x^2}{(a+F^{c+d\,x}\,b)^2}\,dx$$

```
[In] int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^2,x)
[Out] int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^2, x)
```

3.84 $\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \frac{x}{abd\log(F)} - \frac{x}{bd\,(a+bF^{c+dx})\log(F)} - \frac{\log\left(a+bF^{c+dx}\right)}{abd^2\log^2(F)}$$

```
[Out] x/a/b/d/ln(F)-x/b/d/(a+b*F^(d*x+c))/ln(F)-ln(a+b*F^(d*x+c))/a/b/d^2/ln(F)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2222, 2320, 36, 29, 31}

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = -\frac{\log\left(a+bF^{c+dx}\right)}{abd^2\log^2(F)} - \frac{x}{bd\log(F)\,(a+bF^{c+dx})} + \frac{x}{abd\log(F)}$$

```
[In] Int[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^2,x]
```

```
[Out] x/(a*b*d*Log[F]) - x/(b*d*(a + b*F^(c + d*x))*Log[F]) - Log[a + b*F^(c + d*
x)]/(a*b*d^2*Log[F]^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x}{bd\left(a+bF^{c+dx}\right)\log(F)}+\frac{\int \frac{1}{a+bF^{c+dx}}\,dx}{bd\log(F)}\\
&= -\frac{x}{bd\left(a+bF^{c+dx}\right)\log(F)}+\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,F^{c+dx}\right)}{bd^2\log^2(F)}\\
&= -\frac{x}{bd\left(a+bF^{c+dx}\right)\log(F)}-\frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,F^{c+dx}\right)}{ad^2\log^2(F)}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,F^{c+dx}\right)}{abd^2\log^2(F)}\\
&= \frac{x}{abd\log(F)}-\frac{x}{bd\left(a+bF^{c+dx}\right)\log(F)}-\frac{\log\left(a+bF^{c+dx}\right)}{abd^2\log^2(F)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \frac{\frac{dF^{c+dx}x\log(F)}{a+bF^{c+dx}} - \frac{\log(a+bF^{c+dx})}{b}}{ad^2\log^2(F)}$$

```
[In] Integrate[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^2,x]
[Out] ((d*F^(c + d*x)*x*Log[F])/(a + b*F^(c + d*x)) - Log[a + b*F^(c + d*x)]/b)/(
a*d^2*Log[F]^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{x\,\mathrm{e}^{(dx+c)\ln(F)}}{d\ln(F)a\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}-\frac{\ln\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}{\ln(F)^2 b\,d^2 a}$	67
parallelrisch	$\frac{F^{dx+c}xbd\ln(F)-\ln\left(a+b\,F^{dx+c}\right)F^{dx+c}b-\ln\left(a+b\,F^{dx+c}\right)a}{\ln(F)^2 ab\,d^2\left(a+b\,F^{dx+c}\right)}$	79
risch	$\frac{x}{abd\ln(F)}+\frac{c}{\ln(F)b\,d^2 a}-\frac{x}{bd\left(a+b\,F^{dx+c}\right)\ln(F)}-\frac{\ln\left(F^{dx+c}+\frac{a}{b}\right)}{\ln(F)^2 b\,d^2 a}$	87

```
[In] int(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/ln(F)/a*x*exp((d*x+c)*ln(F))/(a+b*exp((d*x+c)*ln(F)))-1/ln(F)^2/b/d^2/a
*ln(a+b*exp((d*x+c)*ln(F)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \frac{F^{dx+c}bdx\log(F) - \left(F^{dx+c}b+a\right)\log\left(F^{dx+c}b+a\right)}{F^{dx+c}ab^2d^2\log(F)^2 + a^2bd^2\log(F)^2}$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
[Out] (F^(d*x + c)*b*d*x*log(F) - (F^(d*x + c)*b + a)*log(F^(d*x + c)*b + a))/(F^
(d*x + c)*a*b^2*d^2*log(F)^2 + a^2*b*d^2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = -\frac{x}{F^{c+dx}b^2d\log(F)+abd\log(F)} + \frac{x}{abd\log(F)} - \frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{abd^2\log(F)^2}$$

```
[In] integrate(F**(d*x+c)*x/(a+b*F**(d*x+c))**2,x)
[Out] -x/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F)) + x/(a*b*d*log(F)) - log(F**
(c + d*x) + a/b)/(a*b*d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \frac{F^{dx}F^cx}{F^{dx}F^cabd\log(F)+a^2d\log(F)} - \frac{\log\left(\frac{F^{dx}F^cb+a}{F^cb}\right)}{abd^2\log(F)^2}$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
[Out] F^(d*x)*F^c*x/(F^(d*x)*F^c*a*b*d*log(F) + a^2*d*log(F)) - log((F^(d*x)*F^c*
b + a)/(F^c*b))/(a*b*d^2*log(F)^2)
```

Giac [F]

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \int \frac{F^{dx+c}x}{(F^{dx+c}b+a)^2}\,dx$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x, algorithm="giac")
[Out] integrate(F^(d*x + c)*x/(F^(d*x + c)*b + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \frac{F^c\,F^{d\,x}\,x}{a\,d\,\ln(F)\,\left(a+F^c\,F^{d\,x}\,b\right)} - \frac{\ln\left(a+F^c\,F^{d\,x}\,b\right)}{a\,b\,d^2\ln(F)^2}$$

```
[In] int((F^(c + d*x)*x)/(a + F^(c + d*x)*b)^2,x)
[Out] (F^c*F^(d*x)*x)/(a*d*log(F)*(a + F^c*F^(d*x)*b)) - log(a + F^c*F^(d*x)*b)/(
a*b*d^2*log(F)^2)
```

3.85 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2} dx = -\frac{1}{bd(a+bF^{c+dx})\log(F)}$$

```
[Out] -1/b/d/(a+b*F^(d*x+c))/ln(F)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2278, 32}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2} dx = -\frac{1}{bd\log(F)(a+bF^{c+dx})}$$

```
[In] Int[F^(c + d*x)/(a + b*F^(c + d*x))^2,x]
```

```
[Out] -(1/(b*d*(a + b*F^(c + d*x))*Log[F]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+bx)^2}\,dx, x, F^{c+dx}\right)}{d\log(F)}$$
$$= -\frac{1}{bd\left(a+bF^{c+dx}\right)\log(F)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{bd\left(a+bF^{c+dx}\right)\log(F)}$$

```
[In] Integrate[F^(c + d*x)/(a + b*F^(c + d*x))^2,x]
[Out] -(1/(b*d*(a + b*F^(c + d*x))*Log[F]))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln\left(F\right)}$	26
default	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln\left(F\right)}$	26
risch	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln\left(F\right)}$	26
parallelrisch	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln\left(F\right)}$	26
norman	$\frac{{\rm e}^{(dx+c)\ln(F)}}{d\ln\left(F\right)a\left(a+b\,{\rm e}^{(dx+c)\ln(F)}\right)}$	36

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/b/d/(a+b*F^(d*x+c))/ln(F)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2}\,dx = -\frac{1}{F^{dx+c}b^2d\log(F)+abd\log(F)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
[Out] -1/(F^(d*x + c)*b^2*d*log(F) + a*b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2}\,dx = -\frac{1}{F^{c+dx}b^2d\log(F)+abd\log(F)}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))**2,x)
[Out] -1/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2}\,dx = -\frac{1}{(F^{dx+c}b+a)bd\log(F)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
[Out] -1/((F^(d*x + c)*b + a)*b*d*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2}\,dx = -\frac{1}{(F^{dx}F^cb+a)bd\log(F)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2,x, algorithm="giac")
[Out] -1/((F^(d*x)*F^c*b + a)*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2}\,dx = \frac{F^{c+d\,x}}{a^2\,d\,\ln(F) + F^{c+d\,x}\,a\,b\,d\,\ln(F)}$$

```
[In] int(F^(c + d*x)/(a + F^(c + d*x)*b)^2,x)
[Out] F^(c + d*x)/(a^2*d*log(F) + F^(c + d*x)*a*b*d*log(F))
```

3.86 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2\,x}\,dx = -\frac{1}{bd\,(a+bF^{c+dx})\,x\log(F)} - \frac{\operatorname{Int}\left(\frac{1}{(a+bF^{c+dx})x^2},x\right)}{bd\log(F)}$$

```
[Out] -1/b/d/(a+b*F^(d*x+c))/x/ln(F)-Unintegrable(1/(a+b*F^(d*x+c))/x^2,x)/b/d/ln
(F)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2\,x}\,dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})^2\,x}\,dx$$

```
[In] Int[F^(c + d*x)/((a + b*F^(c + d*x))^2*x),x]
```

```
[Out] -(1/(b*d*(a + b*F^(c + d*x))*x*Log[F])) - Defer[Int][1/((a + b*F^(c + d*x))
*x^2), x]/(b*d*Log[F])
```

Rubi steps

$$\text{integral} = -\frac{1}{bd\,(a+bF^{c+dx})\,x\log(F)} - \frac{\int \frac{1}{(a+bF^{c+dx})x^2}\,dx}{bd\log(F)}$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} \, dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} \, dx$$

```
[In] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x),x]
[Out] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})^2\,x}dx$$

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x)
[Out] int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} \, dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^2 x} \, dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x, algorithm="fricas")
[Out] integral(F^(d*x + c)/(2*F^(d*x + c)*a*b*x + F^(2*d*x + 2*c)*b^2*x + a^2*x),
 x)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2\, x}\, dx = -\frac{1}{F^{c+dx}b^2dx\log(F)+abdx\log(F)} - \frac{\int \frac{1}{ax^2+bx^2e^{c\log(F)}e^{dx\log(F)}}\, dx}{bd\log(F)}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))**2/x,x)
```

```
[Out] -1/(F**(c + d*x)*b**2*d*x*log(F) + a*b*d*x*log(F)) - Integral(1/(a*x**2 + b
*x**2*exp(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2\, x}\, dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^2x}\, dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x, algorithm="maxima")
```

```
[Out] -1/(F^(d*x)*F^c*b^2*d*x*log(F) + a*b*d*x*log(F)) - integrate(1/(F^(d*x)*F^c
*b^2*d*x^2*log(F) + a*b*d*x^2*log(F)), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2\, x}\, dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^2x}\, dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x} dx = \int \frac{F^{c+dx}}{x\,(a+F^{c+dx}\,b)^2} dx$$

```
[In] int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^2),x)
[Out] int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^2), x)
```

3.87 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2x^2}\,dx=-\frac{1}{bd\left(a+bF^{c+dx}\right)x^2\log(F)}-\frac{2\text{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)x^3},x\right)}{bd\log(F)}$$

```
[Out] -1/b/d/(a+b*F^(d*x+c))/x^2/ln(F)-2*Unintegrable(1/(a+b*F^(d*x+c))/x^3,x)/b/
d/ln(F)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2x^2}\,dx=\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2x^2}\,dx$$

```
[In] Int[F^(c + d*x)/((a + b*F^(c + d*x))^2*x^2),x]
```

```
[Out] -(1/(b*d*(a + b*F^(c + d*x))*x^2*Log[F])) - (2*Defer[Int][1/((a + b*F^(c +
d*x))*x^3), x])/(b*d*Log[F])
```

Rubi steps

$$\text{integral}=-\frac{1}{bd\left(a+bF^{c+dx}\right)x^2\log(F)}-\frac{2\int \frac{1}{\left(a+bF^{c+dx}\right)x^3}\,dx}{bd\log(F)}$$

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} \, dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} \, dx$$

```
[In] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x^2),x]
[Out] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})^2 \, x^2}dx$$

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x)
[Out] int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2} \, dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^2 x^2} \, dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x, algorithm="fricas")
[Out] integral(F^(d*x + c)/(2*F^(d*x + c)*a*b*x^2 + F^(2*d*x + 2*c)*b^2*x^2 + a^2
*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2} dx = -\frac{1}{F^{c+dx}b^2dx^2\log(F)+abdx^2\log(F)} - \frac{2\int \frac{1}{ax^3+bx^3e^{c\log(F)}e^{dx\log(F)}} dx}{bd\log(F)}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))**2/x**2,x)
```

```
[Out] -1/(F**(c + d*x)*b**2*d*x**2*log(F) + a*b*d*x**2*log(F)) - 2*Integral(1/(a*
x**3 + b*x**3*exp(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2} dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x^2} dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/(F^(d*x)*F^c*b^2*d*x^2*log(F) + a*b*d*x^2*log(F)) - 2*integrate(1/(F^(d*
x)*F^c*b^2*d*x^3*log(F) + a*b*d*x^3*log(F)), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2} dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x^2} dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = \int \frac{F^{c+d\,x}}{x^2\,\left(a+F^{c+d\,x}\,b\right)^2}\,dx$$

```
[In] int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^2),x)
[Out] int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^2), x)
```

3.88 $\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx &= -\frac{3x^2}{2a^2bd^2\log^2(F)}+\frac{3x^2}{2abd^2(a+bF^{c+dx})\log^2(F)}+\frac{x^3}{2a^2bd\log(F)} \\ &\quad -\frac{x^3}{2bd(a+bF^{c+dx})^2\log(F)}+\frac{3x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)} \\ &\quad -\frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{2a^2bd^2\log^2(F)}+\frac{3\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)} \\ &\quad -\frac{3x\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}+\frac{3\operatorname{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)}\end{aligned}$$

```
[Out] -3/2*x^2/a^2/b/d^2/ln(F)^2+3/2*x^2/a/b/d^2/(a+b*F^(d*x+c))/ln(F)^2+1/2*x^3/
a^2/b/d/ln(F)-1/2*x^3/b/d/(a+b*F^(d*x+c))^2/ln(F)+3*x*ln(1+b*F^(d*x+c)/a)/a
^2/b/d^3/ln(F)^3-3/2*x^2*ln(1+b*F^(d*x+c)/a)/a^2/b/d^2/ln(F)^2+3*polylog(2,
-b*F^(d*x+c)/a)/a^2/b/d^4/ln(F)^4-3*x*polylog(2,-b*F^(d*x+c)/a)/a^2/b/d^3/l
n(F)^3+3*polylog(3,-b*F^(d*x+c)/a)/a^2/b/d^4/ln(F)^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2222, 2216, 2215, 2221, 2611, 2320, 6724, 2317, 2438}

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx = \frac{3\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)}+\frac{3\,\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)} - \frac{3x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}+\frac{3x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{a^2bd^3\log^3(F)} - \frac{3x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{2a^2bd^2\log^2(F)}-\frac{3x^2}{2a^2bd^2\log^2(F)}+\frac{x^3}{2a^2bd\log(F)} + \frac{3x^2}{2abd^2\log^2(F)\,(a+bF^{c+dx})}-\frac{x^3}{2bd\log(F)\,(a+bF^{c+dx})^2}$$

```
[In] Int[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] (-3*x^2)/(2*a^2*b*d^2*Log[F]^2) + (3*x^2)/(2*a*b*d^2*(a + b*F^(c + d*x))*Lo
g[F]^2) + x^3/(2*a^2*b*d*Log[F]) - x^3/(2*b*d*(a + b*F^(c + d*x))^2*Log[F])
 + (3*x*Log[1 + (b*F^(c + d*x))/a])/(a^2*b*d^3*Log[F]^3) - (3*x^2*Log[1 + (
b*F^(c + d*x))/a])/(2*a^2*b*d^2*Log[F]^2) + (3*PolyLog[2, -((b*F^(c + d*x))
/a)])/(a^2*b*d^4*Log[F]^4) - (3*x*PolyLog[2, -((b*F^(c + d*x))/a)])/(a^2*b*
d^3*Log[F]^3) + (3*PolyLog[3, -((b*F^(c + d*x))/a)])/(a^2*b*d^4*Log[F]^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{3\int\frac{x^2}{\left(a+bF^{c+dx}\right)^2}\,dx}{2bd\log(F)}\\
&= -\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}-\frac{3\int\frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx}{2ad\log(F)}+\frac{3\int\frac{x^2}{a+bF^{c+dx}}\,dx}{2abd\log(F)}\\
&= \frac{3x^2}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^3}{2a^2bd\log(F)}\\
&\quad -\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}-\frac{3\int\frac{x}{a+bF^{c+dx}}\,dx}{abd^2\log^2(F)}-\frac{3\int\frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx}{2a^2d\log(F)}\\
&= -\frac{3x^2}{2a^2bd^2\log^2(F)}+\frac{3x^2}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^3}{2a^2bd\log(F)}\\
&\quad -\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}-\frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{2a^2bd^2\log^2(F)}\\
&\quad +\frac{3\int\frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx}{a^2d^2\log^2(F)}+\frac{3\int x\log\left(1+\frac{bF^{c+dx}}{a}\right)dx}{a^2bd^2\log^2(F)}\\
&= -\frac{3x^2}{2a^2bd^2\log^2(F)}+\frac{3x^2}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^3}{2a^2bd\log(F)}\\
&\quad -\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{3x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}-\frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{2a^2bd^2\log^2(F)}\\
&\quad -\frac{3x\operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}-\frac{3\int\log\left(1+\frac{bF^{c+dx}}{a}\right)dx}{a^2bd^3\log^3(F)}+\frac{3\int\operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)dx}{a^2bd^3\log^3(F)}\\
&= -\frac{3x^2}{2a^2bd^2\log^2(F)}+\frac{3x^2}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^3}{2a^2bd\log(F)}\\
&\quad -\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{3x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}\\
&\quad -\frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{2a^2bd^2\log^2(F)}-\frac{3x\operatorname{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}\\
&\quad -\frac{3\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx,x,F^{c+dx}\right)}{a^2bd^4\log^4(F)}+\frac{3\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx,x,F^{c+dx}\right)}{a^2bd^4\log^4(F)}
\end{aligned}$$

$$= -\frac{3x^2}{2a^2bd^2\log^2(F)} + \frac{3x^2}{2abd^2(a+bF^{c+dx})\log^2(F)} + \frac{x^3}{2a^2bd\log(F)} - \frac{x^3}{2bd(a+bF^{c+dx})^2\log(F)} + \frac{3x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{2a^2bd^2\log^2(F)} + \frac{3\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)} - \frac{3x\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)} + \frac{3\text{Li}_3\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.84

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx = \frac{dx\log(F)\left(bd^2F^{c+dx}(2a+bF^{c+dx})\,x^2\log^2(F)+6\left(a+bF^{c+dx}\right)^2\log\left(1+\frac{bF^{c+dx}}{a}\right)-3d\left(a+bF^{c+dx}\right)x\log\right.}{}$$

```
[In] Integrate[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] (d*x*Log[F]*(b*d^2*F^(c + d*x)*(2*a + b*F^(c + d*x))*x^2*Log[F]^2 + 6*(a +
b*F^(c + d*x))^2*Log[1 + (b*F^(c + d*x))/a] - 3*d*(a + b*F^(c + d*x))*x*Log
[F]*(b*F^(c + d*x) + (a + b*F^(c + d*x))*Log[1 + (b*F^(c + d*x))/a])) - 6*(
a + b*F^(c + d*x))^2*(-1 + d*x*Log[F])*PolyLog[2, -((b*F^(c + d*x))/a)] + 6
*(a + b*F^(c + d*x))^2*PolyLog[3, -((b*F^(c + d*x))/a)])/(2*a^2*b*d^4*(a +
b*F^(c + d*x))^2*Log[F]^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{x^2(\ln(F)adx-3bF^{dx+c}-3a)}{2\ln(F)^2d^2b(a+bF^{dx+c})^2a}-\frac{3c^2x}{2ba^2d^3\ln(F)}-\frac{3\ln\left(1+\frac{bF^{dx}F^c}{a}\right)x^2}{2ba^2d^2\ln(F)^2}-\frac{3\,\text{Li}_2\left(-\frac{bF^{dx}F^c}{a}\right)x}{ba^2d^3\ln(F)^3}-\frac{3cx}{ba^2d^3\ln(F)^2}+\frac{3\ln\left(1+\frac{b}{}\right.}{ba^2d^3}$

```
[In] int(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*(ln(F)*a*d*x-3*b*F^(d*x+c)-3*a)/ln(F)^2/d^2/b/(a+b*F^(d*x+c))^2/a-
3/2/b/a^2/d^3/ln(F)*c^2*x-3/2/b/a^2/d^2/ln(F)^2*ln(1+b*F^(d*x)*F^c/a)*x^2-3
/b/a^2/d^3/ln(F)^3*polylog(2,-b*F^(d*x)*F^c/a)*x-3/b/a^2/d^3/ln(F)^2*c*x+3/
b/a^2/d^3/ln(F)^3*ln(1+b*F^(d*x)*F^c/a)*x+3/b/a^2/d^4/ln(F)^3*ln(1+b*F^(d*x
)*F^c/a)*c+1/2*x^3/a^2/b/d/ln(F)+3/2/b/a^2/d^4/ln(F)^2*ln(1+b*F^(d*x)*F^c/a
)*c^2-3/2/b/a^2/d^4/ln(F)^2*c^2*ln(F^c*F^(d*x)*b+a)+3/2/b/a^2/d^4/ln(F)^2*c
^2*ln(F^(d*x)*F^c)-3/2*x^2/a^2/b/d^2/ln(F)^2-3/b/a^2/d^4/ln(F)^3*c*ln(F^c*F
```

```
^(d*x)*b+a)+3/b/a^2/d^4/ln(F)^3*c*ln(F^(d*x)*F^c)-1/b/a^2/d^4/ln(F)*c^3+3/b
/a^2/d^4/ln(F)^4*polylog(2,-b*F^(d*x)*F^c/a)-3/2/b/a^2/d^4/ln(F)^2*c^2+3/b/
a^2/d^4/ln(F)^4*polylog(3,-b*F^(d*x)*F^c/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(249) = 498.

Time = 0.28 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.21

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx$$

$$= \frac{a^2c^3\log(F)^3 + 3\,a^2c^2\log(F)^2 + \left((b^2d^3x^3+b^2c^3)\log(F)^3 - 3\,(b^2d^2x^2-b^2c^2)\log(F)^2\right)F^{2\,dx+2\,c} + \left(2\,(abd^3x\right.}{}$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*c^3*log(F)^3 + 3*a^2*c^2*log(F)^2 + ((b^2*d^3*x^3 + b^2*c^3)*log(F
)^3 - 3*(b^2*d^2*x^2 - b^2*c^2)*log(F)^2)*F^(2*d*x + 2*c) + (2*(a*b*d^3*x^3
 + a*b*c^3)*log(F)^3 - 3*(a*b*d^2*x^2 - 2*a*b*c^2)*log(F)^2)*F^(d*x + c) -
6*(a^2*d*x*log(F) + (b^2*d*x*log(F) - b^2)*F^(2*d*x + 2*c) + 2*(a*b*d*x*log
(F) - a*b)*F^(d*x + c) - a^2)*dilog(-(F^(d*x + c)*b + a)/a + 1) - 3*(a^2*c^
2*log(F)^2 + 2*a^2*c*log(F) + (b^2*c^2*log(F)^2 + 2*b^2*c*log(F))*F^(2*d*x
+ 2*c) + 2*(a*b*c^2*log(F)^2 + 2*a*b*c*log(F))*F^(d*x + c))*log(F^(d*x + c)
*b + a) - 3*((a^2*d^2*x^2 - a^2*c^2)*log(F)^2 + ((b^2*d^2*x^2 - b^2*c^2)*lo
g(F)^2 - 2*(b^2*d*x + b^2*c)*log(F))*F^(2*d*x + 2*c) + 2*((a*b*d^2*x^2 - a*
b*c^2)*log(F)^2 - 2*(a*b*d*x + a*b*c)*log(F))*F^(d*x + c) - 2*(a^2*d*x + a^
2*c)*log(F))*log((F^(d*x + c)*b + a)/a) + 6*(2*F^(d*x + c)*a*b + F^(2*d*x +
 2*c)*b^2 + a^2)*polylog(3, -F^(d*x + c)*b/a))/(2*F^(d*x + c)*a^3*b^2*d^4*l
og(F)^4 + F^(2*d*x + 2*c)*a^2*b^3*d^4*log(F)^4 + a^4*b*d^4*log(F)^4)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx = \frac{3F^{c+dx}bx^2 - adx^3\log(F) + 3ax^2}{4F^{c+dx}a^2b^2d^2\log(F)^2 + 2F^{2c+2dx}ab^3d^2\log(F)^2 + 2a^3bd^2\log(F)^2} + \frac{3\left(\int\left(-\frac{2x}{a+be^{c\log(F)}e^{dx\log(F)}}\right)dx + \int\frac{dx^2\log(F)}{a+be^{c\log(F)}e^{dx\log(F)}}\,dx\right)}{2abd^2\log(F)^2}$$

```
[In] integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c))**3,x)
```

```
[Out] (3*F**(c + d*x)*b*x**2 - a*d*x**3*log(F) + 3*a*x**2)/(4*F**(c + d*x)*a**2*b
**2*d**2*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*d*
*2*log(F)**2) + 3*(Integral(-2*x/(a + b*exp(c*log(F))*exp(d*x*log(F))), x)
+ Integral(d*x**2*log(F)/(a + b*exp(c*log(F))*exp(d*x*log(F))), x))/(2*a*b*
d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx$$
$$= -\frac{adx^3\log(F) - 3F^{dx}F^cbx^2 - 3ax^2}{2\left(2F^{dx}F^ca^2b^2d^2\log(F)^2 + F^{2dx}F^{2c}ab^3d^2\log(F)^2 + a^3bd^2\log(F)^2\right)}$$
$$+ \frac{d^3x^3\log(F)^3 - 3d^2x^2\log(F)^2}{2a^2bd^4\log(F)^4}$$
$$- \frac{3\left(d^2x^2\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F)^2 + 2dx\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log(F) - 2\,\mathrm{Li}_3(-\frac{F^{dx}F^cb}{a})\right)}{2a^2bd^4\log(F)^4}$$
$$+ \frac{3\left(dx\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\right)}{a^2bd^4\log(F)^4}$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*d*x^3*log(F) - 3*F^(d*x)*F^c*b*x^2 - 3*a*x^2)/(2*F^(d*x)*F^c*a^2*b^
2*d^2*log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*log(F)^2 + a^3*b*d^2*log(F)^2)
 + 1/2*(d^3*x^3*log(F)^3 - 3*d^2*x^2*log(F)^2)/(a^2*b*d^4*log(F)^4) - 3/2*(
d^2*x^2*log(F^(d*x)*F^c*b/a + 1)*log(F)^2 + 2*d*x*dilog(-F^(d*x)*F^c*b/a)*l
og(F) - 2*polylog(3, -F^(d*x)*F^c*b/a))/(a^2*b*d^4*log(F)^4) + 3*(d*x*log(F
^(d*x)*F^c*b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(a^2*b*d^4*log(F)^4)
```

Giac [F]

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx = \int \frac{F^{dx+c}x^3}{(F^{dx+c}b+a)^3}\,dx$$

```
[In] integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)*x^3/(F^(d*x + c)*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^3}\,dx = \int \frac{F^{c+d\,x}\,x^3}{(a+F^{c+d\,x}\,b)^3}\,dx$$

```
[In] int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^3,x)
[Out] int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^3, x)
```

3.89 $\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx = -\frac{x}{a^2bd^2\log^2(F)}+\frac{x}{abd^2(a+bF^{c+dx})\log^2(F)}+\frac{x^2}{2a^2bd\log(F)} -\frac{x^2}{2bd(a+bF^{c+dx})^2\log(F)}+\frac{\log(a+bF^{c+dx})}{a^2bd^3\log^3(F)} -\frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^2\log^2(F)}-\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}$$

```
[Out] -x/a^2/b/d^2/ln(F)^2+x/a/b/d^2/(a+b*F^(d*x+c))/ln(F)^2+1/2*x^2/a^2/b/d/ln(F
)-1/2*x^2/b/d/(a+b*F^(d*x+c))^2/ln(F)+ln(a+b*F^(d*x+c))/a^2/b/d^3/ln(F)^3-x
*ln(1+b*F^(d*x+c)/a)/a^2/b/d^2/ln(F)^2-polylog(2,-b*F^(d*x+c)/a)/a^2/b/d^3/
ln(F)^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2222, 2216, 2215, 2221, 2317, 2438, 2320, 36, 29, 31}

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx = -\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}+\frac{\log(a+bF^{c+dx})}{a^2bd^3\log^3(F)} -\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{a^2bd^2\log^2(F)}-\frac{x}{a^2bd^2\log^2(F)}+\frac{x^2}{2a^2bd\log(F)} +\frac{x}{abd^2\log^2(F)(a+bF^{c+dx})}-\frac{x^2}{2bd\log(F)(a+bF^{c+dx})^2}$$

```
[In] Int[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] -(x/(a^2*b*d^2*Log[F]^2)) + x/(a*b*d^2*(a + b*F^(c + d*x))*Log[F]^2) + x^2/
(2*a^2*b*d*Log[F]) - x^2/(2*b*d*(a + b*F^(c + d*x))^2*Log[F]) + Log[a + b*F
^(c + d*x)]/(a^2*b*d^3*Log[F]^3) - (x*Log[1 + (b*F^(c + d*x))/a])/(a^2*b*d^
2*Log[F]^2) - PolyLog[2, -((b*F^(c + d*x))/a)]/(a^2*b*d^3*Log[F]^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
```

```
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{\int\frac{x}{\left(a+bF^{c+dx}\right)^2}\,dx}{bd\log(F)}\\
&= -\frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}-\frac{\int\frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx}{ad\log(F)}+\frac{\int\frac{x}{a+bF^{c+dx}}\,dx}{abd\log(F)}\\
&= \frac{x}{abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^2}{2a^2bd\log(F)}\\
&\quad -\frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}-\frac{\int\frac{1}{a+bF^{c+dx}}\,dx}{abd^2\log^2(F)}-\frac{\int\frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx}{a^2d\log(F)}\\
&= \frac{x}{abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^2}{2a^2bd\log(F)}-\frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}\\
&\quad -\frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^2\log^2(F)}-\frac{\operatorname{Subst}\left(\int\frac{1}{x(a+bx)}\,dx,x,F^{c+dx}\right)}{abd^3\log^3(F)}+\frac{\int\log\left(1+\frac{bF^{c+dx}}{a}\right)\,dx}{a^2bd^2\log^2(F)}
\end{aligned}$$

$$= \frac{x}{abd^2\left(a+bF^{c+dx}\right)\log^2(F)} + \frac{x^2}{2a^2bd\log(F)} - \frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)} - \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^2\log^2(F)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, F^{c+dx}\right)}{a^2d^3\log^3(F)} - \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, F^{c+dx}\right)}{a^2bd^3\log^3(F)} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, F^{c+dx}\right)}{a^2bd^3\log^3(F)}$$

$$= -\frac{x}{a^2bd^2\log^2(F)} + \frac{x}{abd^2\left(a+bF^{c+dx}\right)\log^2(F)} + \frac{x^2}{2a^2bd\log(F)} - \frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)} + \frac{\log\left(a+bF^{c+dx}\right)}{a^2bd^3\log^3(F)} - \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^2\log^2(F)} - \frac{\text{Li}_2\left(-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx = \frac{bd^2F^{c+dx}\left(2a+bF^{c+dx}\right)x^2\log^2(F) + 2\left(a+bF^{c+dx}\right)^2\log\left(1+\frac{bF^{c+dx}}{a}\right) - 2d\left(a+bF^{c+dx}\right)x\log(F)\left(bF^{c+dx} \cdots\right.}{2a^2bd^3\left(a+bF^{c+dx}\right)^2\log^3(F)}$$

```
[In] Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] (b*d^2*F^(c + d*x)*(2*a + b*F^(c + d*x))*x^2*Log[F]^2 + 2*(a + b*F^(c + d*x
))^2*Log[1 + (b*F^(c + d*x))/a] - 2*d*(a + b*F^(c + d*x))*x*Log[F]*(b*F^(c
+ d*x) + (a + b*F^(c + d*x))*Log[1 + (b*F^(c + d*x))/a]) - 2*(a + b*F^(c +
d*x))^2*PolyLog[2, -((b*F^(c + d*x))/a)])/(2*a^2*b*d^3*(a + b*F^(c + d*x))^
2*Log[F]^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{x\left(\ln(F)adx-2b\,F^{dx+c}-2a\right)}{2\ln(F)^2d^2ab\left(a+b\,F^{dx+c}\right)^2}+\frac{x^2}{2a^2bd\ln(F)}+\frac{cx}{b\,a^2d^2\ln(F)}+\frac{c^2}{2b\,a^2d^3\ln(F)}-\frac{\ln\left(1+\frac{b\,F^{dx}F^c}{a}\right)x}{b\,a^2d^2\ln(F)^2}-\frac{\ln\left(1+\frac{b\,F^{dx}F^c}{a}\right)c}{b\,a^2d^3\ln(F)^2}-\frac{\text{Li}_2 \cdots}{b \cdots}$

```
[In] int(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(ln(F)*a*d*x-2*b*F^(d*x+c)-2*a)/ln(F)^2/d^2/a/b/(a+b*F^(d*x+c))^2+1/
2*x^2/a^2/b/d/ln(F)+1/b/a^2/d^2/ln(F)*c*x+1/2/b/a^2/d^3/ln(F)*c^2-1/b/a^2/d
```

```
^2/ln(F)^2*ln(1+b*F^(d*x)*F^c/a)*x-1/b/a^2/d^3/ln(F)^2*ln(1+b*F^(d*x)*F^c/a
)*c-1/b/a^2/d^3/ln(F)^3*polylog(2,-b*F^(d*x)*F^c/a)+1/b/a^2/d^3/ln(F)^3*ln(
F^c*F^(d*x)*b+a)-1/b/a^2/d^3/ln(F)^3*ln(F^(d*x)*F^c)+1/b/a^2/d^3/ln(F)^2*c*
ln(F^c*F^(d*x)*b+a)-1/b/a^2/d^3/ln(F)^2*c*ln(F^(d*x)*F^c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(177) = 354$.

Time = 0.26 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.08

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx = -\frac{a^2c^2\log(F)^2 + 2a^2c\log(F) - \left((b^2d^2x^2 - b^2c^2)\log(F)^2 - 2(b^2dx + b^2c)\log(F)\right)F^{2dx+2c} - 2\left((abd^2x^2\right.}{}$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c^2*log(F)^2 + 2*a^2*c*log(F) - ((b^2*d^2*x^2 - b^2*c^2)*log(F)^2
 - 2*(b^2*d*x + b^2*c)*log(F))*F^(2*d*x + 2*c) - 2*((a*b*d^2*x^2 - a*b*c^2)
*log(F)^2 - (a*b*d*x + 2*a*b*c)*log(F))*F^(d*x + c) + 2*(2*F^(d*x + c)*a*b
+ F^(2*d*x + 2*c)*b^2 + a^2)*dilog(-(F^(d*x + c)*b + a)/a + 1) - 2*(a^2*c*l
og(F) + (b^2*c*log(F) + b^2)*F^(2*d*x + 2*c) + 2*(a*b*c*log(F) + a*b)*F^(d*
x + c) + a^2)*log(F^(d*x + c)*b + a) + 2*((b^2*d*x + b^2*c)*F^(2*d*x + 2*c)
*log(F) + 2*(a*b*d*x + a*b*c)*F^(d*x + c)*log(F) + (a^2*d*x + a^2*c)*log(F)
)*log((F^(d*x + c)*b + a)/a))/(2*F^(d*x + c)*a^3*b^2*d^3*log(F)^3 + F^(2*d*
x + 2*c)*a^2*b^3*d^3*log(F)^3 + a^4*b*d^3*log(F)^3)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx = \frac{2F^{c+dx}bx - adx^2\log(F) + 2ax}{4F^{c+dx}a^2b^2d^2\log(F)^2 + 2F^{2c+2dx}ab^3d^2\log(F)^2 + 2a^3bd^2\log(F)^2} + \frac{\int \frac{dx\log(F)}{a+be^{c\log(F)}e^{dx\log(F)}}\,dx + \int\left(-\frac{1}{a+be^{c\log(F)}e^{dx\log(F)}}\right)dx}{abd^2\log(F)^2}$$

```
[In] integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c))**3,x)
```

```
[Out] (2*F**(c + d*x)*b*x - a*d*x**2*log(F) + 2*a*x)/(4*F**(c + d*x)*a**2*b**2*d*
*2*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*d**2*log
(F)**2) + (Integral(d*x*log(F)/(a + b*exp(c*log(F))*exp(d*x*log(F))), x) +
Integral(-1/(a + b*exp(c*log(F))*exp(d*x*log(F))), x))/(a*b*d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.11

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx$$
$$= -\frac{adx^2\log(F) - 2F^{dx}F^cbx - 2ax}{2\left(2F^{dx}F^ca^2b^2d^2\log(F)^2 + F^{2dx}F^{2c}ab^3d^2\log(F)^2 + a^3bd^2\log(F)^2\right)} + \frac{x^2}{2a^2bd\log(F)}$$
$$- \frac{x}{a^2bd^2\log(F)^2} - \frac{dx\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)}{a^2bd^3\log(F)^3} + \frac{\log\left(F^{dx}F^cb+a\right)}{a^2bd^3\log(F)^3}$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*d*x^2*log(F) - 2*F^(d*x)*F^c*b*x - 2*a*x)/(2*F^(d*x)*F^c*a^2*b^2*d^
2*log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*log(F)^2 + a^3*b*d^2*log(F)^2) + 1
/2*x^2/(a^2*b*d*log(F)) - x/(a^2*b*d^2*log(F)^2) - (d*x*log(F^(d*x)*F^c*b/a
 + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(a^2*b*d^3*log(F)^3) + log(F^(d*x)*
F^c*b + a)/(a^2*b*d^3*log(F)^3)
```

Giac [F]

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx = \int \frac{F^{dx+c}x^2}{(F^{dx+c}b+a)^3}\,dx$$

```
[In] integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)*x^2/(F^(d*x + c)*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^2}{(a+bF^{c+dx})^3}\,dx = \int \frac{F^{c+d\,x}\,x^2}{(a+F^{c+d\,x}\,b)^3}\,dx$$

```
[In] int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^3,x)
```

```
[Out] int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^3, x)
```

3.90 $\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3} dx = \frac{1}{2abd^2(a+bF^{c+dx})\log^2(F)} + \frac{x}{2a^2bd\log(F)} - \frac{x}{2bd(a+bF^{c+dx})^2\log(F)} - \frac{\log(a+bF^{c+dx})}{2a^2bd^2\log^2(F)}$$

```
[Out] 1/2/a/b/d^2/(a+b*F^(d*x+c))/ln(F)^2+1/2*x/a^2/b/d/ln(F)-1/2*x/b/d/(a+b*F^(d
*x+c))^2/ln(F)-1/2*ln(a+b*F^(d*x+c))/a^2/b/d^2/ln(F)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2222, 2320, 46}

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3} dx = -\frac{\log(a+bF^{c+dx})}{2a^2bd^2\log^2(F)} + \frac{x}{2a^2bd\log(F)} + \frac{1}{2abd^2\log^2(F)(a+bF^{c+dx})} - \frac{x}{2bd\log(F)(a+bF^{c+dx})^2}$$

```
[In] Int[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] 1/(2*a*b*d^2*(a + b*F^(c + d*x))*Log[F]^2) + x/(2*a^2*b*d*Log[F]) - x/(2*b*
d*(a + b*F^(c + d*x))^2*Log[F]) - Log[a + b*F^(c + d*x)]/(2*a^2*b*d^2*Log[F
]^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{\int\frac{1}{\left(a+bF^{c+dx}\right)^2}\,dx}{2bd\log(F)}\\
&= -\frac{x}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{\text{Subst}\left(\int\frac{1}{x(a+bx)^2}\,dx,x,F^{c+dx}\right)}{2bd^2\log^2(F)}\\
&= -\frac{x}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{\text{Subst}\left(\int\left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx,x,F^{c+dx}\right)}{2bd^2\log^2(F)}\\
&= \frac{1}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x}{2a^2bd\log(F)}-\frac{x}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}-\frac{\log\left(a+bF^{c+dx}\right)}{2a^2bd^2\log^2(F)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3}\,dx = \frac{bdF^{c+dx}\left(2a+bF^{c+dx}\right)x\log(F)-\left(a+bF^{c+dx}\right)\left(-a+\left(a+bF^{c+dx}\right)\log\left(a+bF^{c+dx}\right)\right)}{2a^2bd^2\left(a+bF^{c+dx}\right)^2\log^2(F)}$$

```
[In] Integrate[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] (b*d*F^(c + d*x)*(2*a + b*F^(c + d*x))*x*Log[F] - (a + b*F^(c + d*x))*(-a +
 (a + b*F^(c + d*x))*Log[a + b*F^(c + d*x)]))/(2*a^2*b*d^2*(a + b*F^(c + d*
x))^2*Log[F]^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05

method	result
risch	$\frac{x}{2a^2bd\ln(F)}+\frac{c}{2a^2b\,d^2\ln(F)}-\frac{\ln(F)adx-b\,F^{dx+c}-a}{2d^2\ln(F)^2ab\left(a+b\,F^{dx+c}\right)^2}-\frac{\ln\left(F^{dx+c}+\frac{a}{b}\right)}{2a^2b\,d^2\ln(F)^2}$
norman	$\frac{\frac{\mathrm{e}^{(dx+c)\ln(F)}}{2\ln(F)^2a\,d^2}+\frac{x\,\mathrm{e}^{(dx+c)\ln(F)}}{d\ln(F)a}+\frac{bx\,\mathrm{e}^{(2dx+2c)\ln(F)}}{2\ln(F)a^2d}+\frac{1}{2\ln(F)^2b\,d^2}}{\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)^2}-\frac{\ln\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}{2a^2b\,d^2\ln(F)^2}$
parallelrisch	$\frac{b^3F^{2dx+2c}x\ln(F)d+2x\,F^{dx+c}\ln(F)a\,b^2d-\ln\left(a+b\,F^{dx+c}\right)F^{2dx+2c}b^3-2\ln\left(a+b\,F^{dx+c}\right)F^{dx+c}a\,b^2-\ln\left(a+b\,F^{dx+c}\right)a^2b+F^{dx\cdots}}{2\ln(F)^2a^2b^2d^2\left(a+b\,F^{dx+c}\right)^2}$

```
[In] int(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/a^2/b/d/ln(F)+1/2/a^2/b/d^2/ln(F)*c-1/2*(ln(F)*a*d*x-b*F^(d*x+c)-a)/d
^2/ln(F)^2/a/b/(a+b*F^(d*x+c))^2-1/2/a^2/b/d^2/ln(F)^2*ln(F^(d*x+c)+a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.40

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3}\,dx = \frac{F^{2\,dx+2\,c}b^2dx\log\left(F\right)+\left(2\,abdx\log\left(F\right)+ab\right)F^{dx+c}+a^2-\left(2\,F^{dx+c}ab+F^{2\,dx+2\,c}b^2+a^2\right)\log\left(F^{dx+c}b+a\right)}{2\left(2\,F^{dx+c}a^3b^2d^2\log\left(F\right)^2+F^{2\,dx+2\,c}a^2b^3d^2\log\left(F\right)^2+a^4bd^2\log\left(F\right)^2\right)}$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(F^(2*d*x + 2*c)*b^2*d*x*log(F) + (2*a*b*d*x*log(F) + a*b)*F^(d*x + c)
+ a^2 - (2*F^(d*x + c)*a*b + F^(2*d*x + 2*c)*b^2 + a^2)*log(F^(d*x + c)*b +
 a))/(2*F^(d*x + c)*a^3*b^2*d^2*log(F)^2 + F^(2*d*x + 2*c)*a^2*b^3*d^2*log(
F)^2 + a^4*b*d^2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3}\,dx = \frac{F^{c+dx}b - adx\log(F) + a}{4F^{c+dx}a^2b^2d^2\log(F)^2 + 2F^{2c+2dx}ab^3d^2\log(F)^2 + 2a^3bd^2\log(F)^2} + \frac{x}{2a^2bd\log(F)} - \frac{\log\left(F^{c+dx} + \frac{a}{b}\right)}{2a^2bd^2\log(F)^2}$$

```
[In] integrate(F**(d*x+c)*x/(a+b*F**(d*x+c))**3,x)
```

```
[Out] (F**(c + d*x)*b - a*d*x*log(F) + a)/(4*F**(c + d*x)*a**2*b**2*d**2*log(F)**
2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*d**2*log(F)**2) + x
/(2*a**2*b*d*log(F)) - log(F**(c + d*x) + a/b)/(2*a**2*b*d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.42

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3}\,dx = \frac{F^{2\,dx}F^{2\,c}b^2dx\log(F) + (2\,F^c abdx\log(F) + F^c ab)F^{dx} + a^2}{2\left(2\,F^{dx}F^c a^3b^2d^2\log(F)^2 + F^{2\,dx}F^{2\,c}a^2b^3d^2\log(F)^2 + a^4bd^2\log(F)^2\right)} - \frac{\log\left(\frac{F^{dx}F^c b + a}{F^c b}\right)}{2\,a^2bd^2\log(F)^2}$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(F^(2*d*x)*F^(2*c)*b^2*d*x*log(F) + (2*F^c*a*b*d*x*log(F) + F^c*a*b)*F^
(d*x) + a^2)/(2*F^(d*x)*F^c*a^3*b^2*d^2*log(F)^2 + F^(2*d*x)*F^(2*c)*a^2*b^
3*d^2*log(F)^2 + a^4*b*d^2*log(F)^2) - 1/2*log((F^(d*x)*F^c*b + a)/(F^c*b))
/(a^2*b*d^2*log(F)^2)
```

Giac [F]

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3} dx = \int \frac{F^{dx+c}x}{(F^{dx+c}b+a)^3} dx$$

```
[In] integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(F^(d*x + c)*x/(F^(d*x + c)*b + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3} dx = -\frac{\frac{F^c F^{dx}}{2 a d^2 \ln(F)^2} - \frac{F^c F^{dx} x}{a d \ln(F)} + \frac{F^{2c} F^{2dx} b}{2 a^2 d^2 \ln(F)^2} - \frac{F^{2c} F^{2dx} b x}{2 a^2 d \ln(F)}}{a^2 + F^{2c} F^{2dx} b^2 + 2 F^c F^{dx} a b} - \frac{\ln\left(a + F^c F^{dx} b\right)}{2 a^2 b d^2 \ln(F)^2}$$

```
[In] int((F^(c + d*x)*x)/(a + F^(c + d*x)*b)^3,x)
```

```
[Out] - ((F^c*F^(d*x))/(2*a*d^2*log(F)^2) - (F^c*F^(d*x)*x)/(a*d*log(F)) + (F^(2*
c)*F^(2*d*x)*b)/(2*a^2*d^2*log(F)^2) - (F^(2*c)*F^(2*d*x)*b*x)/(2*a^2*d*log
(F)))/(a^2 + F^(2*c)*F^(2*d*x)*b^2 + 2*F^c*F^(d*x)*a*b) - log(a + F^c*F^(d*
x)*b)/(2*a^2*b*d^2*log(F)^2)
```

3.91 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx = -\frac{1}{2bd\,(a+bF^{c+dx})^2\log(F)}$$

```
[Out] -1/2/b/d/(a+b*F^(d*x+c))^2/ln(F)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2278, 32}

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx = -\frac{1}{2bd\log(F)\,(a+bF^{c+dx})^2}$$

```
[In] Int[F^(c + d*x)/(a + b*F^(c + d*x))^3,x]
```

```
[Out] -1/2*1/(b*d*(a + b*F^(c + d*x))^2*Log[F])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
```

```
e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+bx)^3}\,dx, x, F^{c+dx}\right)}{d\log(F)}$$
$$= -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}$$

```
[In] Integrate[F^(c + d*x)/(a + b*F^(c + d*x))^3,x]
[Out] -1/2*1/(b*d*(a + b*F^(c + d*x))^2*Log[F])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
default	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
risch	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
parallelrisch	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
norman	$-\frac{1}{2bd\ln(F)\left(a+b\,e^{(dx+c)\ln(F)}\right)^2}$	28

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/d/(a+b*F^(d*x+c))^2/ln(F)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx = -\frac{1}{2\left(2\,F^{dx+c}ab^2d\log\left(F\right)+F^{2\,dx+2\,c}b^3d\log\left(F\right)+a^2bd\log\left(F\right)\right)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
[Out] -1/2/(2*F^(d*x + c)*a*b^2*d*log(F) + F^(2*d*x + 2*c)*b^3*d*log(F) + a^2*b*d
*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx = -\frac{1}{4F^{c+dx}ab^2d\log\left(F\right)+2F^{2c+2dx}b^3d\log\left(F\right)+2a^2bd\log\left(F\right)}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))**3,x)
[Out] -1/(4*F**(c + d*x)*a*b**2*d*log(F) + 2*F**(2*c + 2*d*x)*b**3*d*log(F) + 2*a
**2*b*d*log(F))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx = -\frac{1}{2\left(F^{dx+c}b+a\right)^2bd\log\left(F\right)}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2/((F^(d*x + c)*b + a)^2*b*d*log(F))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3} \, dx = -\frac{1}{2\,(F^{dx}F^{c}b+a)^2 bd\log{(F)}}$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3,x, algorithm="giac")
[Out] -1/2/((F^(d*x)*F^c*b + a)^2*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3} \, dx = -\frac{1}{2\,b\,d\,\ln{(F)}\,\left(a+F^{c+d\,x}\,b\right)^2}$$

```
[In] int(F^(c + d*x)/(a + F^(c + d*x)*b)^3,x)
[Out] -1/(2*b*d*log(F)*(a + F^(c + d*x)*b)^2)
```

3.92 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x} \, dx = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2 x \log(F)} - \frac{\text{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)^2 x^2}, x\right)}{2bd\log(F)}$$

```
[Out] -1/2/b/d/(a+b*F^(d*x+c))^2/x/ln(F)-1/2*Unintegrable(1/(a+b*F^(d*x+c))^2/x^2
,x)/b/d/ln(F)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x} \, dx = \int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x} \, dx$$

```
[In] Int[F^(c + d*x)/((a + b*F^(c + d*x))^3*x),x]
```

```
[Out] -1/2*1/(b*d*(a + b*F^(c + d*x))^2*x*Log[F]) - Defer[Int][1/((a + b*F^(c + d
*x))^2*x^2), x]/(2*b*d*Log[F])
```

Rubi steps

$$\text{integral} = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2 x \log(F)} - \frac{\int \frac{1}{\left(a+bF^{c+dx}\right)^2 x^2} \, dx}{2bd\log(F)}$$

Mathematica [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx$$

```
[In] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x),x]
[Out] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})^3 x}dx$$

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x)
[Out] int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x} dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^3 x} dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x, algorithm="fricas")
[Out] integral(F^(d*x + c)/(3*F^(d*x + c)*a^2*b*x + 3*F^(2*d*x + 2*c)*a*b^2*x + F
^(3*d*x + 3*c)*b^3*x + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 170, normalized size of antiderivative = 7.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \frac{-F^{c+dx}b-adx\log\left(F\right)-a}{4F^{c+dx}a^2b^2d^2x^2\log\left(F\right)^2+2F^{2c+2dx}ab^3d^2x^2\log\left(F\right)^2+2a^3bd^2x^2\log\left(F\right)^2} - \frac{\int \frac{dx\log\left(F\right)}{ax^3+bx^3e^{c\log\left(F\right)}e^{dx\log\left(F\right)}}\,dx+\int \frac{2}{ax^3+bx^3e^{c\log\left(F\right)}e^{dx\log\left(F\right)}}\,dx}{2abd^2\log\left(F\right)^2}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))**3/x,x)
```

```
[Out] (-F**(c + d*x)*b - a*d*x*log(F) - a)/(4*F**(c + d*x)*a**2*b**2*d**2*x**2*lo
g(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*x**2*log(F)**2 + 2*a**3*b*d**2*x**
2*log(F)**2) - (Integral(d*x*log(F)/(a*x**3 + b*x**3*exp(c*log(F))*exp(d*x*
log(F))), x) + Integral(2/(a*x**3 + b*x**3*exp(c*log(F))*exp(d*x*log(F))),
x))/(2*a*b*d**2*log(F)**2)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.04

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x, algorithm="maxima")
```

```
[Out] -1/2*(a*d*x*log(F) + F^(d*x)*F^c*b + a)/(2*F^(d*x)*F^c*a^2*b^2*d^2*x^2*log(
F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*x^2*log(F)^2 + a^3*b*d^2*x^2*log(F)^2) -
 integrate(1/2*(d*x*log(F) + 2)/(F^(d*x)*F^c*a*b^2*d^2*x^3*log(F)^2 + a^2*b
*d^2*x^3*log(F)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx=\int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x, algorithm="giac")
[Out] integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx=\int \frac{F^{c+d\,x}}{x\,\left(a+F^{c+d\,x}\,b\right)^3}\,dx$$

```
[In] int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^3),x)
[Out] int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^3), x)
```

3.93 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3x^2}\,dx=-\frac{1}{2bd\left(a+bF^{c+dx}\right)^2x^2\log(F)}-\frac{\operatorname{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)^2x^3},x\right)}{bd\log(F)}$$

```
[Out] -1/2/b/d/(a+b*F^(d*x+c))^2/x^2/ln(F)-Unintegrable(1/(a+b*F^(d*x+c))^2/x^3,x
)/b/d/ln(F)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3x^2}\,dx=\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3x^2}\,dx$$

```
[In] Int[F^(c + d*x)/((a + b*F^(c + d*x))^3*x^2),x]
```

```
[Out] -1/2*1/(b*d*(a + b*F^(c + d*x))^2*x^2*Log[F]) - Defer[Int][1/((a + b*F^(c +
 d*x))^2*x^3), x]/(b*d*Log[F])
```

Rubi steps

$$\text{integral}=-\frac{1}{2bd\left(a+bF^{c+dx}\right)^2x^2\log(F)}-\frac{\int \frac{1}{\left(a+bF^{c+dx}\right)^2x^3}\,dx}{bd\log(F)}$$

Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2} \, dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2} \, dx$$

```
[In] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x^2),x]
[Out] Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{\left(a+b\,F^{dx+c}\right)^{3} x^{2}}dx$$

```
[In] int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x)
[Out] int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2} \, dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^3 x^2} \, dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x, algorithm="fricas")
[Out] integral(F^(d*x + c)/(3*F^(d*x + c)*a^2*b*x^2 + 3*F^(2*d*x + 2*c)*a*b^2*x^2
 + F^(3*d*x + 3*c)*b^3*x^2 + a^3*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 7.17

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2} \, dx = \frac{-2F^{c+dx}b - adx\log(F) - 2a}{4F^{c+dx}a^2b^2d^2x^3\log(F)^2 + 2F^{2c+2dx}ab^3d^2x^3\log(F)^2 + 2a^3bd^2x^3\log(F)^2} - \frac{\int \frac{dx\log(F)}{ax^4+bx^4e^{c\log(F)}e^{dx\log(F)}}\, dx + \int \frac{3}{ax^4+bx^4e^{c\log(F)}e^{dx\log(F)}}\, dx}{abd^2\log(F)^2}$$

```
[In] integrate(F**(d*x+c)/(a+b*F**(d*x+c))**3/x**2,x)
```

```
[Out] (-2*F**(c + d*x)*b - a*d*x*log(F) - 2*a)/(4*F**(c + d*x)*a**2*b**2*d**2*x**
3*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*x**3*log(F)**2 + 2*a**3*b*d**2
*x**3*log(F)**2) - (Integral(d*x*log(F)/(a*x**4 + b*x**4*exp(c*log(F))*exp(
d*x*log(F))), x) + Integral(3/(a*x**4 + b*x**4*exp(c*log(F))*exp(d*x*log(F)
)), x))/(a*b*d**2*log(F)**2)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 6.12

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2} \, dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^3 x^2} \, dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*d*x*log(F) + 2*F^(d*x)*F^c*b + 2*a)/(2*F^(d*x)*F^c*a^2*b^2*d^2*x^3*
log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*x^3*log(F)^2 + a^3*b*d^2*x^3*log(F)^
2) - integrate((d*x*log(F) + 3)/(F^(d*x)*F^c*a*b^2*d^2*x^4*log(F)^2 + a^2*b
*d^2*x^4*log(F)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)^3x^2}\,dx$$

```
[In] integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x, algorithm="giac")
[Out] integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^3*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3 x^2}\,dx = \int \frac{F^{c+d\,x}}{x^2\,(a+F^{c+d\,x}\,b)^3}\,dx$$

```
[In] int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^3),x)
[Out] int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^3), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
           'sin','cos','tan','cot','sec','csc',
           'arcsin','arccos','arctan','arccot','arcsec','arccsc',
           'sinh','cosh','tanh','coth','sech','csch',
           'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
       'arctan2','floor','abs'
       ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.a
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```python
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```